The Effects of Different Taxes on Risky and Risk-free Investment and on the Cost of Capital

YU ZHU and IRWIN FRIEND*

ABSTRACT

This paper analyzes the major factors which determine the effects of taxation on the value of risky assets and on the cost of capital, and shows how the magnitudes and even the signs of these effects depend on the values assumed for a few key parameters in the model. Using plausible values for these parameters, it is shown that the results obtained are frequently counter-intuitive.

THE EFFECTS OF CHANGES in different types of tax rates on the economy, especially on the personal and corporate savings rates, business investment, the required rate of return on capital assets, and the composition of investor portfolios are matters of obvious interest to both economists and policy makers. Unfortunately, frequently under uncertainty, these effects are quite different from those obtained under certainty assumptions and as a result are often counter-intuitive. Thus, a reduction in personal or corporate taxes with a symmetric tax treatment of gains and losses would increase the after-tax cash flow of return from stock for individual investors, but at the same time the variance of the return and, therefore, the required rate of return would also increase, so that the effects of tax change may be ambiguous.[1] In recent studies, Friend and Hasbrouck [7] and Blume and Friend [2] used a continuous time model of expected utility maximization to ascertain the effects of a constant percentage reduction in personal and corporate taxes, respectively, on investment in risky and riskless assets.

These earlier papers found that, under plausible assumptions about the parameters of the model they used, the immediate effects of lower corporate tax rates would be a reduction in the firm's before-tax cost of capital and an increase in the after-corporate-tax return required by investors. The reduction in personal tax rates, however, would be associated with lower stock prices and an increase in the before-corporate-tax cost of capital. These papers do not allow for the existence of nonmarketable human wealth nor for different personal tax rates applicable to income from risky and risk-free assets, although they do allow for different tax rates for different wealth or income levels.

It is interesting to see to what extent the above-mentioned findings would hold when these more realistic complications are introduced, and this constitutes the main purpose of this paper. The introduction of both human wealth and varying tax rates would be expected to affect significantly the estimated impact of income

* Both authors are from the Wharton School, University of Pennsylvania.

[1] The assumption of symmetric taxation made in this paper (and in the earlier papers referred to) is, of course, only a rough approximation of the current tax structure. But such symmetry is generally regarded as a desirable feature of tax reform.

taxation on the relative demand for risky assets unless the covariance between returns on human and nonhuman wealth and the variation in tax rates applicable to different assets are very small. This paper differs from the earlier papers not only in incorporating human wealth and differential tax rates, in a model explaining the qualitative effects of changes in corporate and personal taxes, but also in measuring the quantitative effects of the tax changes on capital asset valuation. In this paper, a wide range of plausible assumptions is made about the values of the relevant variables treated as exogenous. In addition, this paper considers some long-run implications of changes in personal as well as corporate income tax rates.

The paper is organized as follows. In Section I, the optimization model of demand for risky assets is derived with the existence of n risky assets, one risk-free asset, and human wealth. Section II analyzes the effects of changes in corporate and personal tax rates on this model incorporating human wealth. Section III expands the model to include different personal tax rates on incomes from risky and risk-free assets. Section IV employs some realistic data to illustrate the results of the extended models and their implications. Section V concludes the paper.

I. The Market Price of Risk With Allowance for Human Wealth

Assume that there are three sectors in the economy: government; firms; and investors. The investors are characterized by multiperiod utility functions which exhibit constant proportional risk aversion, although not necessarily with the same risk aversion for each investor. The investors can invest their wealth in n risky marketable assets and one risk-free asset. The risky assets have before-corporate-tax rates of return, $r_1, \cdots, r_n$, which are governed by Wiener processes

$$dr_j = Er_j dt + \sigma_j y_j \sqrt{dt};\ y_j \sim N(0, 1), j = 1, \cdots, n. \tag{1}$$

The risk-free rate is r_f. Investor k also has nonmarketable human wealth, H_k, with rate of return, r_{rh},[2]

$$dr_{kh} = Er_{kh} dt + \sigma_{kh} y_{kh} \sqrt{dt};\ y_{kh} \sim N(0\ \cdot, 1). \tag{2}$$

The random disturbances are correlated in the following way:

$$\text{Cov}(y_i, y_j) = \rho_{ij};\ \text{Cov}(y_j, y_{kh}) = \rho_{j,kh}. \tag{3}$$

The government taxes the investors at personal tax rate, t_{pk}, and taxes firms at corporate tax rate, t_c. All of the corporate taxes are borne by the investors. Both the corporate and personal taxes are symmetric, i.e., the government would give credits for losses. The personal tax rate is assumed to be determined only by an investor's wealth or income.

