A Note on the Welfare Consequences of New Option Markets
Author(s): Barry Schachter
Source: *The Journal of Finance*, Vol. 41, No. 1 (Mar., 1986), pp. 263-267
Published by: Blackwell Publishing for the American Finance Association
Stable URL: http://www.jstor.org/stable/2328358
Accessed: 26/11/2009 08:26

THE JOURNAL OF FINANCE • VOL. XLI, NO. 1 • MARCH 1986

A Note on the Welfare Consequences of New Option Markets

BARRY SCHACHTER*

THIS NOTE EXTENDS EXISTING results on the conditions under which the opening of option markets permits a Pareto improvement in welfare. Specifically, conditions supplementary to those set forth in Dybvig and Ingersoll [4] and Hakansson [5] are identified. These results thus expand the universe of known cases in which trading in options has no negative welfare consequences.

Welfare effects arise from changes in investors' choice sets. The opening of option markets can affect the choice sets of investors in two ways[1], first, by augmenting the existing financial market structure and second, by altering the equilibrium relative prices of assets. In general, the two are not separable. Thus, welfare effects should be considered within the context of an equilibrium model[2].

Hakansson examines the nature of redistributive effects in an equilibrium model where investors' utilities are arbitrary. He identifies necessary and sufficient conditions for there to be no redistributive welfare effects following the opening of option markets. For this result, either the endowment in the richer market structure must be the equilibrium allocation in the original structure (his Theorem 6) or the two equilibria must share a common implicit state-contingent-claim price vector (Theorem 7). In the model set forth below, the conditions of Theorem 6 are not satisfied, and the conditions of Theorem 7 are satisfied trivially. Dybvig and Ingersoll examine the effect on CAPM equilibrium relative prices of the addition of an option market (Section III). They examine two cases. In the first, equilibrium relative prices are not affected when investors have homogeneous quadratic utilities. In the second, with heterogeneous utilities, the CAPM pricing relationships break down. In the model presented here, equilibrium pricing relationships are preserved even though there is heterogeneity among investors.

I. The Model

Assume a frictionless exchange economy with M possible future states and J primary assets, $J < M$. There are H investors with expected utilities of the

$$u^h = a^{h\prime} Vs^h - s^{h\prime} V'BVs^h, \tag{1}$$

* Faculty of Business Administration and Department of Economics, Simon Fraser University. I would like to thank J. S. Butler, Henry Wan, George Oldfield, and S. C. Tsiang for their assistance.

[1] There can be no welfare effects if options are redundant or unwanted. Thus, models in which options are priced by arbitrage (e.g., Black and Scholes [1]) and models in which markets are complete (e.g., Brennan [2]) are inappropriate.

[2] On a related issue, Ross [7], John [6], and others have derived conditions under which options

form[3] where, V is an $M \times J$ matrix of end-of-period state contingent asset payouts, s^h is a $J \times 1$ vector of assets holdings, a^h ($M \times 1$) and B ($J \times J$) are coefficients. For convenience, define $G = V'BV$.

Budget constraints are

$$p'(s^h - s_e^h) = 0, \tag{2}$$

where, p is a $J \times 1$ vector of equilibrium prices, and s_e^h is a $J \times 1$ vector of endowments. Endowments are set exogenously. Investors are not initially given endowments of assets in zero net supply.

Investors' demand functions are given by

$$s^h = G^{-1}V'a^h - \lambda^h G^{-1}P, \tag{3}$$

where the marginal utility of wealth is $\lambda^h = (p'G^{-1}V'a^h - p's_e^h)(p'G^{-1}p)^{-1}$.

Changes in the market structure caused by the opening of option markets are reflected in the model by adding columns to the payout matrix, V. The vectors of investor endowments (and hence, the vector of aggregate asset supplies) are augmented with zeros. As shown in Appendix 1, there is a closed-form equilibrium price vector for this economy.

II. Analysis of Welfare Effects

The difference in an investor's welfare between two market structures is determined by subtracting the expected utility in the original market structure from that with options. For analytical convenience, the indirect utility function corresponding to Equation (1) is used. Substituting the demand functions (3) into (1) and simplifying gives

$$\nu^h = a^{h\prime}VG^{-1}V'a^h - (\lambda^h)^2 p'G^{-1}p. \tag{4}$$

Given the equilibrium prices and assuming that the introduction of options does not alter the patterns of the state payouts of the previously existing assets, we have

THEOREM 1. *The addition of option markets does not affect the relative prices of the original assets.* (*The proof is given in Appendix* 1.)

This result shows that, within the context of the assumed model, there is no breakdown in the equilibrium pricing relationships for this type of heterogeneity among investors. The result also implies that, for each investor, the original

can be used to generate complete markets. Complete markets guarantee a first-best Pareto optimum. Not all the models used in this area are equilibrium models. In the following analysis, options are traded in an equilibrium model in which markets need not be complete.

[3] The expected utility may be derived from quadratic utilities. Define $C_m^h = v_m{}'s^h$, as h's wealth in state, m. Then h's state contingent utility can be written as $u_m^h = \alpha_m^h C_m^h - \beta_m^h C_m^h)^2$. Expected utility is then $u^h = \Sigma_m \pi_m u_m$. Define $a_m = \pi_m \alpha_m$, and $b_m = \pi_m \beta_m$. Then, expected utility can be rewritten in matrix form as $u^h = a^h C^h - C^{h\prime} B^h C^h$; B is diagonal with elements, b_m. To derive analytical results, B must be proportional across investors. When all investors have von Neumann-Morgenstern utilities, then α_m^h and β_m^h are the same for every state. In this case, α_m^h is free to vary across investors.

equilibrium consumption bundle would be available to all investors in the expanded market structure.

