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ABSTRACT

A novel way of estimating the *expected* as opposed to the *actual* share price fall-off is developed using option prices. This method is applied to the UK Traded Options Market using data from 1979 to 1984. The results show that: (a) the average expected fall-off implicit in option prices is around 55 to 60% of the dividend and significantly different from it. The fall-off also varied inversely with the dividend yield, which is consistent with the prediction of the "tax clientele hypothesis." (b) The estimates of the expected fall-off were not significantly different from the actual fall-off. (c) Finally, the results imply that the usual assumption made in valuing options on dividend-paying stocks, that the fall-off is equal to the dividend, would lead to downward-biased estimates of the option value.

IN A WORLD WITH no taxes or transaction costs, the fall-off in the price of a share on its ex-dividend day should be equal to the amount of the dividend. In reality, however, both dividends and capital gains are taxed with the former being, in general, taxed at a higher rate than the latter. The implications of the differential taxation of dividends and capital gains for the ex-dividend day fall-off have been the subject of numerous papers since as early as 1955. There are two main competing hypotheses about the determination of the fall-off. The so-called "tax-clientele" hypothesis (Elton and Gruber [8]) assumes that shareholder clienteles do not change around the ex-dividend day whereas the "short-term trading hypothesis" (Kalay [12], Lakonishok and Vermaelen [16]) claims that the "normal" clientele of a share changes around the ex-dividend day, so that the fall-off is determined by the activities of short-term traders who are taxed equally on dividends and capital gains.

In the absence of transaction costs, the two hypotheses have different empirical predictions; whereas the tax clientele hypothesis predicts a proportionate fall-off of less than one, the short-term trading hypothesis is consistent only with a proportionate fall-off of one. Furthermore, the tax clientele hypothesis predicts a positive monotonic relationship between the dividend yield and the fall-off. In the presence of transaction costs, *small* deviations of the fall-off from unity can be consistent with the short-term trading hypothesis. Furthermore, under certain circumstances (see Lakonishok and Vermaelen [16, p. 8]), a positive monotonic relationship between the yield and the fall-off can be consistent with the short-term trading hypothesis as well. Sometimes, it is, therefore, difficult to reject one hypothesis against the other because of the similarity of the empirical predictions.

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There is, however, one circumstance which is inconsistent with the short-term trading hypothesis but which can be consistent with the tax clientele hypothesis: that is the case of low fall-offs (e.g., around 50% of the dividend).[1] In some cases (Canada, UK), fall-offs of this magnitude have been observed.

The presence of options in a world characterized by the differential taxation of dividends and capital gains raises the possibility of constructing a portfolio of options and bonds which is perfectly correlated with the underlying share but whose after-tax cashflows are higher than those of the share itself. If this were possible, tax exempt investors or low tax rate investors would sell options short to the high tax investors before the ex-dividend day, repurchasing them after the ex-dividend day. We will call this the "hedge portfolio hypothesis." A necessary condition for this strategy to be profitable is that tax arbitrage in the asset market is not possible in the first place (e.g., by short-term trading). This hypothesis implies an ex-dividend day fall-off close to one, since the underlying share will be held by low tax rate investors.

One drawback of all of the empirical studies aiming to test the presence of tax effects is that the tax clientele hypothesis is formulated in terms of the *expected* fall-off, while it was only possible to use the *actual* fall-off as a proxy for the expected one. The implicit assumption is that of rational expectations on the part of investors. In this paper, we present an alternative method of testing the tax clientele hypothesis which does not have to assume rational expectations and is not subject to the sampling error that arises when ex-post returns are used as proxies for ex-ante ones. The assumption used instead is the validity of the Black-Scholes no-arbitrage model for valuing options. Our method relies on the direct estimation of the expected fall-off implicit in the prices of options which it is not optimal to exercise before the ex-dividend day. Thus, if the expected fall-off was found to be significantly different from the dividend, this would imply that our results would be inconsistent with both the "hedge portfolio" and the short-term trading hypotheses. On the other hand, if in addition we found a positive relationship between the dividend yield and the fall-off, our results would be consistent with the tax clientele hypothesis.

