A Utility-Based Model of Common Stock Price Movements
Author(s): Robert H. Litzenberger and Ehud I. Ronn
Source: *The Journal of Finance*, Vol. 41, No. 1 (Mar., 1986), pp. 67-92
Published by: Blackwell Publishing for the American Finance Association
Stable URL: http://www.jstor.org/stable/2328345
Accessed: 26/11/2009 08:18

THE JOURNAL OF FINANCE • VOL. XLI, NO. 1 • MARCH 1986

A Utility-Based Model of Common Stock Price Movements

ROBERT H. LITZENBERGER and EHUD I. RONN*

ABSTRACT

This paper develops and tests a nonlinear utility-based econometric model of the temporal behavior of aggregate stock price movements based on a constant relative risk aversion utility function and an observable information set consisting of aggregate consumption, aggregate dividends, and past stock prices. The stochastic process derived from time-series analyses of consumption and dividends measured over annual intervals is used to derive and empirically test a closed-form solution for stock-price movements. The endogenization of discount rate changes in the utility-based model is shown to be more consistent with aggregate stock price movements over a twenty-year holdout period than constant discount rate models. The model is also used to estimate the representative investor's relative risk aversion. The estimate of 4.22 is consistent with that used by Grossman and Shiller in their perfect foresight model and is significantly higher than the relative risk aversion of 1.0 implied by logarithmic utility.

THE PURPOSE OF THIS paper is to develop a utility-based econometric model of the time-series behavior of aggregate common stock price movements. A nonlinear time series model is estimated based on a constant relative risk aversion (CRRA) utility function and an observable information set consisting of aggregate consumption, aggregate dividends, and past stock prices. Since spot consumption data is not available, the model accounts for the use of consumption data measured over annual intervals. The stochastic process derived from time-series analyses of consumption and dividends is used to derive a closed-form solution for aggregate stock price movements. This closed-form relation is tested in its conditional expectation form, using time-series data on aggregate stock price movements over the period 1926–1982.

The aggregate CRRA utility function has been extensively analyzed in both theoretical[1] and empirical[2] studies. The present paper also estimates the representative investor's relative risk aversion. The magnitude of the econometric estimate of relative risk aversion is relevant to financial economics for two

* Litzenberger is from the Graduate School of Business, Stanford University, and Ronn is from the Schools of Business Administration, University of California at Berkeley. The authors gratefully acknowledge the helpful suggestions of David Brown, Michael Gibbons, Lawrence Harris, Wayne Ferson, Kenneth Singleton, the participants of the Finance workshops at MIT and Yale, and an anonymous referee. The coauthors are responsible for any remaining errors. The original version of this paper was presented at the June 1984 meetings of the Western Finance Association.

[1] See Cox, Ingersoll, and Ross [7], Breeden and Litzenberger [5], Hakansson [14], Kraus and Litzenberger [17], Merton [20], Rubinstein [22, 23], and Samuelson [24]. Also see Breeden [3] for the development of a consumption-oriented CAPM model in a continuous time framework.

[2] See Grossman and Shiller [12, 13], Hansen and Singleton [15, 16], Brown and Gibbons [6], and Ferson [9].

reasons. First, some theoretical results rely on the assumption of logarithmic utility; by estimating the difference between the relative risk aversion and unity, the logarithmic utility model can be tested. Second, the effects of changes in risk on the demand for risky assets and the savings decision depend on the magnitude of the relative risk aversion parameter.[3]

As recognized by Grossman and Shiller [12], a utility-based econometric time-series model of stock price movements endogenizes intertemporal discount rate changes. The performance of the CRRA model in tracking common stock price movements is compared to constant discount rate models. A constant discount rate model may be derived based on the assumptions of investor risk neutrality. One variant of the constant discount rate model is the limiting case of the utility-based model as relative risk aversion approaches zero. Another variant of a constant discount rate model is the familiar Williams-Gordon model where future expected dividends are assumed to grow at a constant rate. Rubinstein [22, p. 410] has developed a natural uncertainty analogue to the Williams-Gordon model based on CRRA and the assumption that the growth rate in dividends and the growth rate in the marginal utility of aggregate consumption follow stationary random walks with constant stationary correlation. The relative performance of these models in tracking aggregate stock price movements is reflective of the extent to which the available consumption data is able to endogenize discount rate changes.

Previous attempts at using CRRA models to explain stock price movements include Grossman and Shiller [12] and Hansen and Singleton [16]. Grossman and Shiller assume that investors know with certainty the evolution through time of consumption, dividends, and prices. Then, using an assumed value for relative risk aversion, current stock prices are represented as the discounted present value (using marginal rates of substitution as discount factors) of dividends and terminal stock price (1979 is the terminal date). In contrast to the Grossman and Shiller perfect foresight assumption, the current study models stock prices using investors *ex ante* expectations from time-series analyses of observable data.

Hansen and Singleton [16] assume consumption and common stock returns are jointly lognormally distributed. They then fit their model using monthly consumption data and various autoregressive lags. In contrast, the current model does not assume joint lognormality of consumption and stock returns, but employs an empirical characterization of the time series behavior of the marginal utility of dividends.

The current paper uses annual consumption data and incorporates this distinction between interval and spot consumption. The use of annual consumption data has two advantages over monthly consumption data. First, annual consumption data is available over a much longer time period that encompasses a wider range of variability in the growth rate of consumption. Second, reported "consumption" data reflect consumption expenditures rather than consumption. While this problem is mitigated by focusing on nondurables, for short time

[3] See Rothschild and Stiglitz [21].

intervals (such as a month), the relationship between consumption expenditures and consumption is tenuous even for "nondurables." As in the prior studies of Grossman and Shiller and Hansen and Singleton, the treatment of the consumption flows from durables is not considered in the current paper. Implicitly, the marginal utility of consumption is assumed to be proportional to a power function of the consumption of nondurables. This is consistent with either a technological constraint that requires the consumption flow from durables to be extracted in fixed proportion to the consumption of nondurables or additively separable power utility functions for durables and nondurables.[4]

This paper is organized as follows. Section I derives the underlying valuation relation and its implication for the time-series behavior of aggregate stock price movements. Section II discusses the empirical procedures employed in this paper. Section III contrasts the current paper's model with five alternative models: the previously discussed variants of constant discount rate models; a technical mode; the Grossman-Shiller model; and the Hansen-Singleton model. Section IV presents the paper's main conclusions and provides a summary.

I. The Model

A. Introduction

The model analyzes the rate of change in average annual stock prices conditional on the econometrician's limited information set. The econometrician's limited information set is specified as the past *and present* annual average values of dividends, real per capita consumption, and the price level, and *past* averages of stock prices. Note that the limited information set I_t—where I_t is a proper subset of I_t^*, the full-information set—*includes* current dividends and consumption but excludes the current stock price. Thus, the model attempts to explain contemporaneous price movements, not to forecast stock price movements. A forecasting model would use $(P_t, C_t, D_t) \subset I_t$ and derive $E(P_{t+1} \mid I_t)$; here, $(C_t, D_t, P_{t-1}) \subset I_t$, $P_t \notin I_t$, and $E(P_t \mid I_t)$ is derived.

Valuation theory relates spot consumption at points C_t, C_{t+s} (as the determinants of the marginal rate of substitution) to P_t. However, for the entire period 1926–1982, the Commerce Department only estimated the total flows of consumption over an entire year.[5] There thus exists a distinction between spot consumption and the annual quantity actually measured. This issue, denoted the "consumption integral problem," was first studied by Breeden [4]. A different approach than that used by Breeden is required here because of the nonlinear relation that arises from the CRRA assumption in a discrete-time valuation model.

[4] Recently, Dunn and Singleton [8] have utilized an alternative approach to modeling the distinction between consumption purchases and service flows derived therefrom. They posit linear relationships between current-period consumption flows and past periods' expenditures on consumption items.

[5] Quarterly consumption data is available from 1941 and monthly consumption data from January 1959.

B. Derivation of Valuation Model Under Constraints of Available Data

As mentioned in Section I.A, the empirical testing properly distinguishes between spot consumption on which the theory is based and interval consumption for which data is available. Specifically, over the pre-1947 period, data on consumption collected by the Commerce Department was only reported for annual intervals. The following analysis builds from the theoretical relation based on unobservable spot consumption to a relation among observable magnitudes.

Under the assumption that all individuals have identical probability beliefs and CRRA utility functions with identical relative risk aversion and identical subjective rate of time preference, Rubinstein [22] has derived the following valuation relation:

$$P_t = \int_0^{\infty} \beta^{\tau} E\left[\left(\frac{C_{t+\tau}}{C_t}\right)^{-A} D_{t+\tau} \mid I_t^*\right] d\tau \tag{1}$$

where

P_t = the real stock price at time t
$D_{t+\tau}$ = the real dividend at time $t + \tau$
β = the subjective rate of time preference
A = the coefficient of relative risk aversion
$C_{t+\tau}$ = the consumption flow at time $t + \tau$ and
I_t^* = the *full* information set at time t.