Under the above assumptions, the k$^{\text{th}}$ investor's wealth dynamics at time, t, can be written as

$$\begin{aligned} dW_{k,t+dt} = W_{kt}\{(1 - t_k)[(1 - h_k)(r_f dt + \textstyle\sum_j \alpha_{jk}[(Er_j(1 - t_c) - r_f)dt \\ + (1 - t_c)\sigma_j y_j \sqrt{dt}]) + h_k(Er_{kh} dt + \sigma_{kh} y_{kh} \sqrt{dt})]\}, \end{aligned} \tag{4}$$

[2] H_k is assumed exogenous. We use r_{kh} instead of wage flow simply for convenience.

where W_{kt} is the k[th] investor's wealth level at time t, $W_{kt} > 0$. α_{jk} is the proportion of the k[th] investor's total marketable assets invested in the j[th] asset. h_k is the proportion of the k[th] investor's human wealth in his total wealth.

Let $u(W_{kt})$ be the k[th] investor's utility function. Assume $u'(W_{kt}) > 0$, $u''(W_{kt}) < 0$, and the Arrow-Pratt measure of relative risk aversion

$$C_k = -W_{kt}\left(\frac{u''(W_{kt})}{u'(W_{kt})}\right) \tag{5}$$

is independent of time, t. Since investors are risk-averse, we assume that $C_k > 0$ for all investors. The investor is to maximize his expected utility in the next period by choosing an optimal portfolio $\{\alpha_{1k}, \cdots, \alpha_{nk}\}$, i.e.,

$$\text{Max}_{\{\alpha_{ik}\}} Eu\ (W_{k,t+dt}).$$

To solve this optimization problem, we use basically the same methodology as in Friend and Blume [5], except that here we are dealing with n risky assets instead of one.[3] We have

$$Er_i(1-t_c) - r_f = C_k(1-t_k)[(1-h_k)(1-t_c)^2 \textstyle\sum_j \alpha_{jk}\sigma_{ij} + h_k(1-t_c)\sigma_{i,kh}]$$

$$i = 1, \cdots, n. \tag{6}$$

After aggregation, we obtain a micro-level equilibrium relationship between the market price of risk and the demand for risky assets, as shown in Equation (7):

$$\frac{E(r_m)(1 - t_c) - r_f}{\sigma_m^2(1 - t_c)(1 - t_p)} = [(1 - h)(1 - t_c)\alpha + h\beta_{hm}]C, \tag{7}$$

where Er_m and σ_m^2 are, respectively, the expected value and variance of the return from the market portfolio, $(1 - t_p)$ and C are the harmonic means of the corresponding variables, h is the ratio of total human wealth to the total wealth in the economy, α is the proportion of investment in the risky assets for the economy as a whole, and $\beta_{hm} = (\text{cov}(r_h, r_{hm})/\sigma_m^2)$. This equation shows that, although individual investors hold different portfolios from each other as a result of the existence of human wealth, the equilibrium market price of risk is independent of individuals' unique characteristics. Equation (7) is the basic equation to be used to analyze the effects of reduction of taxes on investment decisions in the following sections. We can see clearly from (7) that the market price of risk would increase if the correlation between the rate of return on human wealth and the rate of return on the market portfolio increases.

II. The Effects of Corporate and Personal Income Tax Rate Changes

In order to study the tax effects, we shall make some further simplifying assumptions about the firm (corporate) sector. We assume that corporations are fully equity financed, and their liabilities constitute the only risky assets in the economy. Firms in turn own productive machines which produce single con-

[3] The methodology used in Friend and Blume [5] is a direct and simple way to derive the results. The implication of this approach is that the investor is only concerned with his wealth level in the next period. For the derivation and an alternative approach, see [15].

sumption goods. We also assume that the machines have no further productive capability and are, therefore, valueless after a fixed amount of time, but up to that point their productivity remains unchanged. Thus, the scale of firms can be adjusted over time either upward or downward. Let V_m be the total market value of existing marketable risky assets, V_f the total value of existing riskless assets, and H the total value of human wealth. In the above model, V_m is interpreted as the total market value of corporate capital. Suppose in the period considered the firms' physical capital is fixed, but some exogenous changes such as tax reform can alter the valuation of the existing capital in the short run. In the long run, the value of all productive machines will be their replacement value, but in the short run there is no guarantee that the market value of these machines will be the same as their replacement value.

