Moral Hazard and Adverse Selection: The Question of Financial Structure
Author(s): Masako N. Darrough and Neal M. Stoughton
Source: *The Journal of Finance*, Vol. 41, No. 2 (Jun., 1986), pp. 501-513
Published by: Blackwell Publishing for the American Finance Association
Stable URL: http://www.jstor.org/stable/2328450
Accessed: 26/11/2009 08:28

THE JOURNAL OF FINANCE • VOL. XLI, NO. 2 • JUNE 1986

Moral Hazard and Adverse Selection: The Question of Financial Structure

MASAKO N. DARROUGH and NEAL M. STOUGHTON*

ABSTRACT

This paper looks at the moral hazard and adverse selection problems confronting an entrepreneur offering securities to an uninformed, but competitive financial market. The adverse selection aspect of the problem is generated by the unobservable entrepreneur's ability to transform effort into value. Moral hazard arises because the investment decision is made subsequent to financing. We consider the joint use of both debt and equity, and characterize the equilibrium relation between capital structure and unobservable attributes. It is shown that: (1) investment and financing are not separable; (2) there is an underinvestment problem for "better" firms; and (3) simultaneous use of both debt and equity can resolve this difficulty. We also establish a connection between expected terminal firm value and debt-promised payment level and between share retention and standard deviation.

AGENCY PROBLEMS IN CORPORATE finance originated with the influential papers of Jensen and Meckling [11], Myers [17], Ross [21], and Leland-Pyle [14]. They show that the market imperfections induced by unobservable actions, lack of contracting ability, and information asymmetry generally lead to second-best outcomes in which the distribution of corporate ownership is achieved only at significant cost. These costs take the form of excessive perquisite consumption, overinvestment, underinvestment, and incomplete diversification of personal investment portfolios.

Moral hazard and *adverse selection* comprise two forms in which agency problems may take shape. Arrow [1] equates these two terms with *hidden action* and *hidden information*, respectively. Moral hazard arises when the action undertaken by the agent is unobservable and has a differential value to the agent as compared to the principal. Adverse selection problems arise when the agent has more information than the principal. The resolution of such difficulties has been explored in a number of contexts with both signals and contingent contracting mechanisms. These definitions exclude certain agency problems in the *delegation* literature for which the preference incongruity results from a lack of precommitment rather than some exogenous feature of the model. In corporate finance, one would thus identify the Jensen-Meckling paper with moral hazard because perquisites enter the insider's objective differently from outsiders, while

* Darrough is from Columbia University and Stoughton is from the University of California, Irvine. The second author's research was partially supported by Grant Number 410-83-0786 R-1 from the Social Sciences and Humanities Research Council of Canada. This paper was presented at the 1984 meetings of the Western Finance Association in Vancouver and at the 1985 European Finance Association meetings in Bern. We thank Kose John and Frans Tempelaar for their comments at the meetings.

the underinvestment problem by Myers would be considered in the delegation category since the preference incongruity is induced only by the assumed contractual terms.

While moral hazard and adverse selection have been extensively analyzed as separate problems, an integration of these two questions now seems in order. This paper takes a step towards that objective as related to the capital structure of the firm. In the model described here, a principal-agent formulation of the type pioneered by Mirrlees [15], Spence and Zeckhauser [23], and Ross [20] is employed along with the additional ingredient that the agent is assumed to possess a set of unobservable attributes. Taking the principal in our model to represent the collective financial market and the agent to be an entrepreneur desiring to sell claims on his or her enterprise, the agent's unobservable attributes may be thought of as indicators of quality, productivity, or efficiency at translating effort into output. Thus, it is beneficial to entrepreneurs in this model to communicate their attributes through the indirect means available to them—selecting their capital structure and retained ownership level. This is the adverse selection aspect. Moral hazard arises because the entrepreneur must combine unobservable effort with the proceeds of outside investment in order to produce value that can be shared among the claimants. Our viewpoint differs from those studies that focus on unobservable perquisite consumption, lack of monitoring, or bonding ability with respect to the proceeds from outside investment. Rather, we have chosen to focus on the problem of motivation and not the problem of avoiding temptation.

We consider the moral hazard cum adverse selection problem in a context involving the simultaneous issuance of both debt and equity. The nonlinear "call-option" payout profile of equity has a number of desirable features. For one thing, the convex pattern motivates the entrepreneur to invest his or her own human capital along with the outside financing. In addition, the debt level in conjunction with the equity share can resolve adverse selection by sorting firms according to their characteristics. In considering the debt/equity case, we show existence of a capital structure equilibrium, even though such an equilibrium may not obtain under a simple capital structure. We also derive the form of the equilibrium to show that there is a specific relationship between debt-promised payment level and the mean of the return distribution, while equity retention and standard deviation are inversely related.