I. Implied Fall-offs from Option Prices

Since call options have no claim on dividends, absence of arbitrage opportunities requires that the risk-adjusted return of an option over the ex-dividend interval should not differ from any other day (see Kalay and Subrahmanyam [13]). Nevertheless, if an investor wants to value an option before it goes ex-dividend in order to decide whether to exercise prematurely or not, he has to provide an expectation about how much the share price will fall on the ex-dividend day. The fact that the dividend is known is irrelevant since the evidence suggests that the fall-off is not necessarily equal to the dividend.

Assuming that it was not optimal to exercise the option on the last cum-

[1] As Kalay [12] puts it: "Although the existence of the arbitrageur casts doubt on our ability to infer taxes from the observed fall-off, it cannot explain systematic departures from unity."

dividend day, there are two ways of obtaining estimates of the expected fall-off:[2]

(a) by using some estimate of the standard deviation of the share returns, we can solve for the implied share price that would produce the (observable) last cum-dividend option price;
(b) by a method (described in more detail in the next section) which estimates the fall-off by observing the change in the option price and the share price from the last cum-dividend day to the ex-dividend day.

The disadvantage of the first method is that it is sensitive to the estimate of the standard deviation used and that there is no way to decide which estimate of the standard deviation is more likely to produce a consistent and efficient estimate of the expected fall-off.

The second method also relies on an estimate of volatility to compute the hedge ratio, but in this case, we implicitly have more information to decide on how correct our estimate of the hedge ratio (and hence the volatility) is by comparing the movement of the share price to that of the option price; in other words, this method provides us with a testable implication (this will become clearer in the next section). The disadvantage is that it requires option and share prices to be synchronous and has to assume that the estimated hedge ratio is valid for a discrete interval. In this paper, we use both methods to estimate the expected fall-off but concentrate more heavily on the second one which is developed in the next section.

II. Estimating the Expected Fall-Off Using Cum- and Ex-Dividend Day Prices

Assuming that the option is not exercised on the last cum-dividend day, any unexpected changes in the option price on the ex-dividend instant can be attributed to the fact that the actual ex-dividend share price was different from expected. This relationship can be given by

$$C_x - C_c(1 + E(r_c)) = H(S_x - E(S_x)) \tag{1}$$

where

C_x = ex-dividend option price
C_c = cum-dividend option price
S_x = ex-dividend share price
$H = \dfrac{\partial C}{\partial S}$ (hedge ratio)
r_c = return on option
E = expectation taken at close of last cum-day.

[2] There is also another way, suggested by Manaster and Rendleman [22], which relies on the simultaneous estimation of implied stock prices and implied standard deviations using data from several options written on the same stock. This was not used first because our regression results were not sensitive to the standard deviation used, and second because of computational reasons.

The expected share price, now, can be expressed as follows:[3]

$$E(S_x) = S_c(1 + E(r_s)) - E(a)D \tag{2}$$

where

a = fall-off as a percent of the dividend
r_s = return on share
D = dividend.

Combining (1) and (2), we have

$$C_x - C_c - C_cE(r_c) = HS_x - HS_c - HS_cE(r_s) + HE(a)D. \tag{3}$$

But any expected change in the share price due to market movements (i.e., abstracting from the fall-off) should be equal to the expected change in the option price divided by the hedge ratio, i.e.,

$$C_cE(r_c) = HS_cE(r_s)$$

so that (3) becomes (after dividing both sides by HD)

$$\frac{C_x - C_c}{HD} = E(a) + \frac{S_x - S_c}{D}. \tag{4}$$

Letting now

$$C_i = \frac{C_{xi} - C_{ci}}{H_iD_i}$$

$$F_i = \frac{S_{xi} - S_{ci}}{D_i} \text{ (actual fall-off)}$$

we can estimate the following regression:

$$C_i = b_0 + b_1F_i + u_i \tag{5}$$

assuming that $E(u_i) = E(u_iu_j) = 0$ and $\mathrm{Var}(u_i) = \sigma_u^2$. Note that since all the variables in (5) are directly observable, if our model is well specified, the residual error can only arise because of errors in the measurement of the variables in the equation.

If, then, the model is well specified, we would expect

$$E(b_0) = E(a) \text{ (expected fall-off)}$$

$$E(b_1) = 1.$$

Note that we are assuming here that the expected fall-off ($E(a)$) is the same for all shares. The effect of allowing $E(a)$ to vary is discussed later on in the paper.