The stochastic Euler condition (1) is well-known in the financial literature. Among others, Rubinstein [22] performed an in-depth analysis of the CRRA utility function, Lucas [19] provided a discrete-time derivation, while Grossman and Shiller [12] considered the continuous-time analogue.

Dividing through by $P_{t-1} \in I_t^*$ yields the rate of price appreciation, R_t:

$$R_t = \int_0^{\infty} \beta^{\tau} E\left[\left(\frac{C_{t+\tau}}{C_t}\right)^{-A} \frac{D_{t+\tau}}{P_{t-1}} \mid I_t^*\right] d\tau. \tag{2}$$

Although Equation (2) expresses the equilibrium value of the market portfolio in a competitive exchange economy, the equilibrium condition can be interpreted as the portfolio equilibrium condition for a single individual who holds all assets and consumes all consumption. Price movements over time may then be viewed as the price changes that are required for that "composite individual" to continue to hold all assets and consume all consumption. Note that the term β^{τ} is the sole determinant of intertemporal differences in the marginal utility of a given level of consumption. The specification of this time preference parameter as a continuous function, while analytically convenient in continuous time portfolio models, is not based on any fundamental economic principle. In order to develop a test of the valuation relation based on annual data, an alternative specification is required.

Under the assumption that individuals are indifferent with respect to the within-year timing of consumption, the subjective rate of time preference changes discontinuously at year's end, Equation (2) changes to Equation (3):

$$R_t = (\textstyle\sum_{i=1}^{\infty} \beta^{(2i-1)/2} J_{ti})/(C_t^{-A} P_{t-1})$$

where (3)

$$J_{ti} \equiv \int_{i-1}^{i} E(C_{t+\tau}^{-A} D_{t+\tau} \mid I_t^*) \, d\tau.$$

Note that Equation (3) is equivalent to a discrete-time interpretation of Equation (2) if all annual consumption occurs at a single point within the year. However, Equation (1) would not hold when annual interval consumption is simply substituted for spot consumption.

Based on a first-order Taylor expansion of aggregate intra-year spot consumption around annual interval consumption, the rate of change in the average annual price is

$$\bar{R}_t \simeq k \textstyle\sum_{i=1}^{\infty} \beta^{(2i-1)/2} \bar{J}_{ti}/(\bar{C}_t^{-A} \bar{P}_{t-1}) \tag{4}$$

where

$$\bar{R}_t \equiv \frac{\int_{i-1}^{i} P_{t+\tau} \, d\tau}{\int_{i-1}^{i} P_{t-1+\tau} \, d\tau}$$

$$\bar{J}_{ti} \equiv E(\bar{C}_{t+i}^{-A} \bar{D}_{t+i} \mid I_t^*)$$

$$k \equiv \frac{(1 - Ak_1)(1 + k_3)}{1 + k_2}$$

$$k_1 \equiv E\left[\int_{i-1}^{i} \left(\frac{C_\tau}{\bar{C}_i} - 1\right)\left(\frac{D_\tau}{\bar{D}_i} - 1\right) d\tau\right]$$

$$k_2 \equiv E\left[\int_{i-1}^{i} \left(\frac{p_\tau}{\bar{p}_i} - 1\right)\left(\frac{q_\tau}{\bar{q}_i} - 1\right) d\tau\right]$$

$$k_3 \equiv E\left[\int_{i-1}^{i} \left(\frac{d_\tau}{\bar{d}_i} - 1\right)\left(\frac{q_\tau}{\bar{q}_i} - 1\right) d\tau\right].$$

$\bar{R}_t$ is the rate of change in the average annual (real) stock price between period $t - 1$ and period t.

$\bar{C}_{t+i}$, $\bar{D}_{t+i}$ are, respectively, the cumulative values of real per capita consumption and real dividends over year $t + i$.

$C_{t+\tau}$, $D_{t+\tau}$ are, respectively, the (real, per capita) consumption and dividend flows at instant $t + \tau$.

$d_{t+\tau}$ is the nominal dividend payment at instant $t + \tau$.

$p_{t+\tau}$ is the nominal stock price at instant $t + \tau$.

$q_{t+\tau}$ is the consumption goods price level at instant $t + \tau$.

$\bar{p}_i$, $\bar{q}_i$, $\bar{d}_i$ are, respectively, the measured annual nominal averages of stock prices, consumption goods prices, and dividends over year i.

Proof is relegated to Appendix A.[6] Note that relation (4), with the exception of the k term and the use of annual averages, is identical to the discrete-time valuation relation based on annual dividend payment intervals. The k term should be viewed as a correction factor for intra-year interaction between dividends, consumption prices, stock prices, and nominal consumption.

The use of a first-order Taylor's series to approximate the integral of the intra-year marginal utility is preferable to assuming that all intra-year consumption of dividends occurs at the same point within the year. Such an approach would ignore within-year consumption variation and would implicitly use a "zero-order" Taylor's series expansion. Consequently, the approximation obtained in this model is more precise than those derived from such a discrete-time empirical approximation. Nevertheless, an identical valuation relation to that used in the current paper could be obtained if individuals are assumed to view all consumption within a year as a "composite good" and the time weights on utility could be specified as $k\beta^{(2i-1)/2}$.

Relation (4) contains a term for involving an infinite series of expected marginal utilities of dividends in future years, $\bar{J}_{ti}$, for $i = 1, \cdots$. Undoubtedly, investors use a very large and complex information set to forecast $\bar{J}_{t1}$. However, it is assumed that once $\bar{J}_{t1}$ is formed, *future* expectations are related by the first-order difference equation

$$\bar{J}_{ti} - \bar{J}_{t,i-1} = \delta_t + \phi(\bar{J}_{t,i-1} - \bar{J}_{t,i-2}) \quad i \geq 2$$

THEOREM 1. *If*

$$\bar{J}_{ti} - \bar{J}_{t,i-1} = \delta_t + \phi(\bar{J}_{t,i-1} - \bar{J}_{t,i-2})$$

for $i \geq 2$, *and where* $\delta_t \equiv \delta \bar{C}_t^{-A} \bar{P}_{t-1}$, *then*

$$\bar{R}_t \simeq \frac{\beta k}{1-\beta}\left\{\frac{\beta\delta}{(1-\beta)(1-\beta\phi)} + \frac{1}{1-\beta\phi} E\left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{-A} \frac{\bar{D}_{t+1}}{\bar{P}_{t-1}} \,\middle|\, I_t^*\right] - \frac{\phi\beta}{1-\beta\phi}\frac{\bar{D}_t}{\bar{P}_{t-1}}\right\}. \tag{5}$$

A proof of Equation (5) is provided in Appendix B.

Equation (5) expresses the rate of price appreciation as a function of the expected marginal utility of next period's dividend, the current dividend, last period's stock price, the subjective rate of time preference, and the parameters of the stochastic process governing the evolution through time of the expected marginal utility of future dividends.

C. Description of Empirical Data

The data used in this study cover the period 1926 through 1982. Consumption is measured as total annual per capita real consumption expenditures on non-durables and services. Note that consumer durables, which provide future consumption flows, are not included in the consumption measure. Thus, consumption durables are implicitly assumed to be investment in future consumption flows that are extracted in fixed proportion to the consumption of nondurables. Real annual dividends, $\bar{D}_t$, were calculated as total nominal dividends on the NYSE

[6] Note that the deriving *average* real prices, inflation interactions *may* play a role in the valuation procedure. Specifically, if, in Equation (4), $k_i \neq 0$, for any i, then these interactions *will* affect $\bar{R}_t$.

value-weighted index divided by the average monthly value of the personal consumption expenditures (PCE) deflator for the appropriate year. Finally, the annual average real price level, $\bar{P}_t$, was estimated as the average of that year's month-end NYSE value-weighted index (appropriately standardized) divided by the average PCE deflator. These data are discussed in more detail in Appendix C.

D. Derivation of Testable Implications

While investors may use a complex and extensive information set in setting $\bar{J}_{t+1}$, the econometrician's information set I_t, is assumed to contain only annual averages of *current* year dividends and consumption growth and the previous year's stock price, dividends, and consumption growth. Analytically,

$$I_t \equiv \left\{\frac{\bar{C}_t}{\bar{C}_{t-1}}, \frac{\bar{C}_{t-1}}{\bar{C}_{t-2}}, \bar{D}_t, \bar{D}_{t-1}, \bar{P}_{t-1}\right\}.$$

Thus, I_t is a proper subset of the full information set, I_t^*.

