The Empirical Implications of the Cox, Ingersoll, Ross Theory of the Term Structure of Interest Rates: Discussion
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Source: *The Journal of Finance*, Vol. 41, No. 3, Papers and Proceedings of the Forty-Fourth Annual Meeting of the America Finance Association, New York, New York, December 28-30, 1985 (Jul., 1986), pp. 630-632
Published by: Blackwell Publishing for the American Finance Association
Stable URL: http://www.jstor.org/stable/2328492
Accessed: 26/11/2009 08:30

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DISCUSSION

WAYNE E. FERSON*: Brown and Dybvig [2] examine the single state variable term structure model of Cox, Ingersoll and Ross (CIR) using price data for U.S. Treasury securities of various maturities. Although the work they present is admittedly preliminary, it seems to have been thoughtfully executed and the approach is appealing for several reasons. First, given the possibility of substitution across maturities it seems reasonable that discount bonds with similar maturities should have similar yields; that is, the term structure should be "smooth." Alternative approaches to imposing smoothness in term structure estimation—such as the use of splines—do so in a relatively ad hoc way. The present approach imposes smoothness across the term structure by using a functional form for bond prices that derives from an economic model and which depends on a "small" number of parameters. A second attractive feature is that such an approach has the potential to produce estimates of parameters that may be of broader interest. As CIR and Brown and Dybvig point out, the term structure in this model embodies the information currently available to the market about the future course of events. Expected market risk premiums for example, are reflected in the term structure. Recent empirical work of Campbell [3], Keim and Stambaugh [5] and others suggests that ex ante yield curve information may have some predictive power for future rates of return on other securities. It seems to me that a confluence of these two strains might prove profitable in future research, potentially providing both guidance on functional form for predictive empirical models as well as interesting tests of pricing theory based on conditional moments.

Brown and Dybvig do not exploit an opportunity to address empirically the classical question of the term structure, namely the relation of the yield curve to expectations about future interest rates. The CIR model as estimated here does not seem to allow separate identification of term premiums; that is, the difference between forward rates and expected future spot rates. It does seem to be possible to obtain, at each date, estimates of the parameters needed to forecast the spot rate, except for the speed of adjustment coefficient. Given an estimate of these parameters (say, from a previous period time series) then ex ante forecasts of future interest rates could be formed using term structure information as of the current date. An important caveat here is that nondeterministic variation in such

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forecasts should by assumption depend only on the level of the spot rate. Evidence of stochastic variation in the ϕ (or σ) parameters indicates a misspecification of the model.

Another appealing feature of the approach in this paper (although readers with different tastes may disagree) is that the model can be cast explicitly in terms of nominal bonds—the bonds we observe in U.S. data. Although the CIR general equilibrium theory suggests that a single state variable model should be cast in "real" terms, it is possible to use pure arbitrage arguments and nominal stochastic process assumptions to obtain pricing results. Given a nominal, single state variable model, it is natural to wonder how this factor is related to inflation. Unfortunately, Brown and Dybvig provide no information on this issue.

To interpret the empirical results, it is useful to recall previous studies that have examined multifactor term structure models (e.g., Brennan and Schwartz [1]) and the related issues of duration and immunization using U.S. Treasury security data (e.g., Gultekin and Rogalski [4], Nelson and Schaefer [6], and others). These studies leave one with the impression that a small number of empirical "factors" is adequate to characterize the behavior of the cross-section of default-free bond price changes. Brown and Dybvig fit four parameters to the cross-section of bond prices at each date. Thus, even though the CIR model is a single state variable model, four series of empirical "factors" are generated.

Given that Brown and Dybvig essentially employ a four factor model, a certain amount of success in characterizing the term structure (e.g., the correspondence of fitted with observed slopes and average term premiums) should not be surprising. They find it remarkable that the implicit volatilities seem to predict time series estimates of short rate volatility fairly well, given that the former are derived from a cross-section of bond prices at a point in time. Persuing the analogy in footnote 6 and recalling the empirical properties of implicit volatilities derived from the option pricing model (by observing only the prices of three securities at a point in time), one would probably be discouraged if the volatilities did not predict fairly well.

There is, of course, further evidence of misspecification and much of the paper is devoted to exploring this. To avoid nitpicking, I will limit my discussion to two final comments here. One observation is that the fitted short rate typically overstates the yields of bills with less than two weeks to maturity. Possibly, the prices of the shortest bills are "too high" due to "liquidity" effects or other imperfections beyond the scope of the model. One might even argue that the shortest bills should be excluded from the sample for this reason. Is the fit of the model in the other maturity ranges adversely affected by forcing it to fit the shortest bills? A second issue is the observed systematic pricing errors. Brown and Dybvig suggest these are consistent with a "neglected tax hypothesis." I am skeptical of this explanation. I would rather think that prices of premium and discount bonds are such that a marginal investor, if one exists, does not prefer one over the other. If dealers are marginal investors in this market, then any price effects of differential taxation should be within dealer transaction cost bounds—probably narrower than bid-ask spreads. I am reminded of Schaefer's [7] earlier work in which a linear programming approach indicated that most (British) Treasury bonds seemed efficient for a zero tax bracket investor. This

could not be the case if differential taxation of discount and premium issues (plausibly more severe in the U.K. than in the U.S.) produces important price effects.

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