[3] We could express (2) as follows:

$$(S_c - E(a)D)(1 + E(r_s)). \tag{2a}$$

The difference between (2) and (2a) is that, in the first case, we assume that the fall-off occurs just before the open of the ex-day while in the latter it occurs just after the close of the last cum-dividend day. Clearly choosing between (2) and (2a) is of no significance.

III. Data

In order to perform our test, we collected 360 pairs of cum- and ex-dividend closing offer prices of options written on 14 different British shares[4] from 1979 to 1984 as well as the simultaneous underlying equity (offer) prices. The data were collected from the London Options Clearing House (LOCH). All underlying shares are quoted on the London Stock Exchange. In general, data for three different options on the same stock were collected; where possible, these were chosen to be in, at, and out of the money. In addition, we avoided collecting options with less than three weeks to maturity since the Black-Scholes model has been observed to perform better with options not so close to maturity, both as a result of the reduced impact of measurement errors and less sensitivity to violations of the underlying assumptions (Manaster and Rendleman [22]).

In the UK, shares go ex-dividend on the first Monday of the two-week Account, and it so happens (for administrative reasons) that options cannot be exercised on the last day of the Account. This implies that the cum-dividend option prices on the last day of the Account were determined by using the expected ex-dividend share price as input.

Furthermore, care was taken to include only options with no remaining dividend payments until their maturity. This would enable us to use the Black-Scholes European Call Option valuation formula to determine the hedge ratio. This requirement reduced the average time to maturity of the options in our sample.

Note that, although option and share prices were recorded simultaneously, they may not represent truly synchronous prices as a result of nontrading. This creates an errors-in-variables problem the effects of which are discussed in Section V.

Finally, we excluded all options which did not satisfy the following inequality (PV being the present value operator):

$$S_c - PV(EX) < C_c. \tag{6}$$

If the above inequality did not hold for a particular option, it would imply that it was optimal to exercise the option prematurely. There were 22 cases (spread over 17 different shares) which did not satisfy this condition.[5]

The main characteristics of the options collected for which it was not optimal to exercise prematurely were:

No. of Options	338
Average of (Share price on ex day)/ (PV of exercise price)	0.93
Average Maturity (years)	0.32
Average Hedge Ratio	0.56

[4] The companies included were: BP, Consolidated Goldfields, Courtaulds, Commercial Union, Grand Met, ICI, Land Securities, Marks & Spencer, Shell Transport, Imperial Group, Lonhro, P & O, Racal, and RTZ.

[5] This exclusion criterion may be slightly biasing our sample in that, if there are errors in the cum-dividend option price, we will be including the overestimates and excluding the underestimates.

IV. Results of Estimation Using Cum-Dividend Prices (Method 1)

In order to obtain estimates of the implied share price on the cum-dividend day using the Black-Scholes formula, we used the standard deviation on the ex-dividend day implied by assuming that the actual call option price is equal to the theoretical Black-Scholes value and solving for the standard deviation.

The expected fall-off was then found by subtracting the implied share price from the actual share price on the cum-dividend day and dividing by the amount of the (known) dividend.

It should be pointed out that, in assuming the validity of the Black-Scholes European Call Option valuation formula, we are implicitly assuming that the taxation of gains and losses arising from trading in options does not affect option values or, for that matter, the hedge ratio. As Scholes [27] shows, if the taxation of capital gains arising from share price changes is the same as that of options and interest income, then there would be no tax effect on option prices or the hedge ratios. In the UK, dealings in traded options are subject to income tax rules when performed by traders or dealers, and Capital Gains Tax rules when they are not. For no person, however, is there an *asymmetry* between the tax treatment of gains and losses arising from option dealing and those arising from share dealing. Nevertheless, since interest is charged as ordinary income, the after-tax risk-free rate earned on the riskless portfolio and hence option values will differ across investors in different tax brackets.[6] It is beyond the scope of this paper, however, to discuss the equilibrium implications on option pricing of investor tax heterogeneity.

Furthermore, the Black-Scholes model does not have to be adjusted for dividends in the case of this method. The reason is that the share price used to price an option *which is not optimally exercised on the cum-dividend day* is the actual share price on that day minus the expected fall-off due to the dividend. In other words, as far as the option holder is concerned, the share is treated effectively as a nondividend paying share (but with a lower price) once the decision not to exercise prematurely has been made. Note also that there are no remaining dividend payments in the option's lifetime. The results of our estimation are shown in Table I.