Dropping the bars above the variables for simplicity of notation, the expectation of the rate of price appreciation, conditional on the econometrician's information set, I_t, is obtained by applying the conditional expectations operator to Equation (5):

$$E(R_t \mid I_t) = \frac{\beta k}{1-\beta}\left\{\frac{\beta\delta}{(1-\beta)(1-\beta\phi)} + \frac{1}{1-\beta\phi} E\left[\left(\frac{C_{t+1}}{C_t}\right)^{-A} \frac{D_{t+1}}{P_{t-1}} \mid I_t\right] - \frac{\phi\beta}{1-\beta\phi}\frac{D_t}{P_{t-1}}\right\}. \quad (6)$$

Relation (6) involves the conditional expectation of the marginal rate of substitution between current consumption and future dividends, based on the econometrician's information set, I_t. The econometrician's conditional expectation is assumed to behave in accordance with the following stochastic process:

$$E\left[\left(\frac{C_{t+1}}{C_t}\right)^{-A} D_{t+1} - \left(\frac{C_t}{C_{t-1}}\right)^{-A} D_t \mid I_t\right] = \alpha_0 P_{t-1} + \alpha_1 \left[\left(\frac{C_t}{C_{t-1}}\right)^{-A} D_t - \left(\frac{C_{t-1}}{C_{t-2}}\right)^{-A} D_{t-1}\right]. \quad (7)$$

This linear difference equation is stated in terms of the *econometrician's* (limited) information set, I_t, rather than the full information set, I_t^*. Note that this first-order autoregressive equation is consistent with the preliminary time-series tests which indicated that $\Delta(C_{t+1}/C_t)^{-A}D_{t+1} \equiv (C_{t+1}/C_t)^{-A}D_{t+1} - (C_t/C_{t-1})^{-A}D_t$ follows an AR(1) process or a random walk.[7] These tests consist of an analysis of the autocorrelation and partial autocorrelation functions of $\Delta(C_{t+1}/C_t)^{-A}D_{t+1}$ for positive values of A; see Tables D.I and D.II of Appendix D. In order to prevent the drift term from decreasing in relative importance as real dividends increase

[7] If the marginal rate of substitution between current consumption and next period's dividend followed a random walk, $\alpha_1 = 0$ in Equation (7).

over time, it is assumed to be proportional to the prior period's price. Tables D.III and D.IV of Appendix D present the autocorrelation and partial autocorrelation functions of the error term in the time-series model

$$\left(\frac{C_{t+1}}{C_t}\right)^{-A}\frac{D_{t+1}}{P_{t-1}} - \left(\frac{C_t}{C_{t-1}}\right)^{-A}\frac{D_t}{P_{t-1}} = \alpha_0 + \alpha_1\left[\left(\frac{C_t}{C_{t-1}}\right)^{-A}\frac{D_t}{P_{t-1}} - \left(\frac{C_{t-1}}{C_{t-2}}\right)^{-A}\frac{D_{t-1}}{P_{t-1}}\right] + \varepsilon_{t+1}.$$

These results are consistent with Equation (7).

Combining relations (6) and (7) and rearranging terms gives relation (8):

$$E(R_t \mid I_t) = c_1 + c_2\left(\frac{C_t}{C_{t-1}}\right)^{-A}\frac{D_t}{P_{t-1}} + c_3\left(\frac{C_{t-1}}{C_{t-2}}\right)^{-A}\frac{D_{t-1}}{P_{t-1}} + c_4\frac{D_t}{P_{t-1}} \tag{8}$$

with

$$c_1 \equiv \frac{k\beta}{(1-\beta)(1-\beta\phi)}\left(\frac{\beta\delta}{1-\beta} + \alpha_0\right); \quad c_2 \equiv \frac{k\beta(1-\alpha_1)}{(1-\beta)(1-\beta\phi)};$$

$$c_3 \equiv -\frac{k\beta\alpha_1}{(1-\beta)(1-\beta\phi)}; \quad c_4 \equiv \frac{k\beta^2\phi}{(1-\beta)(1-\beta\phi)}.$$

Note that by ignoring intra-year variations in consumption, k, would be set identically equal to unity. An attempt to estimate the c_i by estimating their components—i.e., by estimating β, ϕ, δ, α_0, and α_1—would misestimate the c_i's as it would ignore the presence of k.

II. Econometric Model and Estimating Procedures

A. The Structural Econometric Model

The risk-averse model, Equation (8), is appended with the dividend-dynamics equation, Equation (7), and an identity defining the rate of price appreciation. The resulting three-equation nonlinear structural model is:

$$R_t = c_1 + c_2\left(\frac{C_t}{C_{t-1}}\right)^{-A}\frac{D_t}{P_{t-1}} + c_2\alpha_1\left[\left(\frac{C_t}{C_{t-1}}\right)^{-A}\frac{D_t}{P_{t-1}} - \left(\frac{C_{t-1}}{C_{t-2}}\right)^{-A}\frac{D_{t-1}}{P_{t-1}}\right] + c_3\frac{D_t}{P_{t-1}} + \varepsilon_t \tag{9}$$

$$\left(\frac{C_{t+1}}{C_t}\right)^{-A}\frac{D_{t+1}}{P_{t-1}} = \left(\frac{C_t}{C_{t-1}}\right)^{-A}\frac{D_t}{P_{t-1}} + \alpha_0 + \alpha_1\left[\left(\frac{C_t}{C_{t-1}}\right)^{-A}\frac{D_t}{P_{t-1}} - \left(\frac{C_{t-1}}{C_{t-2}}\right)^{-A}\frac{D_{t-1}}{P_{t-1}}\right] + u_t \tag{10}$$

$$R_t \equiv \frac{P_t}{P_{t-1}}. \tag{11}$$

This nonlinear system of equations incorporates multiplicative interaction terms as well as nonlinear terms involving the coefficient of relative risk aversion. Due to the nonlinearity on the LHS of Equation (10), the system of equations cannot be estimated using nonlinear least squares. A maximum-likelihood procedure is used to estimate the parameters of this system of equations.

B. Serial Correlation in the Error Term

Note that since these structural models are formulated in terms of a limited information set that included last period's price but not prices for higher order lags, serial correlation is not necessarily inconsistent with rational expectations.

$$\begin{aligned}\mathrm{COV}(\varepsilon_t, \varepsilon_{t-j}) &= E(\varepsilon_t\varepsilon_{t-j}) = E\{[R_t - E(R_t \mid I_t)][R_{t-j} - E(R_{t-j} \mid I_{t-j})]\}\\ &\equiv E\{[E(R_t \mid I_t^*) - E(R_t \mid I_t)][E(R_{t-j} \mid I_{t-j}^*) - E(R_{t-j} \mid I_{t-j})]\}. \quad (12)\end{aligned}$$

The error in forecast at time t, $\varepsilon_t \equiv E(R_t \mid I_t^*) - E(R_t \mid I_t)$, is due to elements in the complete information set, I_t^*, not included in the limited information set, I_t. These elements comprise the set I_t^c, where I_t^c is the complement of the set, I_t. Similarly, the time $t - j\,(j \geq 1)$ error, $\varepsilon_{t-j} \equiv E(R_{t-j} \mid I_{t-j}^*) - E(R_{t-j} \mid I_{t-j})$ is attributable to the set, I_{t-j}^c. Since I_t and I_{t-j} have been narrowly defined as consumption growth and dividends for the current and previous years and stock price for the previous year, I_t^c and I_{t-j}^c, are both non-null. There are *common* elements—the elements in the set $I_t^c \cap I_{t-j}^c$—which affect both ε_{t-j} and ε_t. Consequently, it is quite plausible to expect $\mathrm{COV}(\varepsilon_t, \varepsilon_{t-j}) \neq 0$; since ε_{t-j} and ε_t *are* affected by common elements contained in $I_t^c \cap I_{t-j}^c$, it is reasonable to expect $\mathrm{COV}(\varepsilon_t, \varepsilon_{t-j}) > 0$. Preliminary empirical evidence was consistent with positive covariance for $j = 1$, but could not reject zero autocorrelation for higher order lags. The test results reported in Section III will explicitly account for first-order serial autocorrelation in the error term, ε_t. The correction for autocorrelation effectively expands the econometrician's information set to include the previous year's error term, which implies an expansion of the information set to include the average annual values of the stock price two years ago, and consumption growth rates and dividends two years ago. Each of the return equations is modified by application of the operator $1 - \rho L$, where L is the lag operator, and ρ is the first-order autocorrelation.

C. Estimating Procedures

The econometric system consists of three equations. The first is Equation (9) (with the parameter constraint imposed). The second equation deals with the dividend-dynamics equation, Equation (10). The third equation is the identity $R_t \equiv P_t/P_{t-1}$.

Denoting

y_t as a 1×3 row vector of the jointly independent variables P_t, C_t, and D_t.
β as a 6×1 vector of the unknown behavioral and time-series parameters A, α_0, α_1, c_1, c_2, and c_3.
e_t as a 1×3 row vector of the random disturbances ε_t, u_t, and 0,

the nonlinear system of equations may be expressed as

$$\begin{pmatrix} F_t(y_t, \beta) \\ R_t - P_t/P_{t-1} \end{pmatrix} = e_t \tag{13}$$

where

F_t is a twice differentiable function whose value is a 1×2 vector.

Under the assumption that e_t is distributed according to the multivariate normal probability law, and following the procedure developed in Berndt, Hall, Hall, and Hausman [1], the parameters are estimated by maximizing the concentrated log-likelihood function:

$$L(\beta) = \textstyle\sum_t \log|\det M_t| - ½ \log \det F'F \tag{14}$$

where

M_t denotes the Jacobian matrix of the transformation from the underlying disturbances to the observed random variables is, y_t:

$$M_t = \frac{\partial F_t(y_t, \beta)}{\partial y_t}. \tag{15}$$

The initial values of A and α_1 are set at 1 and 0, respectively. The initial values of the remaining parameters, c_1, c_2, c_3, and α_0 are then estimated based on separate linear OLS estimates of Equations (9) and (10).