To see the impact of change in corporate taxes in the short run, we rewrite (7) as

$$E(R) = \frac{r_f V_m}{1 - t_c} + \sigma_R^2(1 - t_p)C\left[\frac{(1 - t_c) + H\beta_{hR}}{V_m + V_f + H}\right], \tag{8}$$

where R is the total output of existing capital with mean and variance $E(R)$ and σ_R^2. Clearly,

$$E(R) = E(r_m)V_m$$

$$\sigma_R^2 = \sigma_m^2 V_m^2.$$

β_{hR} is the beta coefficient between the total output and the rate of return on human wealth

$$\beta_{hR} = \frac{\mathrm{Cov}(R, r_h)}{\sigma_R^2} = \frac{\beta_{hm}}{V_m}.$$

In the short run, we can assume that exogenous changes will not alter the distribution of R. To see the effect of a corporate tax rate change, we implicitly differentiate V_m in expression (8) with respect to t_c, keeping $E(R)$, σ_R^2, and β_{hR} constant. We then have

$$\frac{\partial V_m}{\partial t_c} = -\frac{\dfrac{r_f}{1-t_c} - C\sigma_m^2(1-t_p)(1-t_c)\alpha_m}{\dfrac{r_f}{1-t_c} - C\sigma_m^2(1-t_p)(1-t_c)\alpha_m^2\left[1 + \dfrac{H\beta_{hm}}{V_m(1-t_c)}\right]} \cdot \frac{V_m}{1-t}, \tag{9}$$

where $\alpha_m = V_m/(V_m + V_f + H)$, i.e., α_m is the proportion of the sum of firms' equities in the total wealth of the economy. Equation (9) represents the effect of a corporate tax change in the valuation of existing capital. If $\partial V_m/\partial t_c$ is positive, then the reduction in corporate taxes would reduce the value of the existing capital, so that the investment would shift away from risky assets. If $\partial V_m/\partial t_c$ is negative, the impact would be reversed.

Two sets of sufficient conditions for $\partial V_m/\partial t_c$ to be negative are as follows:

I.

$$\frac{r_f}{1-t_c} > \sigma_m^2 C(1-t_p)(1-t_c)\alpha_m \tag{10a}$$

and

$$\delta \equiv \alpha_m \left[1 + \frac{H\beta_{hm}}{V_m(1-t_c)}\right] < 1 \tag{10b}$$

or

II.

$$\frac{r_f}{1-t_c} < \sigma_m^2 C(1-t_p)(1-t_c)\alpha_m \tag{11a}$$

and

$$\delta \equiv \alpha_m \left[1 + \frac{H\beta_{hm}}{V_m(1-t_c)}\right] > 1. \tag{11b}$$

Equation (10a) says that the numerator on the right-hand side of (9) is positive. Equation (10b) indicates that the denominator on the right-hand side is also positive. The other set of conditions can be explained in a similar way. In the absence of human wealth, $\alpha_m = \alpha$, $h = 0$. Equation (9) becomes

$$\frac{\partial V_m}{\partial t_c} = -\frac{\dfrac{r_f}{1-t_c} - C\sigma_m^2(1-t_p)(1-t_c)\alpha}{\dfrac{r_f}{1-t_c} - C\sigma_m^2(1-t_p)(1-t_c)\alpha^2} \cdot \frac{V_m}{1-t_c}.$$

This expression is identical with Equation (6) in Blume and Friend [2]. Incorporating human wealth into the model will not change the direction of the inequality (10a) or (11a).[4] But (10b) could be violated if the covariance between the rate of return of human wealth and the rate of return of the market portfolio is positive and relatively large. We will discuss this problem further using some plausible data in Section IV.

The investors' required rate of return after corporate taxes is $(1 - t_c)E(r_m)$. In our model, we assume that $E(R)$ is constant in the short run. Hence, the effect of tax rate change on the required rate of return can be shown as follows:

$$\begin{aligned}\frac{\partial}{\partial t_c}\left[(1-t_c)\frac{E(R)}{V_m}\right] &= E(r_m)\left[-\frac{1-t_c}{V_m}\frac{dV_m}{dt_c} - 1\right] \\ &= E(r_m)\left[\frac{(r_f/(1-t_c)) - C\sigma_m^2(1-t_p)(1-t_c)\alpha_m}{(r_f/(1-t_c)) - C\sigma_m^2(1-t_p)(1-t_c)\alpha_m \cdot \delta} - 1\right].\end{aligned} \tag{12}$$

[4] Recall that $\alpha = V_m/(V_m + V_f)$. So, $C\alpha_m = C(1 - h)\alpha$. It should be noted that the base estimate of $C(1 - h)$ remains the same in this model. See Friend and Hasbrouck [6].