It can be seen that the mean expected fall-off is significantly different from the dividend which casts doubt on the validity of the tax arbitrage hypothesis. In order to see in more detail the sensitivity of the fall-off to the standard deviation used and also to find out whether the expected fall-off is related to the dividend yield, we ranked the observations by dividend yield and divided them into three groups from lowest to highest yield.

As can be seen from the standard deviation of the mean, the noise in the estimate of the expected fall-off diminishes as we consider higher yield stocks. One reason is that, in order to compute the expected proportionate fall-off, we have to divide by the dividend so that any errors in the expected ex-dividend share price become relatively greater for low dividend than high dividend stocks. The results tentatively suggest that the fall-off tends to be higher for high yield

[6] This points to another advantage of Method 2 over Method 1, in that if taxes significantly affect hedge ratios, this should lead to a rejection of the hypothesis $E(b_1) = 1$.

Table I

Method 1: Implied Expected Fall-Offs Using Cum-Dividend Prices

Group	Average (group) Dividend Yield	Expected Fall-Off	Difference of Means	
			Groups	T-Value
1	1.6%	49% (7.2%)*	1–2	1.78
2	3.1%	64% (4.3%)	2–3	1.53
3	5.3%	72% (2.9%)	1–3	2.95**
All Options	3.3%	62% (3.0%)		

* Figures in parentheses are standard errors (of the mean).
** Significant at the 95% level.

shares although the errors on the means of the first two groups prevent us from making strong statements about the relationship between the fall-off and the dividend yield.

V. Results of Estimation of Fall-off Using Cum and Ex-Dividend Prices

We now consider the estimation of the implied expected fall-off using Method 2. In order to estimate (5), we need to have an estimate of the hedge ratio to compute the dependent variable, C_i. For this purpose, we used the implied standard deviation of the option as in Section IV.[7]

This estimate of volatility was also used to compute the implied *expected* share price on the *ex-dividend day*. The hedge ratio associated with this price was used in our tests. Note that had we used the actual *cum-dividend* share price to compute an estimate of the hedge ratio, we would have biased our estimate *downward* since what is required by the model is the hedge ratio associated with the *expected* ex-dividend share price which is in general lower than the cum-dividend price. Similarly, use of the *actual ex-dividend* share price, although perhaps not biasing our estimate, would unnecessarily introduce noise in our hedge ratio estimate, especially since the ex-dividend prices are closing and not opening prices.[8]

A problem with estimating the model is the fact that it holds for an arbitrarily small interval around the ex-dividend instant whereas we only have data from the close of the last cum-dividend day to the close of the ex-dividend day. Moreover, it is probable that the share and option prices are not synchronous as we mentioned in Section I. As shown in Kaplanis [14], this errors-in-variables problem leads to a downward bias for both b_0 and b_1. Another problem with the model is the residual correlation which arises from the fact that more than one

[7] Three other definitions of volatility were also tried: two weighted implied standard deviation definitions and the historic standard deviation of the underlying stock over the previous five years. Our results were not sensitive to the volatility estimate we used.

[8] We also estimated our model using the *actual* ex-dividend share prices. Our results were not very different apart from the heteroskedasticity which was more serious judging from the Glejser test.

option is used on the same stock. This does not bias our coefficients but it does make them less efficient; however, only the knowledge of the full variance-covariance matrix of the errors could help us estimate the efficient (GLS) coefficients—clearly an impossible task. Finally, the three-month Treasury bill rate was used as an estimate of the risk-free rate in the calculation of the present value of the exercise price.

The result of the estimation of (5) was as follows:

$$C_i = \underset{(0.060)}{0.564} + \underset{(0.083)}{1.002}F_i + \hat{u}_i \quad R^2 = 0.65$$

where the figures in parentheses are White's standard errors. The hypothesis, $b_1 = 1$, cannot be rejected at the 95% significance level, whatever the standard deviation used (see footnote 7) in the calculation of the hedge ratio. The intercept (expected fall-off) is around 56% of the dividend and, again, not very sensitive to the standard deviation used to compute the hedge ratio. Just as in the case of Method 1, the average fall-off is significantly different from the dividend, contrary to the predictions of the tax arbitrage and the hedge portfolio hypotheses.

It was encouraging to see that, despite the possible presence of errors-in-variables bias due to the lack of synchrony between options and share prices, the coefficient of b_1 was close to one, suggesting that the bias is not serious.

