Valuation of Risky Assets in Arbitrage Free Economies: Discussion
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$= \{u \in R^t, \mu \in R \mid q(u) \leq \mu\}$ is a convex set. Its lower envelope is the graph of q. The optimal d is a subgradient of q at zero. Thus, the hyperplane $(d, 1)(u, \mu)^T = 0$ separates the set epi(q) from the set $\{(u, \mu) \mid u \geq 0, \mu = 0\} = C$. That is, $(d, 1)(u, \mu)^T \leq 0$ for every $(u, \mu) \in$ epi(h) and $(d, 1)(u, \mu)^T \geq 0$ for every $(u, \mu) \in C$.

Under the strong form of NAF, the set $D = \{(u, \mu) \mid u \geq 0, u \neq 0, \mu = 0\}$ and epi(q) are disjoint, since $q(u) > 0$ for $u \geq 0$ where $u \neq 0$. However, this is not sufficient to prove that the second inequality is strict when C is replaced by D. If such a separation is possible, then obviously $d > 0$. Note that epi(q) is the projection of the closed set $\{(x, u, \mu) \mid x^T P \leq \mu \text{ and } x^T A_j - f_j(x) \geq u_j, \text{ for every } j\}$ on the (u, μ) coordinates, but epi(q) is not necessarily closed. The weakest possible conditions for such a separation are given in Klee [4]. It is easy to show that if the g_j are piecewise linear and convex, then strict separation is possible.

If strict separation is impossible, then every subgradient of q at zero would have at least one component which is zero. There exists no valuation operator, d, which admits no arbitrage in the strong form with $d > 0$. In this economy, an investor would have no arbitrage opportunities if and only if $1 after tax is worthless in some state j. In terms of the expectation operator, the investor would have no arbitrage opportunities if and only if this state j would occur with probability zero. That suggests that the NAF strong form would not ensure the existence of a valuation operator if and only if the state of nature were misspecified.

REFERENCES

1. K. Fan, I. Glicksberg, and A. J. Hoffman. "Systems of Inequalities Involving Convex Functions." *Proceedings of American Mathematical Society* 8 (June 1957), 617–622.
2. M. B. Garman and J. A. Ohlson. "Valuation of Risky Assets in Arbitrage Free Economies with Transaction Costs." *Journal of Financial Economics* 9 (June 1981), 271–280.
3. J. M. Harrison and D. M. Kreps. "Martingales and Arbitrage in Multi-Period Security Market." *Journal of Economic Theory* 20 (June 1979), 381–408.
4. V. L. Klee. "Maximal Separation Theorem for Convex Sets." *Transactions of the American Mathematical Society* 134 (October 1968), 133–148.
5. E. Z. Prisman. "Immunization as a Maximum Strategy, A New Look." Forthcoming, *Journal of Banking and Finance* (1986).
6. ———. "A Unified Approach to Term Structure Estimation, A Methodology to Estimate Term Structure in a Market with Frictions." Unpublished manuscript, Georgia Institute of Technology, 1984.
7. S. Ross. "Risk, Return, and Arbitrage." In: I. Friend and J. Bicksler, eds., *Risk and Return in Finance.* Cambridge: Ballinger Press, 1976.
8. ———. "A Simple Approach to Valuation of Risky Streams." *Journal of Business* (July 1978), 453–457.
9. ———. "Arbitrage and Martingales with Taxation." Unpublished manuscript, Yale School of Organization and Management, 1984.
10. R. T. Rockafellar. *Convex Analysis.* Princeton: Princeton University Press, 1970.

DISCUSSION

EHUD I. RONN*: Eli Prisman's paper applies linear and non-linear programming (LP/NLP) and duality theory to the valuation of risky assets. The analogy of LP

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to the valuation of risky securities is clearly drawn: a risky asset is a complex security making (possibly negative, typically *non*-negative) payoffs in each state of nature. These states of nature materialize subsequent to the current period and are, by convention, mutually exclusive and jointly exhaustive. An important byproduct of the analysis is the dual vector, whose components comprise the set of Arrow-Debreu prices.

The Prisman paper contains a careful and thorough LP/NLP analysis of this problem in the absence or in the presence of frictions. As I find the paper to be technically sound, I will direct my comments to the methodology and economic intuition involved in the analysis.

The application of LP methodology, when appropriately utilized and justified, to problems in finance is one I personally find quite appealing. One example of such an appealing application of LP methods arises in the context of the term structure of interest rates. The original formulation of the problem in the bond context is one of minimizing the cost of a given stream of after-tax cash flows (subject to a short sale prohibition)—see Hodges and Schaefer [2]. In the presence of heterogeneous taxation, the values of coupon bonds may diverge for different investors; these differences cannot be eliminated by arbitrage in the presence of a short-sale restriction. This particular application of the LP methodology has found wide application, in addition to the Hodges and Schaefer [2] piece, in Schaefer [9, 10]; in Prisman's own work [7], Jordan and Prisman [3], and Dermody and Prisman [1]; and in Ronn [8]. One interesting economic interpretation deducible from these analyses involves a tax-clientele effect: due to the preferential treatment of capital gains, high (low) ordinary income investors prefer low (high) coupon bonds. This empirical result has been documented by Schaefer [10] for the U.K. Government market ("gilts") and by Ronn [8] for the U.S. Government market. Prisman's own work in this field is extremely interesting and enlightening.