Aside from the fact that ours is a moral hazard rather than a delegation problem, our paper differs from several existing works with respect to the important issue of risk-sharing. This can be illustrated by the early and somewhat preliminary research undertaken by Haugen and Senbet [9]. They showed that with complete markets and value maximization, the simultaneous problems of perquisite consumption and the adverse selection problem of eliciting return distribution could be *costlessly* solved under a wide variety of circumstances. Most of these dealt with a "moving support" property of the payoff distributions but the results were extended to cover somewhat more general cases as well. Agency problems are often not problems at all with maximization of value because this is tantamount to risk-neutrality on the part of the agent. And it is well known (Shavell [22]) that, with an unrestricted contract space, agency problems can be averted by simply letting the agent absorb all risk. Although this proposition does not extend trivially to the corporate finance environment because limited liability does impose some restrictions on the effective contract space, Haugen and Senbet show that put and call options can still eliminate the agency problem. Therefore, as they note, "the[ir] solutions to the problems of agency and informational asymmetry play no role in the determination of the financial structure of the firm, and hence, in any framework which evokes these assumptions, they cannot be used to rationalize the positive theory of corporate finance."

Another case is illustrated by John and Kalay [12] where they show that precommitment not to pay dividends can costlessly solve the agency problem of debt contracts as well as asymmetry of information regarding the investment opportunity set. Again, the key is the "risk-neutrality" of insiders. The only paper to illustrate the properties of a true second-best situation is that of John and Williams [13] where they show that agency costs can actually be *necessary* to the establishment of a signalling equilibrium. Since their model also abstracts from the risk-sharing incentive of external financing, it would seem that financial structures more complex than those considered in their paper might resolve the agency problem and therefore lead to nonexistence of the signalling equilibrium.[1]

The main purpose of our paper is to provide a characterization of equilibria under moral hazard-adverse selection in which financing externally is rationalized by the incentive to *shed risk.* Financial contracts are optimal in our model (subject to the payout characteristics of debt and equity). Hence, the residual agency costs are real and are not due to some exogenously imposed constraint on external financing requirements. Our results show that the diversity of capital structures commonly observed can be reconciled as a natural response to the presence of heterogeneous investment opportunities and the delegation of financially relevant actions to the manager (or entrepreneur). In this model, each firm can be expected to select the financial policy tailored for it by the outside market. In so doing, a rational tradeoff among incurring excess risk, providing effort incentives and communicating private information, is made.

Our paper starts with the problem formulation in Section I. The entrepreneurial effort inducement is solved in Section II. Properties of the financial market valuation are derived in Section III. The equilibrium is derived in Section IV, and the properties explored in Section V. Conclusions are given in Section VI.

I. Problem Formulation

We consider the problem of a competitive financial market pricing a package of claims on a firm controlled by a single owner-manager. Prior to the offering date,

[1] There are two additional papers that deal with the moral hazard-adverse selection problem in other contexts. Holmstrom and Weiss [10] develop an explicit solution for a model with a two-state outcome space. Baron [3] develops a model of the investment banking contract between a firm and a dealer by assuming that the latter is informed while the firm is not. His analysis is limited by the restriction to contracts *linear in the information* communicated in a direct revelation game. Financial market contracts seldom have this type of linearity.

time zero, the entrepreneur alone is "informed" about the parameters μ and σ (continuously distributed over support $[\mu_L, \mu_H]$ and $[\sigma_L, \sigma_H]$, respectively) which represent the mean and standard deviation of the entrepreneur's marginal product of effort at time one, ρ. Proceeds from the issuance of securities fall under the jurisdiction of the entrepreneur, who chooses the level of investment to adopt and the level of personal effort commitment.[2] Investment returns are assumed to follow a simple additive production function in which effort increases expected values without any effect on the variance. Outside investors in the competitive financial market price the security package based on the observable capital structure at time zero.