THEOREM 2. *The change in any investor's utility is zero or positive.*

The proof is immediate from Theorem 1. If prices remain the same, then the no-option portfolio remains feasible for each investor. Thus, an investor will trade options only if his utility is increased by this action. A formal proof is available from the author.

All investors who trade options in equilibrium realize a gain in utility. There is no change in utility for those who do not trade options. An example is given in Appendix 2.

Note that investors' endowments in the expanded market are the same as in the original market. These are not equilibrium endowments, and thus the assumptions of Hakansson's Theorem 6 are not satisfied. Hakansson's Theorem 7 is satisfied trivially, since the explicit equilibrium asset prices are unchanged.

Hakansson's Corollary 3 following Theorem 7 states that, if some markets are closed, the result will almost always involve either redistributive effects or leave everyone worse off. We see from Theorem 2 above that in the case examined here, closing of some option markets will have no redistributive effects. In addition, only those who traded options would suffer from the closing of those markets.

The predominantly positive effects associated with the opening of option markets is noted by Hakansson in Section XI. He suggests that the opening of option markets cannot result in a decrease in welfare, and that only when initial endowments are not an efficient allocation could redistributive effects be possible. In the model used here, even redistributive effects are ruled out.

The assumption of quadratic utility used here is essential for obtaining the results above. Hakansson notes that for the HARA class of utilities, values are preserved in the augmented market as long as there is a riskless asset available. There is no necessity of a riskless asset here. However, Cass and Stiglitz [3] have shown that two fund separation will occur without a riskless asset if utilities are quadratic.

In summary, the results set forth above extend the work of Dybvig and Ingersoll and Hakansson by showing there is greater scope for Pareto improvements when financial markets are augmented with options than has been suggested elsewhere. In examining a case Dybvig and Ingersoll did not examine, equilibrium pricing relationships were found to be maintained when option trading is permitted. The work of Hakansson is extended by showing that there is at least one nontrivial case in which redistributive effects do not occur even if endowments are not equilibrium allocations.

As to the generalizability of the results, note that assets' state payouts are not specified. Thus, the introduction of a riskless asset (with a constant payout across states) would not affect the results. Nor are the results dependent on the number of initial assets or the number of options introduced into the economy. Some of the original assets could be options. On the other hand, short positions are not restricted, and short sale constraints would affect the results. The assumption that investors agree on end-of-period state contingent payouts is necessary to achieve analytical results.

Appendix 1

Proof of Theorem 1

Market clearing in the original market structure requires

$$S^J = (G^J)^{-1}(\textstyle\sum_h V^{J\prime}A^h) - (\sum_h (\lambda^J)^h(G^J)^{-1}P^J), \tag{A1}$$

where the superscript, J, identifies the number of tradeable assets in that market structure. Equilibrium prices are given by $P^J = G(S - G^{-1}(\sum_h V^{J\prime}A^h))(-X)$, where $X = (P'G^{-1}P)^2 P'G^{-1}(\sum_h V^{J\prime}A^h) - P'S)^{-1}$. Normalizing, we have

$$P^J = (\textstyle\sum_h V^{J\prime} A^h) - G^J S^J. \tag{A2}$$

Equilibrium prices in the expanded market can be written as

$$P^{(J+R)} = \begin{bmatrix} \sum_h V^{-1}A^h - G^J S^J \\ \sum (V^R)'A^h - (V^R)'BV^J S^J \end{bmatrix} \tag{A3}$$

where the first element is the original relative price vector.

Appendix 2

An Illustrative Example

Let $M = 6$, $H = 2$. The parameters of the investors' expected utility functions are

$$A^A = [0.67, 0.1, 0.1, 0.2, 0.3, 0.3], \qquad A^B = [0.67, 0.3, 0.3, 0.2, 0.1, 0.1],$$

$$B = \begin{bmatrix} 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.25 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.25 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{B1}$$

Let $J = 4$. The payout matrix is

$$V = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 5 & 2 \\ 0 & 4 & 2 & 4 \\ 0 & 3 & 4 & 4 \\ 0 & 2 & 1 & 2 \\ 0 & 1 & 3 & 3 \end{bmatrix}. \tag{B2}$$

Endowments are

$$S^A = [0.1, 0.05, 0.03, 0.04], \qquad S^B = [0.4, 0.05, 0.07, 0.06]. \tag{B3}$$

Equilibrium prices are

$$P = [1.29, 1.225, 0.675, 0.95].$$

Utilities for 0.301 for A and 0.618 for B. Adding an option on the second asset with an exercise price of 3 leaves equilibrium prices unchanged. The option's price is 0.35. Option demand by A is -4.96. B's demand has the opposite sign. Utilities become 0.318 for A and 0.634 for B.

REFERENCES

1. F. Black and M. Scholes. "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy* 81 (May 1973), 637–59.
2. M. Brennan. "The Pricing of Contingent Claims in Discrete Time Models." *Journal of Finance* 34 (March 1979), 53–68.
3. D. Cass and J. Stiglitz. "The Structure of Investor Preferences and Asset Returns, and Separability in Portfolio Allocation: A Contribution to the Pure Theory of Mutual Funds." *Journal of Economic Theory* 2 (April 1970), 122–60.
4. P. Dybvig and J. Ingersoll. "Mean-Variance Theory in Complete Markets." *Journal of Business* 55 (April 1982), 231–51.
5. N. Hakansson. "Changes in the Financial Market: Welfare and Price Effects and the Basic Theorems of Value Conservation." *Journal of Finance* 37 (September 1982), 977–1004.
6. K. John. "Market Resolution and Valuation in Incomplete Markets." *Journal of Financial and Quantitative Analysis* 19 (March 1984), 29–44.
7. S. Ross. "Options and Efficiency." *Quarterly Journal of Economics* 90 (February 1976), 75–89.