The Berndt, Hall, Hall, and Hausman [1] algorithm for determining the parameters that maximize the likelihood function is used. This method is based on an iterative procedure that converges to maximum-likelihood estimators for nonlinear structural econometric models. Their algorithm uses a gradient method to search for the optimal direction of change in the vector of estimated coefficients.

D. Parameter Estimates

Table I presents the econometric estimates of the parameters of the valuation model. The parameters reported in Table I comprise the behavior parameter (A—the coefficient of relative risk aversion) and the parameters of the stochastic processes (ρ and α_1).

Table I

Estimation of CRRA Model's Parameters

$\hat{A}$[a]	$\hat{\sigma}(\hat{A})$[b]	$\hat{\rho}$[c]	α[d]
4.22	0.254	0.846	0.297

[a] $\hat{A}$ = estimate of coefficient of relative risk aversion.
[b] $\sigma(\hat{A})$ = estimate of standard error of A.
[c] $\hat{\rho}$ = estimate of first-order autocorrelation in R_t^m.
[d] $\hat{\alpha}_1$ = estimate of first-order autocorrelation parameter in dividend-dynamics process.

The point estimate of relative risk aversion, A, is 4.22, which is significantly different from zero. This indicates that annual data on aggregate expenditures of nondurables and services is useful in endogenizing discount rate changes. A possible interpretation of this point estimate of A involves the implication for the representative individual's attitude towards a fair gamble. If $\hat{A} = 4.22$, this value of the coefficient of relative risk aversion implies that the representative individual would countenance a consumption penalty of 0.0211% to avoid a gamble over 1% of current consumption.

The $\hat{A}$ estimate of 4.22 is also significantly different from unity. Thus, the null hypothesis that price movements are explained by an aggregate logarithmic utility function is rejected. This implies that there is not a one-to-one correspondence between aggregate consumption and aggregate wealth, which under time additive utility and a nonstationary investment opportunity set is true only under log-utility. This suggests that attempts to measure market values of missing assets for purposes of testing asset pricing models may not improve the predictive ability of the market-oriented CAPM.

Previous empirical studies using different approaches have obtained A estimates ranging from 0.07 to 7.29. Friend and Blume [10] and Blume and Friend [2] used cross-sectional data in a single-period model (assuming risky wealth includes common stocks and human capital) to obtain A estimates ranging from 2.5 to 4.0. While these two studies estimated relative risk aversion using wealth data, other studies have estimated the risk aversion using consumption data. The point estimate of A obtained in this study is not significantly different from the value of A which Grossman and Shiller selected for their analysis: 4.0. Hansen and Singleton [15] used monthly consumption and real stock return data to obtain estimates of A ranging from 0.68 to 0.09.[8] The endogenization of discount rate changes appears to be a less important contributing factor to their models' explanatory power. A possible explanation for their low estimate of relative risk aversion is that the problem of distinguishing between expenditures versus consumption is more critical for monthly data (than for annual data), since even nondurables consumption expenditures are, to a large degree, consumed over a period that exceeds the purchase month. Thus, the endogenization of discount rate changes based on monthly consumption expenditure data may not appropriately characterize utility-based discount rate changes. Brown and Gibbons [6] used monthly data and either unconditional moments or lognormality to yield A estimates ranging from 0.09 to 7.29. Thus, the A estimate of 4.22 presented here falls well within the range of previously obtained estimates of relative risk aversion.

E. Stationarity of the Coefficient of Relative Risk Aversion

To examine the reasonableness of the assumption that the composite individual has a utility function that displays time-invariant constant relative risk aversion, the data were divided into two nonoverlapping samples: 1926–1953 and 1955–1982. The estimates of relative risk aversion for these samples are 4.13 (0.711)

[8] See Ferson [9] for a description of additional empirical estimates of relative risk aversion.

and 5.11 (1.31), with the standard errors of the estimates given in parentheses. The null hypothesis that these independent estimates are equal cannot be rejected at the 10% level. Thus, the assumption of time-invariant constant relative risk aversion appears to be a reasonable basis for a valuation model. The correlation of the predicted price changes with the actual price change is 0.72 over the 1929–1955 period and is 0.67 over the 1955–1982 period. The model correctly predicts the direction of the movement in actual prices 70% of the time over the 1929–1955 period and 68% of the time over the 1955–1982 period. Thus, the model fits the data equally well in both time periods.

III. Comparison with Alternative Methods

This section performs comparisons of the explanatory power of this model with alternative specifications: constant discount rate models and previous authors' analyses. In considering comparisons with alternative models, it is important to bear in mind that the current model encompasses more than a single-equation regression model. The current model comprises Equation (9), the equation relating price appreciation to marginal utility of current and past dividend yields, *as well as* Equation (10), the dividend-dynamics equation. Equation (10) constitutes a restriction on the model, and the system of Equations (9), (10), and (11) represents a constrained-maximization estimation procedure for the parameters of interest.

A. Constant Discount Rate Models

Two constant discount rate models are used as benchmarks for comparing the explanatory power of the utility-based valuation model.

A.1. Risk-Neutral Model

The first constant discount rate model considered is the risk-neutral variant of the previous model; i.e., $A = 0$. The model is then Equation (8) with $A = 0$ imposed:

$$E(R_t \mid I_t) = c_1 + c_2 \frac{D_t}{P_{t-1}} + c_3 \frac{D_{t-1}}{P_{t-1}} \tag{16}$$

with

$$c_1 \equiv \frac{k\beta}{(1-\beta)(1-\beta\phi)}\left(\frac{\beta\delta}{1-\beta} + \alpha_0\right); \quad c_2 \equiv \frac{k\beta(1+\alpha_1-\beta\phi)}{(1-\beta)(1-\beta\phi)};$$

$$c_3 \equiv -\frac{k\beta\alpha_1}{(1-\beta)(1-\beta\phi)}.$$

Thus, a comparison of Equation (8) with Equation (16) would contrast the performance of a risk-averse model that endogenizes discount rate changes with a discount rate model that is identical in other respects—specifically, the dividend-dynamics assumption of an AR(1) or random walk process.

The risk-neutral model implies the following econometric two-equation structural model:

$$R_t = c_1 + c_2 \frac{D_t}{P_{t-1}} + c_3 \frac{D_{t-1}}{P_{t-1}} + \varepsilon_t \tag{17}$$

$$\frac{D_{t+1}}{P_{t-1}} = \frac{D_t}{P_{t-1}} + \alpha_0 + \alpha_1 \left(\frac{D_t}{P_{t-1}} - \frac{D_{t-1}}{P_{t-1}} \right) + u_t. \tag{18}$$

With $A = 0$, two terms combine to make the coefficient of the D_t/P_{t-1} term, c_2, equal to $\beta k(1 + \alpha_1 - \beta\phi)/[(1 - \beta)(1 - \beta\phi)]$. The coefficient for the D_{t-1}/P_{t-1} term, c_3, remains $\beta k \alpha_1/[(1 - \beta)(1 - \beta\phi)]$. Thus, $c_2/c_3 = (1 + \alpha_1 - \beta\phi)/\alpha_1$; since $\beta\phi$ is not independently estimated in this model, c_2/c_3 is now unconstrained.

A.2. Williams-Gordon-Rubinstein Model

The second constant discount rate model is the constant growth[9] model, wherein expected dividends are assumed to be increasing at a constant real growth rate through time:

$$E(D_{t+1} \mid I_t) = (1 + g)D_t$$

or

$$E\left(\frac{D_{t+1}}{P_{t-1}} \;\middle|\; I_t \right) = (1 + g) \frac{D_t}{P_{t-1}}. \tag{19}$$

Substituting (19) into the constant growth valuation model, assuming a constant discount rate, yields

$$E(R_t \mid I_t) = c_1 \frac{D_t}{P_{t-1}} \tag{20}$$

where $c_1 \equiv (1 + g)/[E(R) - g]$ under the Williams-Gordon model. Alternatively, under Rubinstein's formulation,

$$c_1 = \frac{(1 + g) + \mathrm{COV}[g_t, (C_t/C_{t-1})^{-A}]/E[(C_t/C_{t-1})^{-A}]}{R_F - g + \mathrm{COV}[g_t, (C_t/C_{t-1})^{-A}]/E[(C_t/C_{t-1})^{-A}]} \tag{21}$$

where:

$R_F =$ the riskless interest rate
$g_t = D_t/D_{t-1}$ and
$g \equiv E(g_t)$.

These familiar valuation models, which assume a constant discount rate, provide useful benchmarks with which to compare the performance of the current model, which *endogenizes* discount rate changes, in predicting the influence of dividend changes on aggregate stock price movements.