In game theoretic terminology, there are three "moves" in this game of incomplete information:

1. Entrepreneur chooses capital structure (s, δ) according to individual ability (μ, σ). This defines a strategy $s(\mu, \sigma)$, $\delta(\mu, \sigma)$;
2. Financial market sets value, v, as a function of security structure (s, δ) following a strategic rule given by $v(s, \delta)$;
3. Entrepreneur commits effort level ρ to production. This defines a mapping $\rho(v, \mu, \sigma, s, \delta)$;

where s denotes the retained equity ownership, and δ represents the payment promised to debtholders next period. Because debt has absolute priority, equity takes the usual form of a call option on the firm's terminal value with a striking price equal to debt-promised payment. The payoffs are given at time one by the (uncertain as of time zero) shares of terminal value held by the entrepreneur and outside investors, respectively. Financial market investors are risk-neutral while entrepreneurs maximize a utility function exhibiting constant absolute risk aversion.[3] We define the following notion of an informationally consistent equilibrium (Riley [19]):

An *Informationally Consistent Nash Equilibrium*[4] $(s, \delta)^*$, v^*, ρ^* is a triple such that ρ^* is utility maximizing given $(s, \delta)^*$ and v^*; $(s, \delta)^*$ is utility maximizing given v^* and ρ^*; while v^* is exactly equal to the expected financial market share of terminal value (a competitive consistency condition).

We now analyze the equilibrium by considering, in reverse order (3, 2, 1), the optimal moves of each player as explained above.

[2] All the capital for the firm is being raised through external financing. That is, the entrepreneur is bringing effort and not capital of his/her own to the venture.

[3] Restriction to risk-neutrality does not involve any substantive content reductions. In most competitive utility-based pricing models, aggregation implies *conditional* risk neutrality anyway (e.g., CAPM $\Rightarrow$ unsystematic risk not priced).

[4] Notice that we are *not* allowing the selection of security structure s to depend on v in a strategic sense. Thus, problems of nonexistence can be avoided (see Weiss [24]) at the cost of introducing multiple equilibria. We restrict ourselves to the complete sorting equilibrium where (s, δ) is an invertible function.

II. The Entrepreneur's Problem

The entrepreneur of type (μ, σ) maximizes expected utility of end-of-period wealth:

$$\max_{\rho,s,\delta} E_{\mu,\sigma}\left[U\left(\tilde{w} - \frac{1}{2a}\rho^2\right)\right] \tag{1}$$

subject to[5]

$$\tilde{w} = \max(0, s(\tilde{y} - \delta)) \tag{2}$$

$$\tilde{y} = vr_f + \rho\mu + \sigma\tilde{\varepsilon} \tag{3}$$

$$\rho \in \operatorname{argmax}_{\hat{\rho}} E_{\mu,\sigma}\left[U\left(\tilde{w} - \frac{1}{2a}\hat{\rho}^2\right)\right], \tag{4}$$

where

U = entrepreneur's utility function,
ρ = effort expended,
v = total value of securities issued by entrepreneur,
r_f = one plus the risk-free rate of return,
$\tilde{w}$ = end-of-period wealth,
s = fractional retained equity ownership,
δ = debt-promised payment,
$\tilde{y}$ = terminal value of the firm after one period,
μ = marginal productivity of effort,
σ = standard deviation of y,
$\tilde{\varepsilon}$ = unit variance, zero mean random variable,
a = risk-aversion parameter.

Equation (1) is the usual expected utility maximization criterion. It embodies first the von Neumann-Morgenstern assumptions and second the additional ingredient that effort disincentive is measured as a direct proportion of a quadratic income loss. The functional form of the effort disincentive is without loss in generality since any other functional form can be accomodated through a rescaling. However, the fact that disutility of effort is separable does impose substantive restrictions. It is known that without such an assumption, randomized strategies may be employed by the agent (Gjesdal [7]). Grossman and Hart [8] discuss the relevance of this type of separability and its relation to exponential utility. The budget constraint [Equation (2)] represents the entrepreneur's end-of-period wealth as his or her share of the terminal value of equity. The investment technology [Equation (3)] is additively separable in capital investment and effort, while exhibiting constant stochastic returns to scale with respect to the effort variable. In the absence of effort, capital investment earns the riskless rate plus a random shock; effort contributes to terminal value with marginal (and average) product μ. The production function, therefore, exhibits

[5] We impose additionally the constraints that $s \in [0, 1]$, $\delta \in [0, \infty)$, $\rho \in [0, \infty)$.