The sign of a change in the required rate of return after corporate taxes when that tax rate increases is the same as the sign of the product of

$$(\delta - 1) \quad \text{and} \quad \left[\frac{r_f}{1-t_c} - C\sigma_m^2(1-t_p)(1-t_c)\alpha_m \cdot \delta\right].$$

If the first set of sufficient conditions (10a) and (10b) is satisfied, then (12) is negative. This means that a reduction in the corporate tax rate would increase the after-tax required rate of return.

In the long run, the amount of physical capital can no longer be assumed constant. The value of the capital should be equal to its replacement cost. To determine the effect of tax rate change in the long run, we assume that $E(r_m)$, σ_m^2, and β_{hm} (but obviously not α) are constant. From (7), we have

$$E(r_m) = \frac{r_f}{1-t_c} + \sigma_m^2 C(1-t_p)[(1-h)(1-t_c)\alpha + h\beta_{hm}]. \tag{13}$$

Implicitly differentiating (13) with respect to t_c, we obtain[5]

$$\frac{\partial\alpha}{\partial t_c} = -\frac{(r_f/(1-t_c)) - \sigma_m^2 C(1-t_p)(1-t_c)\alpha_m}{\sigma_m^2 C(1-t_p)(1-t_c)(1-h)}. \tag{14}$$

Expression (14) shows that if (10a) is satisfied, then $d\alpha/dt_c$ will be negative. This means that a reduction in the corporate tax rate would increase the proportion of investment in marketable risky assets. It is interesting to notice that in the long run, the existence of human wealth does not change the impact on the investment decision caused by tax reduction, whereas it may in the short run.

Using the same methodology as above, we can discuss the impact of changes in personal income tax rate. The immediate effect is the change in the market value of existing capital:

$$\frac{\partial V_m}{\partial t_p} = \frac{C(1-t_c)\sigma_m^2\left[1 + \dfrac{H\beta_{hm}}{V_m(1-t_c)}\right]V_m}{\dfrac{r_f}{1-t_c} - \sigma_m^2 C(1-t_p)(1-t_c)\alpha_m^2\left[1 + \dfrac{H\beta_{hm}}{V_m(1-t_c)}\right]}. \tag{15}$$

Therefore, if inequalities (10a) and (10b) hold, (15) is positive. This means that a higher personal income tax rate would be associated with a rise in stock prices assuming the symmetry of tax effects. Setting $H = 0$ in (15), we get an expression reflecting the effect of the change in income tax rates when only nonhuman wealth (and the associated income) is taken into consideration:

$$\partial V_m/\partial t_p = \frac{(1-t_c)\sigma_m^2 C V_m}{(r_f/(1-t_c)) - (1-t_p)(1-t_c)\sigma_m^2 C\alpha^2}, \tag{16}$$

which is identical to Equation (8) in Friend-Hasbrouck [7].

[5] To analyze the long-run effect, we have to assume that h is independent of tax rates. This is, of course, a strong assumption.

The long-run effect can be expressed in the following equation:

$$\partial\alpha/\partial t_p = \frac{(1 - h)(1 - t_c)\alpha + h\beta_{hm}}{(1 - h)(1 - t_p)(1 - t_c)}. \tag{17}$$

Equation (17) is always positive if β_{hm} is positive or zero.

III. Model with Different Personal Income Tax Rates and Human Wealth

In order to make the model more realistic, we now introduce personal income tax rates which differ as between risky marketable assets as a whole and all other assets, reflecting the different effective tax rates on capital gains and ordinary income. Assume for investor k the tax rate on the returns from marketable risky assets is t_{sk} and the tax rate on the returns from risk-free assets and human wealth is t_{fk}. Both tax rates are assumed to be symmetric. Under the same set of assumptions as in Section I, the wealth dynamics can be written as follows

$$\begin{aligned} dW_{k,t+dt} = {} & W_{kt}\{(1 - h_k)[\textstyle\sum_j \alpha_{jk}(Er_j dt + \sigma_j y_j \sqrt{dt})(1 - t_c)(1 - t_{sk}) \\ & + (1 - \textstyle\sum_j \alpha_{jk})r_f dt(1 - t_{fk})] + h_k(Er_{kh}dt + \sigma_{kh}y_{kh}\sqrt{dt})(1 - t_{fk})\}. \end{aligned} \tag{18}$$