Another potential problem in our estimation was *heteroskedasticity*. There were three possible sources of heteroskedasticity:

(a) *Measurement error in the option prices.* Our sample included a number of low-priced options which contained a lot of noise, since option prices cannot move by less than ¼p. The presence of such measurement error also reduces the R^2 of the regression. Given that low option prices are usually associated with a low hedge ratio (which is in the denominator of the dependent variable), these errors are magnified.

(b) *Misspecification.* Heteroskedasticity would arise if, for example, the expected fall-off was not constant across shares but was correlated with the dividend yield.

(c) *The variance of the actual fall-off (independent variable) is inversely proportional to the dividend.* This occurs if the variance of price changes on the ex-dividend day is more or less constant across securities, so that dividing by the dividend increases the variance of the fall-offs of low yield stocks relative to high yield ones.

We first attempted to eliminate some of the possible error caused by (a). We, therefore, excluded all options in our sample which had a price of less than 5p. Our results showed no significant change in either the estimated coefficients or the R^2. A Glejser heteroskedasticity test showed a significant (negative) relationship between the absolute value of the residuals and the square root of the dividend yield, the regression coefficient being −4.69 with a t-ratio of −8.36. Various weighting schemes were attempted to eliminate the heteroskedasticity but, probably as a result of the correlation between the actual fall-off (independent variable) and the dividend yield, proved unsuccessful. We, therefore, used a robust Maximum Likelihood procedure developed by S. Hodges [10] which effectively gives less weight to outlying observations. The Hodges estimator assumes that the distribution of residuals is non-normal with a kurtosis greater than 3 (the exact value depending on the distribution chosen). The Hodges estimates will be reported along with the OLS estimates.

VI. Varying Intercept

One of the implications of the tax clientele hypothesis in the presence of short selling restrictions is that one should observe higher (lower) fall-offs for high (low) yield shares. If this hypothesis were true, we would expect the intercept (expected fall-off) in our regression to be higher for high yield shares and lower for low yield shares. By construction, the dividend yield in our model is the ratio of the dividend paid to the cum-dividend price of the share.

Suppose we rank our sample by dividend yield and divide it into three groups, from lowest to highest yield. If the tax clientele hypothesis is true, then we would observe:

$$C_i = b_{01} + b_1 F_i + u_i \text{ for the first group}$$

$$C_i = b_{02} + b_1 F_i + u_i \text{ for the second group}$$

$$C_i = b_{03} + b_1 F_i + u_i \text{ for the third group}$$

with $b_{03} > b_{02} > b_{01}$. Assuming that the error variances are identical, we can combine now these three equations into a single equation as follows:

$$C_i = b_{01} D_1 + b_{02} D_2 + b_{03} D_3 + b_1 F_i + u_i \tag{7}$$

where D_1, D_2, and D_3 are dummy variables with D_i taking the value of 1 for the i^{th} third of the observations and zero elsewhere. The null hypothesis, therefore, is that the coefficients of D_1, D_2, and D_3 are positive, and the coefficient of D_3 is greater than that of D_2 and the latter greater than D_1. The results of this regression are shown in Table II.

All dummy coefficients were significantly different from zero and in the direction predicted by the tax clientele hypothesis. Assuming that this hypothesis is true and that the capital gains tax is zero, the implied tax rate on dividends is

Table II

Varying Intercept (Unrestricted)

	b_{01}	b_{02}	b_{03}	b_2
OLS	0.383	0.561	0.740	1.037
	(0.068)	(0.071)	(0.074)	(0.041)
Hodges*	0.439	0.541	0.717	1.005
$R^2 = 0.69$				
Implied Tax Rate	56	46	28	
Yield	Low	Medium	High	
	(1.6%)	(3.1%)	(5.3%)	

* The Hodges Maximum Likelihood method corrects for a form of heteroskedasticity.

given by $1 - E(a)$. These values are also shown in Table II; as expected, the "implied" tax rate rises as the dividend yield falls implying that high yield shares are held by investors with low marginal tax rates.