The Williams-Gordon-Rubinstein model implies the following two-equation

[9] See Gordon and Shapiro [11], Williams [25], and Rubinstein [22].

structural model:

$$R_t = c_1 \frac{D_t}{P_{t-1}} + \varepsilon_t \tag{22}$$

$$\frac{D_{t+1}}{P_{t-1}} = c_2 \frac{D_t}{P_{t-1}} + u_t. \tag{23}$$

B. Comparison of Explanatory Power of Alternative Models

B.1. Tests

Table II presents the results of empirical testing of four models. In addition to the risk-averse, risk-neutral, and Williams-Gordon-Rubinstein models previously described, a simple autoregressive model is included. This model involves the regression of R_t on R_{t-1} (and a constant). Thus, this model embodies the "pure autocorrelation" effect that arises because of the use of annual average prices in calculating R_t. This is a naive alternative that does not rely on any fundamental information and is, therefore, denoted as the "Technical Model."

The parameters reported in Table II comprise the parameters of the stochastic processes: ρ, α_1, or g.

Consider now the explanatory power of the alternative models. The first indicator of this explanatory power consists of $CORR(R_t, R_t^m)$, where R_t^m is the model's conditional prediction of the rate of price appreciation.[10] The correlation of the conditional predictions and actual rate of price appreciation is higher for the utility-based model than for the risk-neutral, Williams-Gordon-Rubinstein, and the technical models. Note that the risk-neutral model is identical in all respects to the risk-averse model except that relative risk aversion is constrained to be zero. In a single-equation model consisting of only Equation (9), the imposition of this constraint would be expected to lower the correlation of the predicted and actual price movements. However, in a simultaneous system, a non-zero estimate of relative risk aversion constrains a coefficient in Equation (9) by a parametric estimate, α_1, from the time-series dynamics of the marginal utility of dividends given in Equation (10). Thus, it does not follow *a priori* that the $CORR(R_t, R_t^m)$ would be lower when relative risk aversion is constrained to be zero.[11]

The ability of the alternative models to predict market turning points is also examined. The utility-based model predicted the market turning point a higher proportion of times than any of the alternative models. These empirical results are presented in Table III.

While this evidence is suggestive of the superiority of the risk-averse model over the fundamental valuation models that do not endogenize discount rate changes, it does not lend itself to a rigorous statistical interpretation, because

[10] Formally, $R_t^m \equiv \hat{E}(R_t \mid I_t)$, where $\hat{E}$ denotes the conditional expectations operator using *estimated* parameter values.

[11] Of course, the correlation of error terms in Equations (16) and (17) enters into the estimation process.

Table II

Estimation of Models' Parameters

Model	$\hat{\rho}$[a]	$\hat{\alpha}_1$[b]	$\hat{g}$[c]
Risk-averse	0.846	0.297	NA[d]
Risk-neutral	0.292	0.174	NA
Williams-Gordon-Rubinstein	0.161	NA	0.0145
Technical model	0.161	NA	NA

[a] $\hat{\rho}$ = estimate of first-order autocorrelation in R_t^m.
[b] $\hat{\alpha}_1$ = estimate of first-order autocorrelation parameter in dividend-dynamics process.
[c] $\hat{g}$ = the estimate of $E(g_t)$.
[d] NA = not applicable.

Table III

Comparison of Models' Conditional Predictions[a]

Model	$CORR(R_t, R_t^m)$[b]	$\hat{p}^{m}$[c]
Risk-averse	0.670	0.675
Risk-neutral	0.567	0.611
Williams-Gordon-Rubinstein	0.550	0.555
Technical model	0.162	0.574

[a] $N = 54$, where N is the number of years in the data sample.
[b] $CORR(R_t, R_t^m)$ = the sample correlation between R_t and R_t^m.
[c] $\hat{p}^m$ = the proportion of successes for the model.

the equations were fitted over the same time period used to calculate the correlation coefficients and the proportion of correctly predicted direction movements. The subsequent subsection compares the alternative models based on a twenty-year holdout period (1963–1982).

B.2. Analysis of Holdout Sample

The three models are first fitted over a thirty-four year period (1929–1962), and the estimated parameters are used to obtain the conditional expected rates of price appreciation for 1963. The models are then fitted over the thirty-five year period (1929–1963), and the estimated parameters are used to obtain the conditional expected rate of price appreciation for 1964, etc. The model did not drop off earlier years because the 1930's experienced the greatest variation in the annual growth rate in real consumption, and this variation is important in fitting a nonlinear model.

The accuracy of the alternative models in predicting actual stock prices is compared based on several criteria. The average forecast error is an indication of the models' bias. The standard error of the difference between the actual and the predicted rate of price appreciation is a measure of the efficiency of the alternative models. The root mean square forecast error and the mean absolute forecast error are used as overall measures of predictive accuracy that are influenced by both biasedness and efficiency. The correlation coefficient is a measure of the explanatory power of the alternative models. Finally, a binomial sign test is used to analyze the ability of the alternative model to predict the direction of the movement in the market.

The proportion of the time that each model correctly predicts the actual market movements is tested for significance relative to the diffuse prior that the market will go up in a given period. This diffuse prior correctly predicts the direction of the market 55% of the time. The comparisons are given in Table IV.

The risk-averse model outperforms all alternative models for the twenty-year holdout period (1963–1982). The risk-averse models mean error in the rate of price appreciation is 0.3%, which is 0.5% lower in absolute value than the least biased of the alternative models' values. The standard error of the forecast error, the root mean square forecast error, and the absolute mean forecast error are all lower in the risk-averse model than in any of the alternatives. In addition, the correlation of R_t with R_t^m is highest for the risk-averse model.

The utility-based model also correctly predicts the direction of aggregate stock price movements a greater proportion of the time than the diffuse prior that always predicted an upward market movement. The performance of the risk-neutral and Williams-Gordon-Rubinstein models is similar to the performance of a diffuse prior. The risk-neutral and Williams-Gordon-Rubinstein models both incorporate fundamental information (in the form of aggregate dividends); however, the introduction of such information significantly improves conditional prediction of the direction of aggregate common stock price movements only when discount rate changes are endogenized.

The rate of growth in consumption over the last two years enters into the econometric model for only the risk-averse model. It is likely that the past growth rate in aggregate consumption conveys information about future dividends.

Table IV

Comparisons of Models' Conditional Predictors Based on the 1963–1982 Holdout Period

Criterion	Risk-Averse	Risk-Neutral	Williams-Gordon-Rubinstein	Technical Model	Diffuse Prior
$\bar{U}$[a]	0.003	0.008	0.027	−0.029	NA[b]
$\hat{\sigma}(U_t)$[c]	0.102	0.119	0.116	0.116	NA
RMSFE[d]	0.100	0.117	0.116	0.117	NA
AMFE[e]	0.082	0.097	0.097	0.090	NA
$CORR(R_t, R_t^m)$[f]	0.493	0.156	0.256	0.075	NA
$\hat{p}^m$[g]	70%	55%	55%	55%	55%
P[h]	8.85%	50%	50%	50%	NA

[a] $\bar{U} = (1/T)\ \Sigma_t(R_t - R_t^m)$.

[b] NA = not applicable.

[c] $\hat{\sigma}(U_t) \equiv \hat{\sigma}(R_t - R_t^m)$.

[d] RMSFE = root mean square forecast error.

[e] AMFE = absolute mean forecast error.

[f] $CORR(R_t, R_t^m)$ = the sample correlation between R_t and R_t^m.

[g] $\hat{p}^m$ = proportion of successes for the model. $T = 20$, where T is the number of years in the holdout sample.

[h] P = the probability of obtaining a value of $\hat{p}^m$, or greater, by random chance. These P tests are conditional on $p = 0.55$.

However, Equation (10) defines the process that the marginal utility of dividends follows. The existence of this equation constrains the form that past growth rates enter into Equation (9). Thus, it seems unlikely that the superior predictive ability of the risk-averse model is attributable to the information concerning future dividends that is contained in past consumption growth rates.

B.3. Comparison with Grossman-Shiller

An interesting comparison may be performed with the Grossman and Shiller model. A comparison of price *levels* with the Grossman-Shiller model is inappropriate, since the model in this paper is conditioning on past prices, whereas the G & S model conditions on a perfect foresight information set. However, a comparison of the *rates of price appreciation* may be performed across the two models. The correlation obtained in their model (using stock *price appreciation* rather than prices) was 0.500.[12] Thus, predicted aggregate common stock movements based on conditional expectations have a higher association with actual aggregate stock movements than do the corresponding predictions based on the Grossman and Shiller perfect foresight model. This is attributable to the investor not being clairvoyant and hence using an information set more similar to the econometrician's than to the perfect foresight information set. Nevertheless, our estimate of relative risk aversion, 4.22, is very close to that used by Grossman and Shiller, i.e., 4.0.

IV. Conclusions and Summary

This study has considered the question of whether aggregate real stock return movements can be tracked by a model based on a simple aggregate utility function and an observable information set. The primary assumptions of this model were an aggregate utility function of the CRRA class, and marginal utility of aggregate dividends following a first-order autoregressive process. In addition, the model took explicit cognizance of the temporal aggregation implicit in reported consumption data.