The investor maximizes his next period expected utility of wealth by choosing optimal coefficients of his investment portfolio α_{jk}, $j = 1, \cdots, n$. The first-order conditions for the maximization are

$$\begin{aligned} & Er_i(1 - t_c)(1 - t_{sk}) - r_f(1 - t_{fk}) \\ & = C_k(1 - t_c)(1 - t_{sk})[(1 - h_k)(1 - t_c)(1 - t_{sk}) \textstyle\sum_j \alpha_{jk}\sigma_{ij} + h_k(1 - t_{fk})\sigma_{i,kh}] \\ & \qquad i = 1, \cdots, n. \end{aligned} \tag{19}$$

Equation (19) is more complicated than Equation (6), hence the aggregation is more difficult. But under some plausible assumptions, we can obtain the following approximate equation:[6]

$$\frac{Er_m(1 - t_c)(1 - t_s) - r_f(1 - t_f)}{C\sigma_m^2(1 - t_c)^2(1 - t_s)^2} = \alpha(1 - h) + \frac{(1 - t_f)h\beta_{hm}}{(1 - t_s)(1 - t_c)}, \tag{20}$$

where $(1 - t_s)$ is the harmonic mean of $(1 - t_{sk})$, and t_f is the arithmetic mean of t_{fk}. It can be readily shown by the same approach as in the previous section that

$$\frac{\partial V_m}{\partial t_c} = -\frac{\dfrac{r_f(1-t_f)}{(1-t_c)(1-t_s)} - C\sigma_m^2(1-t_s)(1-t_c)\alpha_m}{\dfrac{r_f(1-t_f)}{(1-t_c)(1-t_s)} - C\sigma_m^2\left[(1-t_c)(1-t_s) + \dfrac{H}{V_m}(1-t_f)\beta_{hm}\right]\alpha_m^2} \cdot \frac{V_m}{1-t_c}. \tag{21}$$

Notice in (20) that t_c and t_s are in symmetric positions. So, by interchanging t_c

[6] See [15].

and t_s in Equation (21), we get

$$\frac{\partial V_m}{\partial t_s} = -\frac{\dfrac{r_f(1-t_f)}{(1-t_c)(1-t_s)} - C\sigma_m^2(1-t_s)(1-t_c)\alpha_m}{\dfrac{r_f(1-t_f)}{(1-t_c)(1-t_s)} - C\sigma_m^2\left[(1-t_s)(1-t_c) + \dfrac{H}{V_m}(1-t_f)\beta_{hm}\right]\alpha_m^2} \cdot \frac{V_m}{1-t_s}. \tag{22}$$

Finally, the effect of the change in t_f, which is the personal tax rate on the incomes from risk-free assets and human wealth, can be written as

$$\frac{\partial V_m}{\partial t_f} = \frac{\dfrac{r_f V_m}{(1-t_c)(1-t_s)} + C\sigma_m^2 H\beta_{hm}\alpha_m}{\dfrac{r_f(1-t_f)}{(1-t_c)(1-t_s)} - C\sigma_m^2\left[(1-t_s)(1-t_c) + \dfrac{H}{V_m}(1-t_f)\beta_{hm}\right]\alpha_m^2}. \tag{23}$$

IV. Numerical Illustrations

In this section, we use some realistic data to illustrate the results we obtained in the previous sections on the effects of tax rate changes. The focus is on the short-run effect, i.e., the change in the current market value of capital with respect to the changes in tax rates.

First, let us examine the model in Section II which does not distinguish between the different personal tax rates on risky and risk-free incomes. Assume the average corporate tax rate, $t_c = 0.45$, the (harmonic) average personal income tax rate $t_p = 0.25$, and the standard deviation of annual market return $\sigma_m = 0.15$—all rough estimates but at least reasonable orders of magnitude. Let the ratio of marketable risky assets to total marketable assets $\alpha = V_m/(V_m + V_f) = 0.85$, again a reasonable approximation of reality. So given the risk-free rate, the coefficient of relative risk aversion of investors, the ratio of human wealth to the total wealth in the economy, and the beta coefficient relating the return on human wealth to the market rate of return, we can estimate all the derivatives which reflect the effects of tax rate changes on the investment decisions. The calculation shows that if the investors' (harmonic) average relative risk aversion, $C = 6$, with human wealth (see Friend and Blume [5] and Friend and Hasbrouck [6]), then for $r_f > 0.01$, $0 < \beta_{hm} < 0.5$, both conditions (10a) and (10b) are satisfied.[7] Hence, for a fairly wide range of plausible data, $\partial V_m/\partial t_c$ is negative, while $\partial V_m/\partial t_p$ is positive.