These findings suggest that the fall-off varies monotonically with the dividend yield as predicted by the tax clientele hypothesis. As a result, we model the expected fall-off as a continuous function of the dividend yield. We assume the linear form:

$$E(a_i) = b_0 + b_1 d_i. \tag{8}$$

Since, in the absence of a dividend payment the share price is not expected to fall, we would expect, if the above linear form is correct, $E(b_0) = 0$. Therefore, we suppressed the constant in the regression. The estimated equation was as follows:

$$C_i = \underset{(1.4)}{15.82} d_i + \underset{(0.048)}{1.047} F_i + \hat{u}_i. \quad R^2 = 0.70$$

The coefficient of the dividend yield is positive and significant, implying that high yield stocks have high fall-offs and hence low "implied tax rates" as predicted by the tax clientele hypothesis.

This method's predicted implied tax rates for the low (1.6%), medium (3.1%), and high (5.6%) yield groups were 76%, 53%, and 20%, respectively. The implied tax rates for the medium and high yield groups are not very different from those of the dummy variable technique, although the tax rate of the low yield group is 20% higher and is in fact greater than the statutory maximum tax rate. (This is similar to the findings of some other ex-day studies of Poterba and Summers [25] and Lakonishok and Vermaelen [16].) In our case, it suggests that the expected fall-off may be a *nonlinear* function of the dividend yield.

The tests reported so far assumed that clienteles are formed on the basis of the dividend yield during the ex-dividend period as opposed to the annual dividend yield of a share. Since in the UK companies usually pay a low interim and a high final dividend in a year, this implies that the clientele of a share may change every six months. To see whether this is a reasonable proposition, we performed the following test. First, we ranked the observations we had on each individual share by the dividend yield of each observation. We then divided each share's set of observations into two groups, the "high yield" and the "low yield" groups. If the clienteles of shares never changed, we would expect the average fall-off of the high yield groups to be the same as that of the low yield groups. To test this, we calculated, using Method 1 (Section I), the average expected fall-off of the high yield groups as well as that of the low yield groups. The average proportionate fall-off of the high yield groups turned out to be significantly higher (67.3% with a standard error or 4.5%) than that of the low yield groups (43.5% with a standard error of 5.8%), suggesting that, if the tax clientele hypothesis is the explanation of these low fall-offs, then the clientele of a share is not constant but changes according to the expected size of the next dividend payment. To test this further, we went on to see whether the change in the fall-off from one dividend payment to the next could be explained by the change in the dividend yield over that period. We, therefore, regressed the change in the fall-off against the change in the dividend yield. In this test, we had on average 7 observations per share making a total of 98 observations. The coefficient of the change in the dividend yield was −10.98, implying that for a percentage rise (fall) in the dividend yield the price drop is higher (lower) by around 11% of the dividend, a result which is consistent with our estimates of the fall-off for subgroups of stocks when they are ranked by yield. Unfortunately, however, the coefficient was not significant at the 95% level (its t-ratio was −1.5) which is partly caused by the small number of observations in the test.

VII. Actual Fall-off and Comparison with Expected Fall-off

The intercept of the estimated equation in the previous section was shown to be the *expected* ex-dividend day fall-off of the share. It is interesting to compare our estimates with the actual market adjusted fall-off. This comparison can be interpreted as a joint test of the rational expectations hypothesis and the Black-Scholes model. This hypothesis would be rejected if the average actual fall-off was shown to be significantly different from the expected one. The return over the ex-dividend day is given as follows:

$$\bar{R}_i - r_f(1 - t) = \beta_i(R_m - r_f(1 - t)) \tag{9}$$

where

$$\bar{R}_i = \frac{P_{ex} + aD - P_{cum}}{P_{cum}}$$

and a (in equilibrium) should be equal to the ex-dividend day fall-off as a percentage of the dividend. In the context of the after-tax CAPM (see Brennan [2]), a is one minus the weighted average of investors' marginal tax rates. Letting R_i denote the capital gains return on the share during the ex-dividend pay period, i.e.,

$$R_i = \frac{P_{ex} - P_{cum}}{P_{cum}}$$

we can express (9) as follows:

$$R_i = (1 - \beta_i)(1 - t)r_f + \beta_i R_m - ad_i \tag{9a}$$

where d_i = dividend/(P_{cum}) (dividend yield).

