Some Aspects of Equilibrium for a Cross-Section of Firms Signalling Profitability with Dividends: A Note
Author(s): Anil K. Makhija and Howard E. Thompson
Source: *The Journal of Finance*, Vol. 41, No. 1 (Mar., 1986), pp. 249-253
Published by: Blackwell Publishing for the American Finance Association
Stable URL: http://www.jstor.org/stable/2328356

THE JOURNAL OF FINANCE • VOL. XLI, NO. 1 • MARCH 1986

Some Aspects of Equilibrium for a Cross-Section of Firms Signalling Profitability with Dividends: A Note

ANIL K. MAKHIJA and HOWARD E. THOMPSON*

BHATTACHARYA [1] HAS SUGGESTED THAT firms use dividends to signal profitability so that, in equilibrium, in a cross-section of comparable firms, those that have higher value pay higher dividends. We bring out the pivotal role of the characterization of the cross-section of firms. We show that the dividend and profitability of the least and the most profitable firm affect the nature and feasibility of such signalling equilibria. Under Bhattacharya's characterization, the least profitable firm has zero profits and pays no dividends. This leads to a linear relationship between dividends and maximum earnings across the cross-section. However, substitution of reasonable cost and tax parameters in his model reveals that all firms would pay greater than 100% of maximum (*not average*) earnings in dividends and that dividends would exceed the value of the firm. We argue that equilibria are not possible in such cases. We show that, when the earnings of the least profitable firm are nonzero, the relationship between dividends and maximum earnings and value and dividends is nonlinear. Furthermore, to ensure feasible equilibria, the dividend of the most profitable firm must be limited. In sum, we argue that, for dividends to signal profitability, both the least profitable firm and the most profitable firm must satisfy conditions that have not been recognized in the literature.

I. The Framework

We start by outlining the Bhattacharya model. This signalling framework, also used by Ross [3], follows Spence [4].

Existing shareholders, who are risk neutral and have a single-period horizon, attempt to maximize the market value "assigned" to the firm by outsiders. Outsiders use dividends as the only signal to distinguish the profitability of assets held by a cross-section of firms. Dividends are declared at the beginning of the period. The cashflows generated by the assets of the firm become known at the end of the period, when dividends are paid, and are taken to be intertemporally i.i.d. perpetual streams. Dividends are subject to personal taxes. There are sufficient investment opportunities to reinvest any excess of cashflow over dividends. And, if the assets do not generate sufficient cashflows to meet their

* University of Pittsburgh and University of Wisconsin-Madison. We wish to thank James C. Hickman, Jacob J. Levin, and Donald R. Schuette for stimulating discussions concerning the fundamental differential equation analyzed in this note. We are, of course, responsible for any remaining errors.

dividend commitments, outside financing is available at a penalty interest rate.

The uncertain cashflows, X_t, are characterized by

$$P(X_t \leq a) = F(a) = \int_0^a \frac{1}{t}\,dZ$$

where t denotes both the firm and its maximum earnings. The objective function of the existing shareholders is

$$\underset{D}{\text{Max}}$$

$$\left\{\frac{1}{1+r}\left[V(D) + \alpha D + \int_D^t (Z - D)\frac{1}{t}\,dZ + \int_0^D (1+\beta)(Z - D)\frac{1}{t}\,dZ\right]\right\}. \quad (1)$$

The term $V(D)$ is the value of the firm at the end of the single-period horizon, and αD is the after-personal taxes dividend. The third term is the average excess of earnings over dividends. The last term involves the cases where dividends exceed earnings and β can be interpreted as the penalty interest rate when emergency financing is employed. Equation (1) can be reworked as

$$\underset{D}{\text{Max}}\left\{\frac{1}{1+r}\left[\frac{t}{2} + V(D) - (1-\alpha)D - \frac{\beta D^2}{2t}\right]\right\} \quad (2)$$

where the negative terms indicate the costs of paying dividends, and the benefits are manifested in $V(D)$.

Then, optimizing (2) and using the condition that in equilibrium the value of the firm will be correctly signalled, Bhattacharya finds that dividends must satisfy

$$\frac{dD}{dt} = \frac{1}{2}\frac{K}{K+1}\left[\frac{1 + \beta(D/t)^2}{1 - \alpha + \beta(D/t)}\right],\ t \in [\underline{t}, \infty). \quad (3)$$

For $\underline{t} = 0$, Bhattacharya concludes that $D^*(t) = 0$ and the solution to (3) is

$$D^*(t) = A_1 t \quad (4)$$

where

$$A_1 = \frac{(1-\alpha)}{\beta}\frac{K+1}{K+2}\left[-1 + \sqrt{-1 + \frac{\beta K(K+2)}{(1-\alpha)^2(K+1)^2}}\right] \quad (5)$$

which implies that

$$V(D^*(t)) = [(1-\alpha) + \beta A_1]D^*(t). \quad (6)$$

Now consider some reasonable values for the parameters. Let $\alpha = 0.65$ (the tax rate is 0.35), $K = 10$ (interest rate is 10%), and $\beta = 0.05$ (the penalty for emergency borrowing is 5% over the nominal rates). Then, using (4) and (5), $D^*(t) = 1.1866t$ and, using (6), $V(D^*(t)) = 0.4094D^*(t)$. These results are unreasonable, and the model needs modification.