Defining the corporate tax rate elasticity of the value of capital as

$$e_{tc} = \frac{t_c}{V_m}\frac{\partial V_m}{\partial t_c}$$

[7] Fama [3] suggested that the real risk-free interest rate is in the neighborhood of 0.01, although it clearly has been much higher in the United States for a number of years. Fama and Schwert [4] presented some evidence showing that the relationship between the return on human capital and that on the marketable assets is weak. On the other hand, Friend and Blume [5] provided evidence that the covariance between the returns on human wealth and the market is significantly positive.

and the personal income tax rate elasticity as

$$e_{tp} = \frac{t_p}{V_m} \frac{\partial V_m}{\partial t_p},$$

Table I shows that the absolute values of e_{tc} and e_{tp} increase with β_{hm}. For a given risk-free rate, this means that the higher the covariance between the returns on human wealth and on the market, the more sensitive the market value of capital with respect to the tax rate changes. From Table I, it seems that introducing human wealth into the model does not change the sign of the elasticities. It is interesting to note that, for a given set of parameters, the sensitivity of corporate tax rate change increases with the risk-free rate while the sensitivity of personal tax rate change decreases with the risk-free rate.

In the model with different personal tax rates on incomes from risky and risk-free assets, we assume that the personal tax rate on the income from stocks is $t_s = 0.20$ and the personal tax rate on risk-free assets is $t_f = 0.30$.[8] By Equations (21) to (23), we can calculate the elasticities of the market value of capital with respect to t_c, t_s, and t_f. The results are shown in Table II. We can see that e_{tc} and e_{ts} have the same sign which is negative if r_f is greater than 0.0105. For the given set of parameters, e_{tf} is always positive. Thus, under the parameters assumed, capital gain taxation or the lower taxation associated with risky assets has a qualitatively different effect on the prices of stocks (and other risky assets) than the taxation of income from risk-free asset and from human wealth.

A notable feature of the above numerical illustration is that the changes in corporate and personal income tax rates have opposite effects on the market value of capital. The reason for that is basically the asymmetry of the tax structure; i.e., the corporate income tax is only imposed on income from risky marketable assets, while the personal income tax is imposed on both risky and riskless income.

To get further insight into the different effects of corporate and personal taxes, let us look at Equation (8) again. Equation (8) is, in fact, a market equilibrium equation. $E(R)$ is the expected total cash flow from the capital capacity of all firms, which is assumed not to be changed in the short run. The right-hand side is the required total return (before taxes) from risky marketable assets. The required total return consists of two parts: the certainty return, R_c, which is the first item on the right-hand side of (8), and the aggregate risk premium, π, which is the second item in that equation.

When t_c increases, the required certainty return R_c will increase and the risk premium π will decrease. For the set of parameters we used, particularly assuming r_f is reasonably large, the change in R_c is dominant. Since $E(R)$ is constant in the short run, the increase in required return caused by the increase in the

[8] This assumes that half of the return on stocks comes from dividends with a tax rate of 0.30 and half from capital gains, with an effective tax rate of 0.10. It should be noted that t_s is the complement of the harmonic mean of $(1 - t_s)$, while t_f is the arithmetic mean. There is some evidence that the effective tax rates are somewhat lower, but this would not affect any of our qualitative results, except in the case without human wealth and when the real interest rate is well below 0.02.

Table I

Elasticity of Value of Capital with Respect to Corporate Income Tax Rate and Personal Income Tax Rate

	No Human Wealth		With Human Wealth					
			e_{tc}			e_{tp}		
			β_{hm} =			β_{hm} =		
Risk-free Rate	e_{tc}	e_{tp}	0.00	0.25	0.50	0.00	0.25	0.50
0.01	−0.412	1.102	−0.143	−0.220	−0.474	0.384	1.219	3.979
0.02	−0.734	0.229	−0.528	−0.621	−0.754	0.165	0.401	0.739
0.03	−0.771	0.128	−0.633	−0.700	−0.783	0.105	0.240	0.408
0.04	−0.786	0.089	−0.683	−0.734	−0.794	0.077	0.171	0.281
0.05	−0.793	0.068	−0.711	−0.753	−0.800	0.061	0.133	0.215
0.06	−0.798	0.055	−0.730	−0.765	−0.803	0.050	0.109	0.174

Notes: $e_{tc} = (t_c/V_m)(\partial V_m/\partial t_c)$; $e_{tp} = (t_p/V_m)(\partial V_m/\partial t_p)$. Parameters: $\alpha = 0.85$; $t_c = 0.45$; $t_p = 0.25$; $\sigma_m = 0.15$.