The econometric model gives an estimate of the coefficient of relative risk aversion of 4.22, with an estimated standard error of 0.254. The t-value thus obtained is substantially higher than the t-values for relative risk aversion obtained based on unconditional tests of CRRA models using the method of moments.[13] The correlation of the model rates of return with actual rates of return over a twenty-year holdout period is 0.493, which compares with a correlation of 0.156 for the risk-neutral model, of 0.256 for the Williams-Gordon-Rubinstein constant discount rate model, and of 0.075 based on a simple autoregressive model of aggregate common stock price movements. Over the same holdout period, the utility-based model correctly predicts the direction of aggre-

[12] This correlation was obtained by duplicating their perfect foresight calculation, obtaining their predicted prices, deflating these prices by the previous year's actual price, and then correlating these predicted returns with the true, observed returns.

[13] See, e.g., Hansen and Singleton [15].

gate common stock price movements 70% of the time, which compares with a 55% for the risk-neutral model, for the Williams-Gordon-Rubinstein model, for the simple technical model, and for the diffuse prior of an upward price movement. The endogenization of discount rate changes using a utility-based valuation model and observable consumption data results in superior performance compared to constant discount rate models.

Appendix A

Relationship between Stock Price and Observed Consumption Data

Equation (1) may be rewritten

$$P_t = C_t^A \int_0^\infty \beta^\tau E(C_{t+\tau}^{-A} D_{t+\tau} \mid I_0^*)\, d\tau.$$

Assuming that the subjective rate of time preference changes discontinuously at period's end, the valuation relation implies

$$P_t = \left(\textstyle\sum_{i=1}^{\infty} \beta^{(2i-1)/2} J_{ti}\right)/C_t^{-A}$$

where

$$J_{ti} \equiv \int_{i-1}^{i} E(C_{t+\tau}^{-A} D_{t+\tau} \mid I_t^*)\, d\tau.$$

Approximating C_t^{-A} by a first-order Taylor expansion series,

$$C_t^{-A} \simeq \bar{C}_i^{-A} - A\bar{C}_i^{-A-1}(C_t - \bar{C}_i)$$

where $\bar{C}_i \equiv \int_{i-1}^{i} C_t\, dt$.

Loistl [18] has demonstrated that the interval of convergence for the above expansion is $0 \leq C_t \leq 2\bar{C}_i$. Since spot consumption is unobservable, this constraint cannot be precisely empirically tested; however, it is quite plausible to assume that instantaneous annualized consumption rates never exceed twice the annual average consumption rate.[14]

Thus, using $D_t = D_t + \bar{D}_i - \bar{D}_i$, where $\bar{D}_i \equiv \int_{i-1}^{i} D_t\, dt$,

$$J_{ti} \simeq \int_{i-1}^{i} E\{[\bar{C}_i^{-A} - A\bar{C}_i^{-A-1}(C_t - \bar{C}_i)](\bar{D}_i + D_t - \bar{D}_i) \mid I_t^*\}\, dt$$

$$= E(\bar{C}_i^{-A}\bar{D}_i \mid I_t^*) - A\int_{i-1}^{i} E[\bar{C}_i^{-A-1}(C_t - \bar{C}_i)(D_t - \bar{D}_i) \mid I_t^*]\, dt$$

$$+ \int_{i-1}^{i} E[\bar{C}_i^{-A}(D_t - \bar{D}_i) \mid I_t^*]\, dt - A\int_{i-1}^{i} E[\bar{C}_i^{-A-1}\bar{D}_i(C_t - \bar{C}_i) \mid I_t^*]\, dt.$$

[14] Using post-1959 monthly data, one can certainly verify that annualized *monthly* consumption rates never exceeded twice the annual average.

Assuming that integration and conditional expectation are interchangeable operators, the last two expressions on the RHS of J are zero from the definition of $\bar{C}_i$ and $\bar{D}_i$.

For annual data,

$$\bar{C}_i \equiv \int_{i-1}^{i} C_t \, dt, \qquad \bar{D}_i \equiv \int_{i-1}^{i} D_t \, dt$$

are the measured annual values of (real, per capita) consumption and dividends, respectively. Thus,

$$J_{ti} \simeq E(\bar{C}_i^{-A}\bar{D}_i \mid I_t^*) - AE(\bar{C}_i^{-A}\bar{D}_i H_{1i} \mid I_t^*)$$

where

$$H_{1i} \equiv \int_{i-1}^{i} \left(\frac{C_t}{\bar{C}_i} - 1\right)\left(\frac{D_t}{\bar{D}_i} - 1\right) dt.$$

The interpretation of unconditional expectation, H_{1i}, is the covariability of the spot consumption growth rate (relative to annual average consumption) with the spot dividend growth rate (once again, in relation to the annual average dividends). It is assumed that the process generation intra-year variations in consumption and dividend implies that the magnitude of H_{1i} for a fixed period of time is uncorrelated with $\bar{C}_i^{-A}\bar{D}_\iota$, and that its conditional expectation is an intertemporal constant:

$$\begin{aligned} E(\bar{C}_i^{-A}\bar{D}_i H_{1i} \mid I_t^*) &= E(\bar{C}_i^{-A}\bar{D}_i \mid I_t^*)E(H_{1i} \mid I_t^*) \\ &= E(\bar{C}_i^{-A}\bar{D}_i \mid I_t^*)E(H_{1i}) \end{aligned}$$

where the latter equality follows from the assumed intertemporal stationarity of $E(H_{1i} \mid I_t^*)$.

Then, using $k_1 \equiv E(H_{1i})$,

$$J_{ti} \simeq (1 - k_1 A)E(\bar{C}_i^{-A}\bar{D}_i \mid I_t^*).$$

Substituting for J_{ti} yields

$$P_t \simeq (1 - k_1 A)C_t^A \textstyle\sum_{i=t+1}^{\infty} \beta^{(2\iota-1)/2} E(\bar{C}_i^{-A}\bar{D}_i \mid I_t^*).$$

Define I_t^a to be the set of all *annual* information, i.e., all information which is only known at year's end.[15] This implies $I_t^a = I_{t+\tau}^a \; \forall \tau \in [0,1)$.

Taking the conditional expectation $E(\cdot \mid I_t^a)$:

$$E(P_t \mid I_t^a) \simeq (1 - k_1 A)C_t^A \textstyle\sum_{i=t+1}^{\infty} \beta^{(2i-1)/2} E(\bar{C}_i^{-A}\bar{D}_i \mid I_t^a).$$

For $\tau \in [0,1)$,

$$E(P_{t+\tau} \mid I_{t+\tau}^a) \simeq (1 - k_1 A)C_{t+\tau}^A \textstyle\sum_{i=t+1}^{\infty} \beta^{(2i-1)/2} E(\bar{C}_i^{-A}\bar{D}_\iota \mid I_{t+\tau}^a).$$

[15] For example, in addition to the economic data, such information as total (or average) annual rainfall.

Now, note that $E(\bar{C}_i^{-A}\bar{D}_i \mid I_{t+\tau}^a) = E(\bar{C}_i^{-A}\bar{D}_i \mid I_t^a)$. This follows from $\tau \in [0,1)$ and the definition of $I_{t+\tau}^a$: $I_{t+\tau}^a$ comprises only *annual* information, and since annual data for the year will only be available at year's end, $I_t^a = I_{t+\tau}^a \ \forall\tau \in [0,1)$. Thus,

$$E(P_{t+\tau} \mid I_t^a) = (1 - k_1 A)C_{t+\tau}^A \textstyle\sum_{i=t+1}^{\infty} \beta^{(2i-1)/2} E(\bar{C}_i^{-A}\bar{D}_i \mid I_t^a).$$

Apply the operator $\int_{i-1}^{i} (\cdot)\, dt$, and assume once again the interchangeability of conditional expectation and integration:

$$E(\bar{P}_t \mid I_t^a) = (1 - k_1 A)[\textstyle\sum_{i=t+1}^{\infty} \beta^{(2i-1)/2} E(\bar{C}_i^{-A}\bar{D}_i \mid I_t^a)] \displaystyle\int_{i-1}^{i} C_t^A \, dt.$$

Since, to a first-order approximation, $\int_{i-1}^{i} C_t^A\, dt \simeq \int_{i-1}^{i} [\bar{C}_i^A + A\bar{C}_i^{A-1}(C_t - \bar{C}_i)]\, dt = \bar{C}_i^A$, we then have

$$E(\bar{P}_t \mid I_t^a) = (1 - k_1 A)\bar{C}_t^A \textstyle\sum_{i=t+1}^{\infty} \beta^{(2i-1)/2} E(\bar{C}_i^{-A}\bar{D}_i \mid I_t^a).$$

The next stage constitutes adjustment for consumption goods inflation. By definition,

$$\bar{P}_t \equiv \int_{t-1}^{t} \frac{P_\tau}{q_\tau}\, d\tau = \int_{t-1}^{t} \frac{p_\tau}{q_\tau}\, d\tau.$$