On the other hand, the application of LP methodology to the risky asset case is not, to my mind, as economically appealing as in the bond case. The Prisman analysis has the power of greater generality induced by the arbitrage-based valuation which relaxes previous utility-based valuation techniques. However, it lacks some of the elegant economic intuition provided by the latter analysis. For example, it should be noted that one of the advantages of utility-based state-preference theory is the multi-period insights it lends in deriving Merton-type [5] hedging demands. While the author has deferred multi-period considerations to a later paper, it is natural to question the extent to which LP/NLP will successfully address this issue.

I am somewhat concerned that the paper fails to take proper cognizance of previous work in this area. The paper is seemingly unaware of the foundations of state-preference theory dating back to Mossin [6], who related the valuation duals, d_j, to the marginal rate of substitution between current wealth and state-j wealth. The paper states that "economies with frictions have received little attention." In contrast, however, Litzenberger and Sosin [4] carefully derive equilibrium conditions for capital markets under complete and incomplete markets, with and without short sale restrictions, and in the presence and absence of taxation.

The Litzenberger and Sosin [4] piece derives capital market equilibrium conditions in the context of individual utility-maximization subject to a budget constraint and capital market conditions. A consequence of their analysis is that, under short sale restrictions,

$$P_i \geq m_{ik} \equiv \sum_s m_{sk} X_{is}$$

where

P_i = price of security i
m_{ik} = generalized marginal rate of substitution (MRS) of individual k's current consumption expenditures for security i
m_{sk} = individual k's MRS between current consumption and unit consumption in state s

and

X_{is} = payoffs of the i^{th} security in state s.

The Litzenberger and Sosin discussion gives rise to a clientele effect: $P_i > m_{ik}$ for some k, implying individual k will not hold security i. By contrast, Prisman's analysis implies that "the valuation operator of any individual can be used to value assets." Prisman's NLP-derived frictions model does not incorporate short sale restrictions which give rise to a clientele effect. Consequently, I find the Litzenberger and Sosin discussion more intuitively appealing; I readily admit, however, that this may be a matter of taste.

To come full circle, then, I find LP methods highly desirable in bond analysis, less so in state-preference models. In term structure models, LP models have good intuitive foundation (minimize the cost of given stream of after-tax cash flows for particular tax bracket) and are empirically testable. In the risky asset case, my own preference is for the state-preference analyses pioneered by Mossin and extended by such works as Litzenberger and Sosin.

In conclusion, I have thoroughly enjoyed Prisman's LP/NLP-based analyses of riskless and risky asset. My own preference lies in his work and extensions of term structure models.

REFERENCES

1. J. C. Dermody and E. Z. Prisman. "Term Structure Multiplicity and Clientele in Markets with Transaction Costs and Taxes." Unpublished manuscript, Arizona State University College of Business, 1986.
2. S. D. Hodges and S. M. Schaefer. "A Model for Bond Portfolio Improvement." *Journal of Financial and Quantitative Analysis* 12 (1977), 243–260.
3. J. V. Jordan and E. Z. Prisman. "Tests for Tax-Clientele and Tax-Option Effects in U.S. Treasury Bonds: A New Interpretation." Unpublished manuscript, Arizona State University College of Business, 1985.
4. R. H. Litzenberger and H. B. Sosin. "The Theory of Recapitalizations and the Evidence of Dual Purpose Funds." *Journal of Finance* 32 (December 1977), 1433–1455.
5. R. C. Merton. "Optimum Consumption and Portfolio Rules in a Continuous-Time Model." *Journal of Economic Theory* (December 1971), 373–413.
6. J. Mossin. "Equilibrium in a Capital Asset Market." *Econometrica* (October 1966), 768–783.
7. E. Z. Prisman. "Theory and Empirics of the Term Structure: A Duality Linkage." Unpublished manuscript, Arizona State University College of Business, 1985.

8. E. I. Ronn. "A New Linear Programming Approach to Bond Portfolio Management." Unpublished manuscript, University of California, Berkeley, November 1985.
9. S. M. Schaefer. "Measuring Tax-Specific Term Structure of Interest Rates in the Market for British Government Securities." *Economic Journal* 91 (1981), 415–438.
10. ———. "Tax Induced Clientele Effects in the Market for British Government Securities." *Journal of Financial Economics* 10 (1982), 121–159.