Table II

Elasticity of Value of Capital with Respect to Corporate Income Tax Rate and Different Personal Income Tax Rates

Risk-free Rate	No Human Wealth			With Human Wealth								
				e_{tc}			e_{ts}			e_{tf}		
				$\beta_{hm} =$			$\beta_{hm} =$			$\beta_{hm} =$		
	e_{tc}	e_{ts}	e_{tf}	0.00	0.25	0.50	0.00	0.25	0.50	0.00	0.25	0.50
0.01	0.470	0.144	4.252	0.068	0.113	0.340	0.021	0.034	0.104	0.612	2.032	9.171
0.02	−0.700	−0.214	0.779	−0.453	−0.543	−0.677	−0.139	−0.166	−0.207	0.504	0.903	1.497
0.03	−0.756	−0.231	0.612	−0.588	−0.657	−0.734	−0.180	−0.201	−0.227	0.476	0.707	0.998
0.04	−0.776	−0.237	0.553	−0.651	−0.704	−0.767	−0.199	−0.215	−0.234	0.463	0.625	0.816
0.05	−0.787	−0.240	0.523	−0.686	−0.730	−0.779	−0.210	−0.233	−0.238	0.456	0.581	0.723
0.06	−0.793	−0.242	0.504	−0.709	−0.746	−0.787	−0.217	−0.228	−0.240	0.451	0.553	0.665

Notes: $e_{tc} = (t_c/V_m)(\partial V_m/\partial t_c)$; $e_{ts} = (t_s/V_m)(\partial V_m/\partial t_s)$; $e_{tf} = (t_f/V_m)(\partial V_m/\partial t_f)$. Parameters: $\alpha = 0.85$; $t_c = 0.45$; $t_s = 0.20$; $t_f = 0.30$; $\sigma_m = 0.15$.

corporate tax rate forces the market value of capital to fall in order to restore the market equilibrium.

When t_p increases, however, the only direct impact is that π will decrease. If the risk-free rate is large enough, Equation (8) shows that market value, V_m, has to increase to restore the equilibrium.

From Equation (8), we can see clearly that the corporate income tax and personal income tax have different impacts on the aggregate risk premium as well as on the certainty return required by investors. It is worth pointing out that although the magnitudes of both elasticities e_{t_c} and e_{t_p} are increasing in β_{hm} as shown in Table I, the elasticity of the personal tax rate e_{t_p} is more sensitive to β_{hm} than is e_{t_c}. This can be accounted for by the different impacts of t_c and t_p on the contribution of human wealth to the aggregate risk premium in Equation (8).

V. Summary

In this paper, we have discussed the short-run effects of the different tax rate changes on the market value of risk capital and the rate of return required by investors, and the long-run effect on the ratio of risky to total marketable assets, with the model allowing for the existence of human wealth. If tax changes decrease the corporate tax rate or the tax rate on the returns from stocks and, to maintain tax revenue, correspondingly increase the personal tax rate on returns from risk-free assets and human wealth, then for plausible values of the relevant parameters, this model implies that stock prices would go up in the short run, while in the long run the proportion of wealth of the economy invested in risky assets such as equities would also go up. But it should be pointed out that, if in the long run the proportion of wealth devoted to risky assets does increase, it does not necessarily guarantee an increase in the total amount of investment in risky assets. The investment in risky assets depends not only on the proportion of wealth invested in such assets, but also on the total savings available for investment which is subject to change if tax changes are implemented. However, there is substantial evidence that private savings are not very sensitive to changes in the after-tax rate of return (see Friend and Hasbrouck [8]).

Whether allowance is or is not made for human wealth in our model within a reasonable range of real risk-free interest rates, say 0.02 to 0.05, an increase in corporate tax rates is associated with a decrease in the market value of risky assets at least in the short run, whereas an increase in personal tax rates which are not dependent on the type of asset is associated with an increase in the market value of risky assets (Table I). The larger the risk-free rate, the greater the corporate tax effect and the weaker the personal tax effect (again within a reasonable range of interest rates). When human wealth is provided for in the model, the larger the covariance between income from human and nonhuman wealth, the greater the effect of tax changes, again negative for corporate tax increases and positive for personal tax changes.[9]

[9] The impact of human wealth on the market is reflected in the change in the market price of risk. It is obvious that the larger the covariance between incomes from human and nonhuman wealth, the greater the impact on the market price of risk.

Once more, when plausible income tax rates which are different for risky marketable assets and other assets are introduced, increased corporate tax rates generally retain the same negative effect on the market value of risky assets, while increased personal tax rates on income from risk-free assets and human wealth continue to have a positive effect on market value, which is now higher than under a uniform schedule of personal income taxes for risky marketable and other assets (Table II). Personal tax changes for income from risky marketable assets have effects which are intermediate between those for corporate taxes and for taxes on income from other personal assets. As noted earlier, however, an increase in personal tax rates on risky marketable assets will generally decrease the market value of such assets.