II. Alternative Descriptions of a Least Profitable Firm in a Cross-Section

In this section, we modify the above model by considering different descriptions of a least profitable firm. In addition, even though the fundamental Equation (3) can be solved analytically,[1] we provide a qualitative solution which is more revealing of some aspects of equilibrium.

A. Initial Condition $(\underline{t}, D(\underline{t})) = (0, 0)$

This is the Bhattacharya case discussed above. However, note that (3) has two solutions for $(\underline{t}, D(\underline{t})) = (0, 0)$. These are

$$D(t) = A_1 t \text{ and } D(t) = A_2(t)$$

where

$$A_1, A_2 = \frac{(1-\alpha)}{\beta}\frac{K+1}{K+2}\left[-1 \pm \sqrt{1 + \frac{\beta K(K+2)}{(1-\alpha)^2(K+1)^2}}\right]. \tag{7}$$

Only the solution $D(t) = A_1 t$ was considered by Bhattacharya, the case of $D(t) = A_2 t$ being meaningless in the context.[2] In Figure 1, we draw two straight lines corresponding to these solutions.

B. Initial Condition $(\underline{t}, D(\underline{t})) = (\underline{t}, 0)$, $\underline{t} \neq 0$

For all initial conditions, $\underline{t} \neq 0$ and $D(\underline{t}) = 0$

$$\frac{dD}{dt}\Big|_{D=0} = \frac{K}{2(K+1)(1-\alpha)}.$$

Thus, for all cross-sections which have least profitable firms with nonzero earnings, the slope of $D(t)$ on the horizontal axis will be the same regardless of the value of $\underline{t}$. That is, the solutions for these cross-sections all cross the horizontal axis at the same slope.

By Taylor's expansion of (7), it can be shown that

$$A_1 < \frac{dD}{dt}\Big|_{D=0} = \frac{K}{2(K+1)(1-\alpha)}.$$

Furthermore, the slope of any solution will decline if $D/t < 1/(1-\alpha)$. This will surely be the case for t near $\underline{t}$. It can also be shown that any solution curve arising from an initial condition on the horizontal axis will approach $A_1 t$ as an asymptote.

[1] The equation is separable using the transformation $y = D/t$. However, the solution is an implicit function in y and t, and therefore, does not allow easy analysis of the effects of changes or the initial conditions or parameter values.

[2] There are cases where the line $A_2 t$ is relevant to analysis. Since these cases include models that deal with growth situations, they are beyond the scope of this note. The parameter A_1 becomes negative while A_2 becomes positive and relevant when the term $\int_0^t (Z - D)\frac{1}{2}\,dZ$ in Equation (1) has a coefficient differing from one. More details are found in Makhija and Thompson [2].