first-order stochastic dominance with respect to effort, as is commonly assumed in principal-agent models. Moral hazard is incorporated into Equation (4), which shows that the overall optimization problem is constrained by a suboptimization problem in which the entrepreneur chooses effort "after" capital structure. In accordance with Leland-Pyle [14], we assume that utility is negative exponential:

$$U(\cdot) = -\frac{1}{a}e^{-a(\cdot)},$$

and that production "shocks" ($\tilde{\varepsilon}$) are normal. It is common knowledge that all entrepreneurs have the same utility function and risk-aversion coefficient, a. Under these conditions, expected utility can be written as:

$$E_{\mu,\sigma}U = -\frac{1}{a}e^{1/2\rho^2}\left\{\int_{\tilde{y}\geqq\delta} e^{-as(\tilde{y}-\delta)}f(\tilde{y})\,d\tilde{y} + \int_{\tilde{y}<\delta} f(\tilde{y})\,d\tilde{y}\right\}, \tag{5}$$

where f represents the density function. Following techniques similar to those in Mood et al. (1974, 164), these two integrals can be evaluated as

$$E_{\mu,\sigma}U = -\frac{1}{a}e^{1/2\rho^2}\{e^{-asvr_f+\mu\rho-\delta+1/2a^2s^2\sigma^2}N(\Delta - as\sigma) + N(-\Delta)\} \tag{6}$$

where $N(\cdot)$ denotes the cumulative distribution function of the standard (zero mean, unit variance) normal density and

$$\Delta \equiv \frac{vr_f + \mu\rho - \delta}{\sigma} \tag{7}$$

is defined as the *default margin.* We employ this terminology because Δ is a measure of the normalized margin in expected terminal value above the promised payment level. Notice that $\text{Prob}(\tilde{y} < \delta) = N(-\Delta)$, the probability of default. $N(\Delta - as\sigma)$ is a "risk-adjusted" probability of remaining solvent; the risk term accounts for the variation in payoff conditional on $\tilde{y} \geqq \delta$. An alternative method of expressing expected utility is obtained using the following cumulative likelihood ratio:

$$L(z) \equiv \frac{N'(z)}{N(z)}. \tag{8}$$

$L(-z) = N'(z)/(1 - N(z))$ is "Mill's ratio," a frequently encountered parameter in reliability theory (see Barlowe and Proschen [2]). The cumulative likelihood ratio, L, is decreasing, convex, asymptotic to $-z$ as $z \rightarrow -\infty$ and asymptotic to zero as $z \rightarrow \infty$. It can be shown that another way of writing Equation (6) is

$$E_{\mu,\sigma}U = -\frac{1}{a}e^{1/2\rho^2}\left\{\frac{L(-\Delta)}{L(\Delta - as\sigma)} + 1\right\}N(-\Delta). \tag{9}$$

This gives expected utility as the utility under default plus the ratio of likelihood ratios times the probability of default.

To get the optimal effort expenditure conditional on capital structure (s, δ), and financing proceeds, v, differentiate (9) partially with respect to ρ. This determines the (implicit) relation

$$\rho = \frac{as\mu}{1 + \dfrac{L(\Delta - as\sigma)}{L(-\Delta)}} \tag{10}$$

The level of effort expenditure in (10) may be contrasted with a "first-best" case where ρ is observable, no equity is retained, and the entrepreneur becomes a hired-manager receiving fixed compensation equal to his or her expected product. In this case, expected utility is

$$E_{\mu,\sigma} U = -\frac{1}{a} e^{-a\rho\mu + 1/2\rho^2},$$

and the necessary condition on effort expenditure is $\rho = a\mu$. Because L is a strictly positive function and $s \leqq 1$, $\rho < as\mu \leqq a\mu$. That is, less effort is expended by the entrepreneur who finances with debt and equity under asymmetric information. Nevertheless, as the marginal product of effort increases, so does effort itself, *ceteris paribus*.

III. Financial Market Valuation

We assume that the financial market is composed of a large number of risk neutral individuals (see footnote 3 above) who act in a value-maximizing *symmetric* way and satisfy the following competitive consistency condition: each security issue is priced as it would have been in a world where full information is costlessly obtainable. This implicitly means that we have restricted the entrepreneurs from realizing any rents from the fact that they have superior information.