Similarly,

$$\bar{D}_i \equiv \int_{i-1}^{i} \frac{d_t}{q_t}\, dt.$$

Using $\bar{p}_t \equiv \int_{t-1}^{t} p_t\, dt$ and $\bar{q}_t \equiv \int_{t-1}^{t} q_t\, dt$ and expanding $1/q_t$ by a first-order Taylor's series expansion yields:

$$\begin{aligned}
\bar{P}_t &\simeq \int_{t-1}^{t} (p_\tau - \bar{p}_t + \bar{p}_t) \left[\frac{1}{\bar{q}_t} - \frac{1}{\bar{q}_t^2}(q_\tau - \bar{q}_t)\right] d\tau \\
&= \int_{t-1}^{t} \frac{\bar{p}_t}{\bar{q}_t} \left[1 + \left(\frac{p_\tau}{\bar{p}_t} - 1\right)\right]\left[1 - \left(1 - \frac{q_\tau}{\bar{q}_t}\right)\right] d\tau \\
&= \frac{\bar{p}_t}{\bar{q}_t}\left[1 + \int_{t-1}^{t} \left(\frac{p_\tau}{\bar{p}_t} - 1\right)\left(\frac{q_\tau}{\bar{q}_t} - 1\right) d\tau\right] \\
&\equiv \frac{\bar{p}_t}{\bar{q}_t}(1 + H_{2i}).
\end{aligned}$$

Substituting this expression into $E(\bar{P}_t \mid I_t^a)$ yields $E[(\bar{p}_t/\bar{q}_t)(1 + H_{2i}) \mid I_t^a)]$. Following an argument similar to the one made with respect to H_{1i}, assume that H_{2i} is independent of $\bar{p}_t/\bar{q}_t$. The rationale for this assumption lies in the fact that H_{2i} consists of the covariability of intra-year nominal stock returns and consumption goods inflation. Note that both these quantities—nominal stock *returns* and inflation—are *adjusted* for the average annual levels in stock and consumption goods prices.

$$E\left[\frac{\bar{p}_t}{\bar{q}_t}(1 + H_{2i}) \mid I_t^a\right] = E\left(\frac{\bar{p}_t}{\bar{q}_t} \,\middle|\, I_t^a\right) E(1 + H_{2i} \mid I_t^a)$$

$$= E\left(\frac{\bar{p}_t}{\bar{q}_t} \,\middle|\, I_t^a\right) E(1 + H_{2i})$$

$$\equiv \left[E\left(\frac{\bar{p}_t}{\bar{q}_t} \,\middle|\, I_t^a\right)\right](1 + k_2).$$

By an identical argument for nominal dividends,

$$\bar{D}_i \equiv \int_{i-1}^{i} \frac{d_t}{q_t}\, dt \simeq \int_{i-1}^{i} (d_t - \bar{d}_i + \bar{d}_i)\left[\frac{1}{\bar{q}_i} - \frac{1}{\bar{q}_i^2}(q_t - \bar{q}_i)\right] dt$$

$$= \frac{\bar{d}_i}{\bar{q}_i}\left[1 + \int_{i-1}^{i}\left(\frac{d_t}{\bar{d}_i} - 1\right)\left(\frac{q_t}{\bar{q}_i} - 1\right) dt\right]$$

$$\equiv \frac{\bar{d}_i}{\bar{q}_i}(1 + H_{3i}).$$

Thus, substituting into the equation and using $E(H_{3i}) \equiv k_3$ yields

$$(1 + k_2)E(\bar{P}_t \mid I_t^a) = (1 - k_1 A)\bar{C}_t^A \textstyle\sum_{i=t+1}^{\infty} \beta^{(2i-1)/2}(1 + k_3)E(\bar{C}_i^{-A}\bar{D}_i \mid I_t^a)$$

where $\bar{P}_t$ is now *redefined*[16] as the average nominal price deflated by average consumption goods prices, and $\bar{D}_i$ is the total nominal dividends deflated by average consumption goods prices.

The final step constitutes application of the operator $E(\cdot \mid I_t)$, where I_t is the econometrician's information set defined in the text. Clearly, $I_t \subset I_t^a$, and thus application of $E(\cdot \mid I_t)$ is permissible, if it exists. Then, assuming $E(\cdot \mid I_t)$ exists, rewriting the previous equation, and using

$$k \equiv \frac{(1 - k_1 A)(1 + k_3)}{1 + k_2}$$

implies Equation (4). QED

Appendix B

Proof of Reduced Form Equation [Equation (5)]

Define $[s] \equiv (C_{t+s}^{-A} D_{t+s})/(C_t^{-A} P_{t-1}) \mid I_t^*$.

Now, for $s = 2$,

$$E([2] - [1]) = \delta + \phi E([1] - D_t/P_{t-1} \mid I_t^*)$$

$$= \delta + \phi E([1]) - \phi D_t/P_{t-1}.$$

[16] The same symbol was used for parsimonious notation.

For $s = 3$,

$$E([3] - [2]) = \delta + \phi E([2] - [1])$$
$$= \delta + \phi[\delta + \phi E([1]) - \phi D_t/P_{t-1}]$$
$$= \delta(1 + \phi) + \phi^2 E([1]) - \phi^2 D_t/P_{t-1}.$$

For $s = 4$,

$$E([4]) - [3]) = \delta + \phi E([3] - [2])$$
$$\delta + \phi[\delta + \phi\delta + \phi^2 E([1]) - \phi^2 D_t/P_{t-1}]$$
$$= \delta(1 + \phi + \phi^2) + \phi^3 E([1]) - \phi^3 D_t/P_{t-1}.$$

Thus, in general for $s \geq 2$,

$$E([s]) - [s - 1]) = \delta\left(\textstyle\sum_{i=0}^{s-2} \phi^i\right) + \phi^{s-1}E([1]) - \phi^{s-1}D_t/P_{t-1}.$$

Further,

$$E([s]) = \textstyle\sum_{i=2}^{s} E([i]) - [i - 1]) + E([1])$$
$$= \textstyle\sum_{i=2}^{s} \left[\delta \sum_{j=0}^{i-2} \phi^j + \phi^{i-1}E([1]) - \phi^{i-1}\dfrac{D_t}{P_{t-1}}\right] + E([1])$$
$$= \delta \textstyle\sum_{i=2}^{s} \sum_{j=0}^{i-2} \phi^j + E([1])(1 + \sum_{i=2}^{s} \phi^{i-1}) \dfrac{D_t}{P_{t-1}} \sum_{i=2}^{s} \phi^{i-1}.$$

Now, using

$$\textstyle\sum_{i=2}^{2} \sum_{j=0}^{i-2} \phi^i = \dfrac{s - 1}{1 - \phi} - \dfrac{\phi}{1 - \phi}\dfrac{1 - \phi^{s-1}}{1 - \phi}$$

and

$$1 + \textstyle\sum_{i=2}^{s} \phi^{i-1} = 1 + \dfrac{\phi(1 - \phi^{s-1})}{1 - \phi} = \dfrac{1 - \phi^s}{1 - \phi}$$

we then have

$$E([s]) = \delta\left(\frac{s-1}{1-\phi} - \frac{\phi}{1-\phi}\frac{1-\phi^{s-1}}{1-\phi}\right) + \frac{1-\phi^s}{1-\phi}E([1]) - \phi\frac{1-\phi^{s-1}}{1-\phi}\frac{D_t}{P_{t-1}}.$$

Then,

$$R_t/(1-k) = \textstyle\sum_{s=1}^{\infty} \beta^s E([s])$$
$$= \sum_{s=1}^{\infty} \beta^s \left\{\delta\left(\frac{s-1}{1-\phi} - \frac{\phi}{1-\phi}\frac{1-\phi^{s-1}}{1-\phi}\right) + \frac{1-\phi^s}{1-\phi}E([1]) - \phi\frac{1-\phi^{s-1}}{1-\phi}\frac{D_t}{P_{t-1}}\right\}$$

$$= \frac{\delta}{1-\phi}\left(\sum_{s=1}^{\infty} s\beta^s - \sum_{s=1}^{\infty}\beta^s\right) - \frac{\phi\delta}{(1-\phi)^2}\left(\sum_{s=1}^{\infty}\beta^s - \sum_{s=1}^{\infty}\beta^s\phi^{s-1}\right)$$

$$+ \frac{E([1])}{1-\phi}\left(\sum_{s=1}^{\infty}\beta^s - \sum_{s=1}^{\infty}\beta^s\phi^s\right) - \frac{D_t}{P_{t-1}}\frac{\phi}{1-\phi}\left(\sum_{s=1}^{\infty}\beta^s - \sum_{s=1}^{\infty}\beta^s\phi^{s-1}\right)$$

$$= \frac{\delta}{1-\phi}\left[\frac{\beta}{(1-\beta)^2} - \frac{\beta}{1-\beta}\right] - \frac{\phi\delta}{(1-\phi)^2}\left(\frac{\beta}{1-\beta} - \frac{\beta}{1-\beta\phi}\right)$$

$$+ \frac{E([1])}{1-\phi}\left(\frac{\beta}{1-\beta} - \frac{\beta\phi}{1-\beta\phi}\right) - \frac{D_t}{P_{t-1}}\frac{\phi}{1-\phi}\left(\frac{\beta}{1-\beta} - \frac{\beta}{1-\beta\phi}\right).$$

Simplifying and combining terms yields (5).

Appendix C

Calculation of Empirical Proxies for Dividends and Prices

Define

$VWRETD_j$ = total rate of return, including dividends, on value-weighted NYSE portfolio during month j.