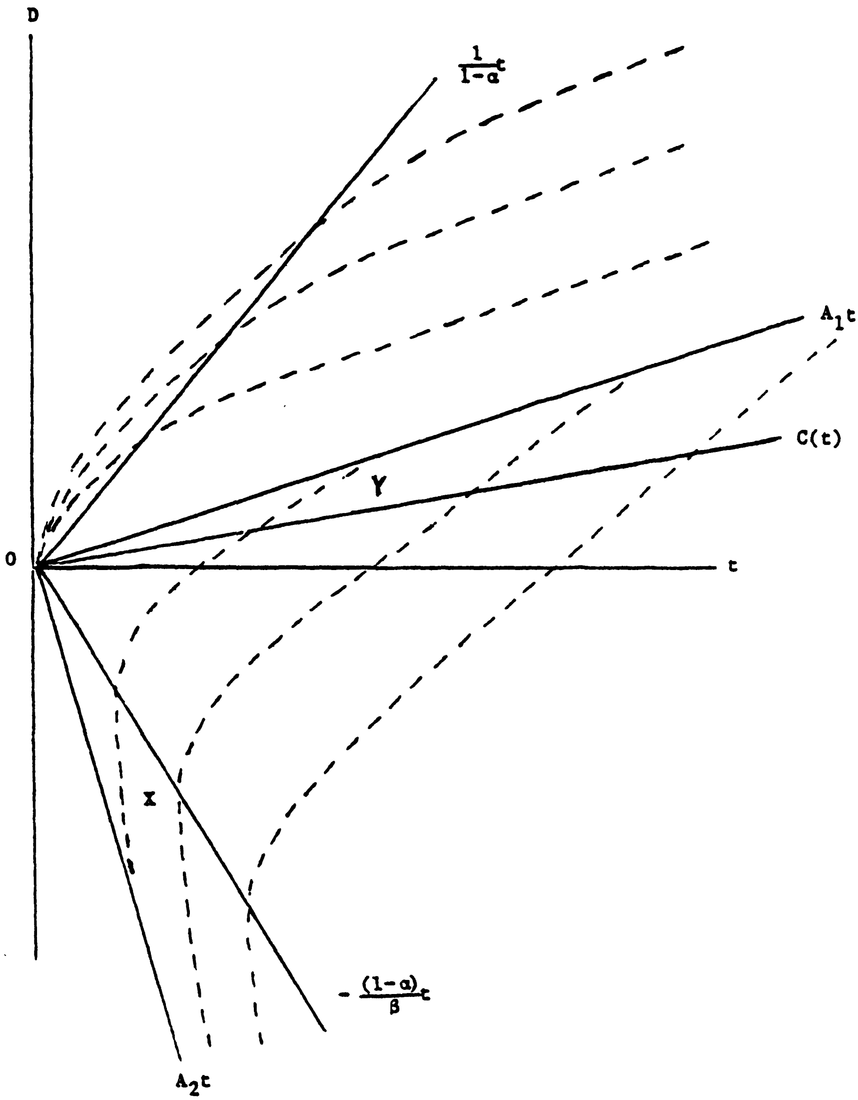

Figure 1. Solution Families for Equation (3)

Also, all solution curves passing through $D = -(1 - \alpha)t/\beta$ do so with infinite slope. The line $D = -(1 - \alpha)t/\beta$ will always lie between A_1t and A_2t and will have negative slope. Using these facts, the families of solution are shown in Figure 1 by the dashed lines of the type XY.

C. Initial Condition $\underline{t} \neq 0$, $D(\underline{t}) \neq 0$

Start with noting that if $D/t = 1/(1 - \alpha)$ then $dD/dt = K/2(K + 1)(1 - \alpha)$. Thus, any solution that crosses $D = (1/(1 - \alpha))t$ will cross it at the same slope as solutions crossing the horizontal axis. Combining this fact with the fact that the slope will decline to approach A_1, we know that the solution curves in the region above $A_1 t$ and below $(1/(1-\alpha))t$ will eventually become parallel to $A_1 t$. So, solutions for $\underline{t} > 0$ and $D(\underline{t}) > A_1 \underline{t}$ are curves with large slopes which decline to slope $K/(2(K+1)(1-\alpha))$ upon passing through $(1/(1-\alpha))t$ and end up with slope A_1. Figure 1 illustrates these cases and represents a complete solution to Equation (3).[3]

The initial condition, $(\underline{t}, D(\underline{t})) = (0, 0)$, is not the only case relevant to the analysis of firms since not all cross-sections will have a lowest firm with a maximum earnings of zero. It remains to be shown that some of the solutions detailed above lead to feasible equilibria. For that purpose, we turn to the analysis of the most profitable firm.

III. Constraints on a Most Profitable Firm

Given the dividends of the least profitable firm, more profitable firms, to distinguish themselves, must pay higher dividends. Without a constraint on the amount of dividends that firms can pay, higher profitability will cause firms to move up the solution curves in the region between the horizontal axis and the line $A_1 t$ and the dividends of the most profitable firm may approach $A_1 t$. As in our example ($K = 10$, $\alpha = 0.65$, $\beta = 0.05$), some cases will have A_1 greater than one. Therefore, some profitable firms may be required to pay dividends in excess of *maximum* earnings according to the model. They will then always be in the emergency borrowing situation! But lenders would not lend if there are no prospects of repayment. In equilibrium, dividends must be limited by the ability of the firm, on the average, to cover these dividends by earnings. That is, we must have

$$\frac{t}{2} - D(t) - \frac{\beta D^2(t)}{2t} > 0 \qquad \text{or} \qquad D(t) \leq C(t) = \frac{-1 \pm \sqrt{1+\beta}}{\beta} < \frac{1}{2}\, t. \tag{8}$$

Constraint (8) is drawn in Figure 1 where it shows that it may limit dividends of the most profitable firm. The nonlinear solution curves between the horizontal axis and $C(t)$ coupled with $C(t)$ in Figure 1 are feasible signalling equilibria.

IV. Conclusions

We have shown that dividends can serve to signal the profitability of a cross-section of firms, provided the least profitable and the most profitable firm are properly characterized. Additionally, we have shown that the relation between dividends and earnings is likely to be nonlinear.

[3] These are again relevant under conditions described in footnote 2.

REFERENCES

1. S. Bhattacharya. "Imperfect Information, Dividend Policy, and 'The Bird in the Hand' Fallacy." *The Bell Journal of Economics* 10, (Spring 1979), 259–70.
2. A. K. Makhija and H. E. Thompson. "Imperfect Information and Signalling with Dividends: Some Additional Results." Wisconsin Working Paper No. 2-85-7 (February 1985).
3. S. A. Ross. "The Determination of Financial Structure: The Incentive Signalling Approach." *The Bell Journal of Economics* 8, (Spring 1977), 23–40.
4. A. M. Spence. "Competitive Optimal Responses to Signals: Analysis of Efficiency and Distribution." *Journal of Economic Theory* 7 (March 1974), 296–332.