Viewing the market as holding both debt and equity, we see here that for terminal values below δ, the market will receive the entire value of the firm, whereas their claim is δ plus a fraction $(1 - s)$ of anything above δ otherwise. The market is, therefore, valuing a claim that pays

$$\begin{cases} \tilde{y} & \text{if} \quad \tilde{y} < \delta \\ \delta + (1 - s)(\tilde{y} - \delta) & \text{if} \quad \tilde{y} \geqq \delta. \end{cases}$$

This claim is equivalent to holding the entire firm and writing s call options on the assets with striking price equal to the promised payment. Letting the call option value be denoted by $C(\mu, \sigma, \rho, \delta)$, we obtain the following expression for the value of proceeds invested by outsiders:

$$v = \frac{vr_f + \mu\rho}{r_f} - sC$$

which implies that

$$\frac{\mu\rho}{r_f} = sC. \tag{11}$$

Brennan [4] has derived a call option formula under assumptions that the underlying asset is normally distributed and individuals possess negative exponential utility functions. His formula (adapted to risk-neutrality) is

$$C(\mu, \sigma, \rho, \delta) = \frac{(vr_f + \mu\rho - \delta)N(\Delta) + \sigma N'(\Delta)}{r_f}$$

$$= \frac{\sigma(\Delta N(\Delta) + N'(\Delta))}{r_f}. \tag{12}$$

While this formula satisfies many of the same properties of the well-known Black-Scholes lognormal formula, we emphasize that increases in the variance increase the value of the call, as do increases in Δ, the default margin. Combining Equations (11) and (12) gives the implicit relation between the value of proceeds and the parameters μ and σ:[6]

$$\mu\rho = s\sigma(\Delta N(\Delta) + N'(\Delta)). \tag{13}$$

We will return to this expression once the utility maximization problem is solved.

IV. Capital Structure Equilibrium

The necessary condition for an interior optimal capital structure choice is that, for an entrepreneur with marginal product described by μ and σ, expected utility must be stationary at the chosen v, s, and δ. An equilibrium obtains when the set of choices coincide with Equation (13) as μ and σ vary. Totally differentiating expected utility yields

$$dE_{\mu,\sigma}U = \frac{\partial EU}{\partial \rho}\,d\rho + \frac{\partial EU}{\partial v}\,dv + \frac{\partial EU}{\partial s}\,ds + \frac{\partial EU}{\partial \delta}\,d\delta$$

$$= \frac{\partial EU}{\partial v}\,dv + \frac{\partial EU}{\partial s}\,ds + \frac{\partial EU}{\partial \delta}\,d\delta,$$

because of the "envelope" condition that effort is optimized subsequent to the capital structure decision. Performing the differentiation of Equation (9) yields[7]

$$0 = \frac{1}{a}\left\{\frac{L(-\Delta)}{L(\Delta - as\sigma)} + 1\right\}N'(-\Delta)\,d\Delta$$

$$- \frac{1}{a}\left\{\frac{L(-\Delta)(-\Delta + L(-\Delta))\,d\Delta}{L(\Delta - as\sigma)}\right\}N(-\Delta)$$

$$- \frac{1}{a}\,\frac{L(-\Delta)(\Delta - as\sigma + L(\Delta - as\sigma))\,(d\Delta - a\sigma\,ds)}{L(\Delta - as\sigma)}\,N(-\Delta).$$

[6] It is worth noting that this equation can be expressed in terms of the cumulative likelihood ratio as

$$\mu\rho = s\sigma(\Delta + L(\Delta))N(\Delta).$$

[7] We have employed the following relation:

$$L'(z) = -L(z)(z + L(z)).$$

Making the substitution of optimal effort [Equation (10)] and a number of algebraic operations yields

$$s\sigma\rho\, d\Delta + \rho\sigma(\Delta - as\sigma + L(\Delta - as\sigma))\, ds = 0$$

giving the necessary condition for optimality that must hold in the parameter space describing the firm's capital structure:

$$s(r_f\, dv - d\delta) + \sigma(\Delta - as\sigma + L(\Delta - as\sigma))\, ds = 0. \tag{14}$$

In order to obtain an equilibrium solution such that (14) is satisfied for all μ and σ given by (13), we employ the conceptual device known in the literature as the "revelation principle" (attributed to Myerson [18], among others). In the context of this model, we reinterpret the choice of financial structure by the entrepreneur as the consequence of a direct report of entrepreneurial type. Let $(\hat{\mu}, \hat{\sigma})$ denote this report. An honest mediator then translates this report into a point on a two-dimensional manifold in the capital structure parameters v, s, and δ such that Equation (14) is satisfied. An equilibrium occurs when this translation gives the entrepreneur the incentive to report the "true" values of μ and σ.

For the special case where it is common knowledge that $\mu^2 = \sigma$, the debt/equity equilibrium can be solved for explicitly. The following is the main result of the paper.