$VWRETX_j$ = total rate of return, excluding dividends, on value-weighted NYSE portfolio during month j.

$$INDEX_i \equiv \begin{cases} 1 & \text{for } i = 1 \\ \prod_{j=2}^{i}(1 + VWRETX_i) & \text{for } i \geq 2 \end{cases}$$

$B \equiv$ base for standardizing price and dividend series defined below. Then

$$d_t = \frac{1}{B}\sum_{j=2}^{13} INDEX_{12(t-1)+j-1}(VWRETD_{12(t-1)+j} - VWRETX_{12(t-1)+j}).$$

The nominal dividend series thus obtained excludes effects of intra-year compounding (or reinvestment) of dividends. Real dividends, D_t, are then given by d_t/q_t.

The remaining empirical datum is the real price series, P_t. Using $INDEX_i$ as previously defined, nominal prices, p_t, are given by

$$p_t = \frac{1}{13B}\sum_{j=1}^{13} INDEX_{12t-j+2}.$$

Thus, p_t is the average nominal price level of stocks in year t. The real price level, P_t, is then given by $P_t \equiv p_t/q_t$. Finally, to allow comparability with other data, P_t was standardized to have a value equal to the S & P 500 average in 1982. While this standardization permits comparability with the Standard & Poor's price series, it in no way reduces the generality of the analysis. Indeed, B is arbitrary. Thus, $P_{1982} = 119.71$, and all other P_t and D_t were scaled up accordingly.

Appendix D

Time-Series Diagnostic Tests

Table D.I

Autocorrelation Function for $\Delta(C_{t+1}/C_t)^{-A}D_{t+1}$

Lag	$A=0$	$A=1$	$A=4$	$A=6$	$A=8$
1	0.200	0.239	0.061	−0.234	−0.326*
2	−0.208	−0.224	−0.045	0.065	0.115
3	−0.162	−0.151	−0.169	−0.198	−0.210
4	−0.101	−0.113	−0.124	−0.105	−0.094
5	−0.071	−0.111	−0.114	−0.060	−0.011
6	−0.063	0.006	0.035	0.009	0.002

* Significant at the two standard error level.

Table D.II

Partial Autocorrelation Function for $\Delta(C_{t+1}/C_t)^{-A}D_{t+1}$

Lag	$A=0$	$A=1$	$A=4$	$A=6$	$A=8$
1	0.200	0.239	0.061	−0.234	−0.326
2	−0.258	−0.298	−0.049	0.011	0.010
3	−0.065	−0.012	−0.176	−0.190	−0.190
4	−0.113	−0.151	−0.155	−0.212	−0.252
5	−0.088	−0.101	−0.166	−0.149	−0.136
6	−0.099	−0.003	−0.047	−0.098	−0.101

Table D.III

Autocorrelation Function for Error Term in

$$\left(\frac{C_{t+1}}{C_t}\right)^{-A}\frac{D_{t+1}}{P_{t-1}} - \left(\frac{C_t}{C_{t-1}}\right)^{-A}\frac{D_t}{P_{t-1}} = \alpha_0 + \alpha_1\left[\left(\frac{C_t}{C_{t-1}}\right)^{-A}\frac{D_t}{P_{t-1}} - \left(\frac{C_{t-1}}{C_{t-2}}\right)^{-A}\frac{D_{t-1}}{P_{t-1}}\right] + \varepsilon_{t+1}$$

Lag	$A=0$	$A=1$	$A=4$	$A=6$	$A=8$
1	0.030	−0.281*	−0.090	−0.116	−0.109
2	−0.252	0.033	−0.065	0.047	0.093
3	−0.121	−0.210	−0.160	−0.234	−0.274*
4	−0.094	−0.205	−0.137	−0.190	−0.222
5	0.042	0.227	−0.113	−0.040	0.003
6	−0.099	−0.143	−0.012	−0.026	−0.021

* Significant at the two standard error level.

Table D.IV

Partial Autocorrelation Function for Error Term in

$$\left(\frac{C_{t+1}}{C_t}\right)^{-A}\frac{D_{t+1}}{P_{t-1}} - \left(\frac{C_t}{C_{t-1}}\right)^{-A}\frac{D_t}{P_{t-1}}$$

$$= \alpha_0 + \alpha_1\left[\left(\frac{C_t}{C_{t-1}}\right)^{-A}\frac{D_t}{P_{t-1}} - \left(\frac{C_{t-1}}{C_{t-2}}\right)^{-A}\frac{D_{t-1}}{P_{t-1}}\right] + \varepsilon_{t+1}$$

Lag	$A = 0$	$A = 1$	$A = 4$	$A = 6$	$A = 8$
1	0.030	−0.281	−0.090	−0.116	−0.109
2	−0.253	−0.050	−0.074	0.034	0.083
3	−0.111	−0.233	−0.175	−0.228	−0.260
4	−0.164	−0.382	−0.184	−0.258	−0.303
5	−0.019	0.020	−0.193	−0.099	−0.022
6	−0.201	−0.192	−0.132	−0.109	−0.068

REFERENCES

1. E. K. Berndt, B. H. Hall, R. E. Hall, and J. A. Hausman. "Estimation and Inference in Nonlinear Structural Models." *Annals of Economic and Social Measurement* 3 (October 1974), 653–65.
2. M. Blume and I. Friend. "The Asset Structure of Individual Portfolios and Some Implications for Utility Functions." *Journal of Finance* 30 (May 1975), 585–604.
3. Douglas Breeden. "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities." *Journal of Financial Economics* 7 (September 1979), 265–96.
4. ———. "Biases in Some Estimates of Consumption Betas." Research Paper 632, Graduate School of Business, Stanford University, February 1983.
5. ——— and Robert Litzenberger. "Prices of State Contingent Claims Implicit in Option Prices." *Journal of Business* 51 (October 1978), 621–51.
6. David P. Brown and Michael R. Gibbons. "A Simple Econometric Approach for Utility-Based Asset Pricing Models." *Journal of Finance* 40 (June 1985), 359–81.
7. John Cox, Jon Ingersoll, and Stephen Ross. "A Theory of the Term Structure of Interest Rates." *Econometrica* 53 (May 1985), 385–407.
8. Kenneth Dunn and Kenneth Singleton. "Modeling the Term Structure of Interest Rates under Nonseparable Utility and Durability of Goods." Working Paper, Graduate School of Industrial Administration, Carnegie-Mellon University, June 1984.
9. W. E. Ferson. "Expectations of Real Interest Rates and Aggregate Consumption: Empirical Tests." *Journal of Financial and Quantitative Analysis* 18 (December 1983), 477–97.
10. I. Friend and M. Blume. "The Demand for Risky Assets." *American Economic Review* 65 (December 1975), 900–22.
11. M. J. Gordon and E. Shapiro. "Capital Equipment Analysis: The Required Rate of Profit." *Management Science* 3 (October 1956), 102–110.
12. Sanford J. Grossman and Robert J. Shiller. "The Determinants of the Variability of Stock Market Prices." *American Economic Review* 71 (May 1981), 222–27.
13. ———. "Consumption Correlatedness and Risk Measurement in Economies with Non-Traded Assets and Heterogeneous Information." *Journal of Financial Economics* 10 (July 1982), 195–210.
14. N. Hakansson. "Optimal Investment and Consumption Under Risk for a Class of Utility Functions." *Econometrica* 50 (September 1970), 587–607.
15. Lars P. Hansen and Kenneth J. Singleton. "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models." *Econometrica* 50 (September 1982), 1269–86.

16. ———. "Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Asset Returns." *Journal of Political Economy* 91 (April 1983), 249–65.
17. A. Kraus and Robert Litzenberger. "Market Equilibrium in a Multiperiod State Preference Model with Logarithmic Utility." *Journal of Finance* 30 (December 1973), 1213–27.
18. Otto Loistl. "The Erroneous Approximation of Expected Utility by Means of a Taylor's Series Expansion: Analytic and Computational Results." *American Economic Review* 66 (December 1976), 904–10.
19. Robert Lucas, Jr. "Asset Prices in an Exchange Economy." *Econometrica* 46 (November 1978), 1429–46.
20. Robert Merton. "Optimum Consumption and Portfolio Rules in a Continuous Time Model." *Journal of Economic Theory* 3 (December 1971), 373–418.
21. M. Rothschild and J. Stiglitz. "Increasing Risk II: Its Economic Consequences." *Journal of Economic Theory* 3 (March 1971), 66–84.
22. Mark Rubinstein. "The Valuation of Uncertain Income Streams and the Pricing of Options." *Bell Journal of Economics and Management Science* 7 (Autumn 1976), 407–25.
23. ———. "The Strong Case for the Generalized Logarithmic Utility Model as the Premier Model of Financial Markets." In H. Levy and M. Sarnat (eds.), *Financial Decision-Making Under Uncertainty.* NY: Academic Press, 1977.
24. Paul Samuelson. "Lifetime Portfolio Selection by Dynamic Stochastic Programming." *Review of Economics and Statistics* 51 (August 1969), 239–46.
25. J. B. Williams. *Theory of Investments.* MA: Harvard University Press, 1938.