Obviously, these results depend on the assumptions made in the model including the invariance of the before-tax real risk-free interest rate to personal as well as to corporate income taxes[10] and the validity of the range of values estimated or assumed for such parameters as the actual value of the risk-free rate, the effective tax rates on the three major categories of assets, the ratio of human wealth to total wealth including marketable assets, the covariance of income from human wealth and from marketable assets, the initial ratio of risky marketable assets to total marketable assets, and the measure of relative risk aversion for the market as a whole. The short-run results also depend on the assumption that the supply of physical plant and capacity output cannot be changed in a short period of time, whereas the long-run analysis does permit such changes but does not explicitly introduce the supply conditions. As a result, the short-run results are more satisfactory than the long-run results.

In spite of these serious deficiencies, some of which we hope to remedy in subsequent work, we believe the results are adequate to indicate some of the major factors on which the effect of taxation on the value of risky assets and on the cost of risk capital depends, although with the problem of the determination of the risk-free rate, the results may be more germane to the relative than to the absolute value and cost of risk capital.

[10] The invariance of the before-tax real risk-free interest rate to income taxes is, of course, a questionable assumption which will be modified in future work. However, it should be noted that the empirical evidence is not strongly inconsistent with this assumption. Thus, Tanzi [14] finds no empirical evidence of income tax effects on risk-free interest rates implying some form of fiscal illusion. Melvin [10], however, interprets the same evidence as pointing to significant, although not the complete, tax effects implied by traditional theory.

Under the assumption of stationarity of the after-tax risk-free rate, it can be shown that the effect of personal income tax on the market value of existing capital is ambiguous. Since the empirical evidence suggests something intermediate between the two assumptions on the effect of tax on the risk-free rate, it would appear that the sign of the result we get is correct.

REFERENCES

1. K. J. Arrow. *Essays in the Theory of Risk-Bearing.* Amsterdam: North Holland, 1971.
2. M. Blume and I. Friend. "The Effect of Reduction in Corporate Taxes on Investment in Riskfree and Risky Asset." Rodney L. White Center for Financial Research, Working Paper 3-84, Wharton School, University of Pennsylvania, 1984.
3. E. F. Fama. "Short-Term Interest Rates as Predictors of Inflation." *American Economic Review* 65 (June 1975), 269–82.

4. ——— and G. Schwert. "Human Capital and Capital Market Equilibrium." *Journal of Financial Economics* 4 (January 1977), 95–125.
5. I. Friend and M. Blume. "The Demand for Risky Assets." *American Economic Review* 65 (December 1975), 900–22.
6. I. Friend and J. Hasbrouck. "Effect of Inflation on the Profitability and Valuation of U.S. Corporations." In Marshall Sarnet and Giorgo Szego (eds.), *Saving and Investment Incentives in an Inflationary Environment.* Cambridge, MA: Ballinger Publishing Company, 1981.
7. ———. "Inflation and the Stock Market: Comment." *American Economic Review* 72 (March 1982), 237–42.
8. ———. "Saving and After-Tax Rates of Return." *Review of Economics and Statistics* 65 (November 1983), 537–43.
9. D. Mayers. "Non-Marketable Assets and Capital Market Equilibrium Under Uncertainty." In M. C. Jensen (ed.), *Studies in the Theory of Capital Markets.* NY: Praeger Publishers, Inc., 1972.
10. M. Melvin. "Expected Inflation, Taxation, and Interest Rates: The Delusion of Fiscal Illusion." *American Economic Review* 72 (September 1982), 841–45.
11. R. C. Merton. "Lifetime Portfolio Selection Under Uncertainty." *Review of Economics and Statistics* 51 (August 1969), 247–57.
12. ———. "Optimum Consumption and Portfolio Rules in a Continuous Time Model." *Journal of Economic Theory* 3 (December 1971), 373–413.
13. ———. "An Intertemporal Capital Asset Pricing Model." *Econometrica* 41 (September 1973), 867–87.
14. V. Tanzi. "Inflationary Expectations, Economic Activity, Taxes and Interest Rates." *American Economic Review* 70 (March 1980), 12–21.
15. Y. Zhu and I. Friend. "The Effects of Different Taxes on Risky and Risk-free Investment and on the Cost of Capital." Rodney L. White Center for Financial Research, Working Paper 6-85, Wharton School, University of Pennsylvania, 1985.