PROPOSITION. *Suppose it is common knowledge that $\mu^2 = \sigma$ for all types μ and σ. A Nash equilibrium in the debt + equity game exists and is characterized by the following three conditions*:

(*i*) *Equity retention is inversely proportional to standard deviation*

$$s\sigma = G. \tag{15}$$

(*ii*) *The default margin is also identical for every firm. This implies*

$$(vr_f + \rho\mu - \delta)s = \Delta G = \text{constant}. \tag{16}$$

(*iii*) *The constants G and Δ are selected so that*

$$L(\Delta - aG) = aG. \tag{17}$$

Proof: The proof applies the three conditions above as "conjectures" by the financial market in order to obtain the translation between the strategies $\hat{\mu}$ and $\hat{\sigma}$ of the direct revelation game and the parameters v, s, and δ of the original game. Hence, if the share retention s is chosen, it is equivalent to a report of $\hat{\sigma}$ where

$$s\hat{\sigma} = G. \tag{18}$$

In order to translate v and δ into a report of $\hat{\mu}$, a number of steps are needed. First, notice that the equilibrium conditions (15) and (16) imply that $as\sigma$ and Δ are identical for all firm types. Hence, $L(-\Delta)$ and $L(\Delta - as\sigma)$ are constants as well. The effort choice must then be equal to

$$\rho = Kas\mu$$

where K is a constant between zero and one. Moreover, in equilibrium, it is conjectured that $\rho = GKa\mu/\sigma$ because of condition (i). And finally, this implies that $\rho\mu = GKa\mu^2/\sigma$ is conjectured to be constant because of the market's valuation relation, Equation (13). Therefore, upon receiving a report of $\hat{\mu}$ and $\hat{\sigma}$, the mediator will choose v and δ so that

$$(vr_f + \rho\hat{\mu} - \delta)s = \Delta G$$

or

$$vr_f - \delta = \frac{\Delta G}{s} - \rho\hat{\mu} = \Delta\hat{\sigma} - \frac{GKa\hat{\mu}^2}{\hat{\sigma}}. \tag{19}$$

Totally differentiating Equations (18) and (19) yields

$$ds = -\frac{G}{\hat{\sigma}^2}\,d\hat{\sigma} \tag{20}$$

and

$$r_f\,dv - d\sigma = \Delta\,d\hat{\sigma} - d\left(\frac{Gka\hat{\mu}^2}{\hat{\sigma}}\right). \tag{21}$$

Substitute both (20) and (21) into the expected utility maximization condition [Equation (14)]; we obtain:

$$s\Delta\,d\hat{\sigma} - sd\left(\frac{GKa\hat{\mu}^2}{\hat{\sigma}}\right) - \sigma(\Delta - as\sigma + L(\Delta - as\sigma))\,\frac{G}{\hat{\sigma}^2}\,d\hat{\sigma} = 0.$$

Rearranging yields

$$-sd\left(\frac{GKa\hat{\mu}^2}{\hat{\sigma}}\right) + \left(s\Delta - \frac{s\sigma}{\hat{\sigma}}\,\Delta\right)d\hat{\sigma} - (-as\sigma + L(\Delta - as\sigma))\,\frac{G}{\hat{\sigma}^2}\,d\hat{\sigma} = 0.$$

The third term on the LHS is zero because of condition (iii) in the proposition. The second term on the LHS is zero if and only if $\hat{\sigma} = \sigma$. And the first term is zero because of the common knowledge assumption that $\mu^2 = \sigma$ for all firms. Therefore, the entrepreneur will truthfully report the investment opportunities parameters and the equilibrium will be sustained. Q.E.D.

The equilibrium exists because the available set of financial market instruments is sufficiently rich. Any agent deviating from what the market expects may benefit from either the debt offering or the equity offering. Nevertheless, the entrepreneur loses more on the other security and hence will not deviate. Suppose the entrepreneur selects a low equity share. This is tantamount to announcing that the riskiness of the firm is larger than it actually is. In order to control for risk, the market then reduces the acceptable promised payment level in the debt that is issued. The entrepreneur thus faces a more nearly linear payoff profile, but one with a shallower slope. Effort expenditure is below optimal, to such an extent that the entrepreneur is worse off than if he or she had chosen the higher equity share. In this model, equilibrium is sustained both by the desire to communicate attributes as well as a lack of precommitment regarding the expenditure of effort.