As a test of the specification of the model, we estimated the following equation:

$$R_i = b_0 + b_1 x_1 + b_2 x_2 + u_i \tag{10}$$

where $x_1 = \beta_i R_m$ and $x_2 = d_i$. The beta of each share was estimated by the trade-to-trade method using the previous five years of monthly data. The betas were also Bayesian-adjusted, using a method described by Vasicek [30]. (The results were very similar when the unadjusted betas were used.) One of the hypotheses of the model, which is testable against a known constant, is $b_1 = 1$. The result of the estimation was as follows:

$$R_i = 0.009 + 0.871\ \beta_i R_m - 0.812 d_i + u_i. \quad R^2 = 0.62$$
$$(0.003) \quad (0.152) \qquad (0.077)$$

A Glejser test (regression of absolute residuals against the square root of the dividend yield) showed no evidence of heteroskedasticity (the coefficient was −0.01 with a t-ratio of −0.54).

When we restricted b_1 to unity and suppressed the constant, the estimated fall-off was 63%, which is not significantly different from our estimates of the expected fall-off using either the direct method (62%) or the regression method (56%).

In order to compare the expected and the actual fall-off, one has to take into account the monotonic relationship between yield and fall-off and examine whether the expected fall-off for a certain range of yields is close to the actual fall-off observed. In Table III, we present a comparison of the estimates of the expected fall-off from the option model with those of the actual fall-off for low, medium, and high yield shares. The expected fall-off is not statistically different from the mean actual fall-off whichever methods are used.

In Section VI, we estimated a linear functional relationship between the expected fall-off and the dividend yield. For reasons of comparison, we estimated the parameters of the same relationship with the *actual* fall-off as the dependent

Table III

Comparison of Expected with Mean Actual Fall-off (%)

Yield*	Dummy	Continuously Varying	Direct
	Option Model-Expected Fall-off		
Low (1.6%)	44	24	39
Medium (3.1%)	54	47	64
High (5.3%)	72	80	72
Yield	Unrestricted	Equation (10)	Equation (10) No-Constant
	Actual Fall-off		
Low (1.7%)	24	33	23
Medium (3.4%)	43	51	60
High (5.9%)	79	77	67

* The slight difference in the yields of the two models arises from the fact that we excluded observations associated with option prices less than 5p and also those not satisfying the early exercise condition (6) in the option model. Although the *same* ex-days were used in both cases, in the case of the actual fall-off, each ex-day received the same weight (since only one observation was associated with it) whereas in the case of the expected fall-off, each ex-day received a weight of ⅓, ⅔, or 1, depending on how many option prices were available.

variable; we regressed, that is, the actual fall-off (F_i) against the dividend yield (d_i).

Since F_i will normally be negative and d_i positive, a negative regression coefficient implies that high fall-offs are associated with high yield stocks. For comparability with the coefficients in Section VII, we report the estimated parameter multiplied by -1.

The estimated dividend yield coefficient when the constant is suppressed (12.104) is lower but not significantly different from the one in the case of the expected fall-off (15.820).[9]

Our results, therefore, indicate that the expected fall-off implicit in option prices is very close to the average observed fall-off and also that the ex-ante relationship between fall-off and dividend yield is very close to the ex-post relationship. Thus, our results do not appear to reject the joint hypothesis of rational expectations and the validity of the Black-Scholes model.

VIII. Conclusions

In this paper, we estimate the *expected* as opposed to the *actual* fall-off of share prices on their ex-dividend day using option prices. This provides an alternative test of an implication of the tax clientele hypothesis, namely, that the fall-off should be less than the dividend and that there should be a positive monotonic relationship between the expected fall-off and the dividend yield. Our method avoids the sampling problem present in previous studies where the ex-post fall-off was used as a proxy for the ex ante one under the assumption of rational expectations. Its disadvantage, however, is that it exhibits heteroskedasticity which cannot be removed.

Three major conclusions emerge from our study. First, our results are consistent with the tax clientele hypothesis and inconsistent with the short-term trading and the "hedge portfolio" hypothesis; i.e., the average expected proportionate fall-off was significantly lower than unity (around 55%) and exhibited a positive relationship with the dividend yield. Second, a comparison of our estimates of the expected fall-off with the actual fall-off shows that these two are not statistically significantly different from each other, thus not rejecting the joint hypothesis of rational expectations and the validity of the Black-Scholes model. Finally, our results show that the usual assumptions made in valuing options on dividend paying stocks, that the fall-off is equal to the dividend, is unrealistic and would lead to downward-biased estimates of the option value.

[9] The standard errors are 1.4 and 4.1, respectively.

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