V. Properties of the Equilibrium

Before discussing the equilibrium relationships in this model, it is helpful to review as a frame of reference what would happen if only μ were uncertain and δ were fixed at zero for all firms. This all-equity case is the generalization of Leland-Pyle [14] to moral hazard and was analyzed in a full version of this paper (Darrough and Stoughton [5]).[8] It was shown in that paper that retained ownership shares would be distributed at levels greater than one-half (as opposed to zero in the Leland-Pyle model). As a consequence, equilibrium proceeds from the issuance of securities were *lower* for firms with better prospects than for those with less favorable opportunities. Hence, investment and financing conflict with one another in the all-equity version of this model.

Returning to the debt/equity model considered here, the most important relation between capital structure and the characteristics of the entrepreneur is given by condition (ii) of the proposition. This states that $s(vr_f + \rho\mu - \delta)$ is constant, i.e., the entrepreneur's share of the differential between expected output and promised payment is identical for all firms. Those firms with low shares have correspondingly greater expected returns. Because $\rho = Kas\mu = GKa/\mu$, effort is actually decreasing with respect to expected return. However, the product $\rho\mu$ is constant and independent of entrepreneurial type. Since $vr_f + \rho\mu - \delta$ is increasing in μ, it must be the case that $vr_f - \delta$, the difference between external investment and promised payment, increases with μ as well. Therefore, firms with higher expected return raise relatively more capital than those with low returns. This can be contrasted with the negative relation between proceeds and productivity in the all-equity model. Notice that there is no specific prediction in this model about whether the absolute level of promised payments are large or small. This follows from the constant returns to scale assumption. A dollar increase in the future value of proceeds is equivalent to a dollar decrease in promised payment. It is only the *difference* between proceeds and promised payment that matters.

The constancy of Δ, the default margin, has the further implication that all firms are equally likely to go bankrupt. Firms with higher expected returns raise greater proceeds in the capital market as a fraction of promised payment. But the market recognizes that such firms have greater risks as well, because of the association between μ and σ; hence, the *average* interest rate is identical in this simple economy. On the other hand, the *marginal* interest rates differ, depending upon the prior choices made by the individual owner-managers.

Although this agency model began with heterogeneity of agents, we have seen that the action of the financial market has tended to equilibrate the diversity of agents. As evidence of this, we cite the constancy of the default margin and the expected contribution of the entrepreneur, $\rho\mu$. The nature of the equilibrium also imposes certain testable restrictions concerning the relation of financial structure and ex ante unobservables. Since s and σ are inversely proportional, the equity

[8] For a discussion of linear and piecewise linear contracts with both moral hazard and adverse selection in a context of managerial compensation, see Darrough and Stoughton [6].

retention reflects information about risk rather than return. The choice of promised payment, or more accurately, the debt coverage, provides an indicator of expected return. Therefore, one would associate debt with the mean and equity with the variance.

The Nash equilibrium we have defined is not unique. The family of such equilibria may be parameterized by the constant G which indirectly determines Δ through Equation (17). It can be seen that, as G is increased, Δ tends toward zero (debt gets more risky). But because there is strict underinvestment in effort as compared to a first-best situation, effort increases. Since the extra effort does not impose any additional risk, the entrepreneur's expected utility increases. Therefore, the *unique efficient* Nash equilibrium within the class covered by the proposition is given by choosing G as large as possible subject to the constraint that $s \leqq 1$. This implies that $G = \sigma_L$ where $\sigma_L = \mu_L^2$, corresponding to the entrepreneur with minimal expected return. All the equity is retained by this entrepreneur; financing is achieved solely through the issuance of debt. There is one element in common between the debt + equity and the all-equity models. Full diversification by zero equity retention cannot be an equilibrium; the moral hazard problem places a restriction on the number of shares that can be issued.

VI. Summary and Conclusions

This paper has examined the moral hazard-cum-adverse selection problem in a corporate finance context via an analysis of the debt/equity financial market structure. The complex capital structure obtained from issuing debt and equity does a better job of resolving these combined problems than does an all-equity structure. We examined this market structure under the assumption that means and standard deviations are positively correlated. While the technical details make the equilibrium derivation difficult in general, we were able to establish a set of market-based conjectures that lead to an informationally consistent complete sorting equilibrium. These can be summarized as follows. Entrepreneurs with low variances of terminal value choose high ownership shares. Entrepreneurs with greater expected marginal productivity work less, but generate identical expected returns and increase their debt-promised payment levels accordingly. Outside investment is indeterminate under the constant returns to scale assumption; nevertheless, the difference between aggregate external investment and debt-promised payment is increasing in productivity, *ceteris paribus*. The end result is that capital structure leaves every firm with the same probability of defaulting on its bonds. Therefore, this model would predict that defaults on corporate debt are not failures of the informational efficiency of financial markets; rather, they are due to market-wide events that are outside the firm's control.

Some final remarks concerning the debt-equity equilibrium are in order. While the analytical form was derived only for the special case where $\mu^2 = \sigma$, the arguments can be extended in a straightforward manner to cover the case where μ is known but σ is uncertain. If both μ and σ are uncertain, then existence of an equilibrium might require an even more general form of financial structure, one involving side payments between the entrepreneur and financial market. Such payments, which may also be interpreted as dividends, help compensate for the differences in risk imposition among agents.

REFERENCES

1. K. J. Arrow. "The Economics of Agency." Stanford University IMSSS Technical Report No. 451, October 1984.
2. R. E. Barlowe and F. Proschen. *Mathematical Theory of Reliability*. New York: John Wiley and Sons, 1965.
3. D. Baron. "A Model of the Demand for Investment Banking and Advising and Distribution Services for New Issues." *Journal of Finance* 37 (September 1982), 955–76.
4. M. Brennan. "The Pricing of Contingent Claims in Discrete-Time Models." *Journal of Finance* 24 (March 1979), 53–68.
5. M. Darrough and N. Stoughton. "Moral Hazard and Adverse Selection: The Question of Financial Structure." Working Paper, Columbia University, July 1985.
6. ———. "Managerial Compensation: Linear Sharing vs. Bonus Incentive Plans under Moral Hazard and Adverse Selection," In G. A. Feltham, A. H. Amershi, and W. T. Ziemba (eds.), *Economic Analysis of Information and Contracts: Essays in Honour of John E. Butterworth*. Forthcoming, 1986.
7. F. Gjesdal. "Accounting for Stewardship." *Journal of Accounting Research* (Spring 1981), 208–31.
8. S. Grossman and O. Hart. "An Analysis of the Principal-Agent Problem." *Econometrica* 51 (January 1983), 7–45.
9. R. A. Haugen and L. W. Senbet. "New Perspectives on Informational Asymmetry and Agency Relationships." *Journal of Financial and Quantitative Analysis* 14 (November 1979), 671–94.
10. B. Holmstrom and L. Weiss. "Managerial Incentives, Investment and Aggregate Implications: Scale Effects." *Review of Economic Studies* 52 (July 1985), 403–25.
11. M. Jensen and W. Meckling. "The Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure." *Journal of Financial Economics* 3 (October 1976), 305–60.
12. K. John and A. Kalay. "Informational Content of Optimal Debt Contracts," In E. Altman and M. Subrahmanyam (eds.), *Recent Advances in Corporate Finance*. Homewood, IL: Richard D. Irwin, Inc., 1985, 133–61.
13. K. John and J. Williams. "Financial Signalling with Agency Costs." N.Y.U. Working Paper, November 1982.
14. H. Leland and D. Pyle. "Informational Asymmetries, Financial Structure and Financial Intermediation." *Journal of Finance* 32 (May 1977), 371–87.
15. J. A. Mirrlees. "An Exploration in the Theory of Optimum Income Taxation." *Review of Economic Studies* 38 (April 1971), 175–208.
16. A. Mood, F. Graybill, and D. Boes. *Introduction to the Theory of Statistics*, New York: McGraw-Hill, 1974.
17. S. Myers. "Determinants of Corporate Borrowing." *Journal of Financial Economics* 5 (November 1977), 147–76.
18. R. B. Myerson. "Incentive Compatibility and the Bargaining Problem." *Econometrica* 47 (January 1979), 61–74.
19. J. Riley. "Informational Equilibrium." *Econometrica* 47 (March 1979), 331–60.
20. S. A. Ross. "The Economic Theory of Agency: The Principal's Problem." *American Economic Review* 63 (May 1973).
21. ———. 'The Determination of Financial Structure: The Incentive Signalling Approach." *The Bell Journal of Economics* 7 (Spring 1977), 23–40.
22. S. Shavell, "Risk Sharing and Incentives in the Principal and Agent Relationship." *The Bell Journal of Economics* 10 (Spring 1979), 55–73.
23. M. Spence and R. Zeckhauser. "Insurance, Information and Individual Action." *American Economic Review* 61 (May 1971), 380–89.
24. A. Weiss. "A Sorting-cum Learning Model of Education." *Journal of Political Economy* 91 (June 1983), 420–42.