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Author(s): Mark Latham
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Informational Efficiency and Information Subsets

MARK LATHAM*

ABSTRACT

This paper proposes a new definition of the Efficient Markets Hypothesis with respect to information, which is more formal and precise than those of Rubinstein [13], Fama [4], Jensen [6], and Beaver [1], and which fits well as a framework for interpreting the many tests of the Efficient Markets Hypothesis in the literature. Security markets are here considered "efficient with respect to information set ϕ" if and only if revealing ϕ to all agents would change neither equilibrium prices nor portfolios. In addition to other desirable features, this definition has the "subset property": efficiency with respect to ϕ implies efficiency with respect to any subset of ϕ.

EFFICIENCY HAS MANY DIFFERENT connotations in economics and finance. This paper examines the concept called "market efficiency" in the finance literature. It will be referred to here as informational efficiency, to distinguish it from other connotations. The concept was developed to describe and explain some observed statistical properties of capital-asset prices. The empirical informational-efficiency literature is vast; for surveys, see Fama [3] and Jensen [6]. The theoretical literature is comparatively small and, I will argue, lacking at a fundamental level: no unambiguous definition of informational efficiency has yet been offered.

It is well known that the mathematical definition of a random walk is too stringent to serve as a characterization of informational efficiency. It would imply that security returns are serially independent, and that their probability distributions are constant through time. It is also clear that the submartingale model ("expected returns are positive") is not stringent enough. Fama's [3] attempt to formalize the notion that in an efficient market, prices "fully reflect" available information, was appropriately criticized by LeRoy [10] as empty and tautological. Fama's [4] revised definition, that the market correctly uses all relevant information in determining security prices, is unclear if investors have heterogeneous information, since Fama did not explain what "the market uses ⋯ the information" would mean in such a case. Jensen's [6] statement in terms of the absence of "economic profits" is incomplete because, like Fama [3], he gives no guidance on how to distinguish abnormal returns from normal returns to risk-bearing.

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Grossman and Stiglitz [5, p. 404], and later Jordan [7], reinterpret the informational efficiency idea, as expressed by "prices fully reflect the [specified] information," to mean that prices *reveal* the information (or at least a sufficient statistic for it). That is different. Information can be fully reflected without being revealed in prices. Suppose a piece of information were given to everyone (e.g., published in the *Wall Street Journal*). Then it is generally accepted (e.g., by the sufficient conditions of Fama [3, p. 387]) that prices would fully reflect it and the market would be efficient with respect to it. Yet the information may not be discernible from prices alone (see example in Section I), so that by the Grossman-Stiglitz definition, the market would *not* be efficient with respect to it. Clearly, the Grossman-Stiglitz concept is sufficient but not necessary for what is normally meant by market (informational) efficiency in the finance literature.

Among the several informational-efficiency definitions suggested by Rubinstein [12] and [13], the one "closest in spirit to the implicit definitions used in most of the empirical literature" is that prices would not change if all private information were publicized. This explicitly recognizes heterogeneity of information and beliefs in an equilibrium setting, but applies only to one information set: all information known. So, it can be taken to define only strong-form efficiency. Following a suggestion by William Sharpe, Beaver [1] extended this idea to apply to any information set: the market is efficient with respect to an information set if revealing that information to all investors (while they keep the information they already have) would not change equilibrium prices. This seems to capture the notions of informational efficiency implicit in the empirical and theoretical literatures, while avoiding the main pitfalls of tautology and ambiguity. It is thus taken as the "state of the art," and the starting point for this paper.

It turns out that there is still some ambiguity in Beaver's definition, which is easily avoided by an amendment. But an important problem remains: none of Beaver's definitions has the "subset property," a widely accepted feature of informational efficiency. The property is that efficiency with respect to an information set necessarily implies efficiency with respect to any subset of that information set. It is by this principle that if the market is semi-strong-form efficient (i.e., with respect to all public information), then it must be weak-form efficient (i.e., with respect to past security prices), since past prices are public. It is by this principle that an event study based on published earnings figures is a test of semi-strong-form efficiency. Not *all* public information is used, only a small subset; yet associated abnormal returns are taken to imply semi-strong-form inefficiency. That logical step assumes the subset property.

The amended version of Beaver's efficiency definition only violates the subset principle in those unusual cases where revealing information would change equilibrium portfolios but not prices.[1] This naturally suggests the inclusion of a portfolio benchmark in a market-efficiency definition, as one way to obtain the subset property. A new definition, "E-efficiency," is proposed: the market is E-efficient with respect to some information set if revealing it to all agents would change neither equilibrium prices nor portfolios. I point out that this is only

[1] Neither of the two definitions offered by Beaver has the subset property.

slightly stronger than previous, price-oriented definitions, that it has the subset property, that it provides an appropriate framework for interpreting existing empirical work, and that it may offer opportunities for efficiency tests involving trading volume.

Section I presents the model and notation. Section II is a critique of Beaver [1], amending his definition to eliminate an ambiguity, and giving a counterexample to the subset property. Section III contains the definition of E-efficiency, the subset property theorem, and a discussion of its relevance to the empirical literature. Section IV offers conclusions and mentions some important remaining questions about informational efficiency.

I. The Model

I choose the simplest setting rich enough to demonstrate the necessary ideas. (Notation is summarized in Table I). It is a pure-exchange one-period economy. At $t = 0$, there is no consumption; investors (indexed by i) just trade their endowments $\bar{z}_{ik}$ of securities (indexed by k) to arrive at equilibrium security holdings, z_{ik}. At $t = 1$, each security, k, pays a liquidating (terminal) dividend P_k^1, and the investors consume those payoffs. The amount of each security's

Table I

Notation

Ω the space of possible states of nature at $t = 1$

ω a state, usually taken as the one that is going to occur at $t = 1$

ζ a state; may or may not occur at $t = 1$

i index for investors

k index for securities

$\bar{Z}$ endowment matrix $[\bar{z}_{ik}]$

Z equilibrium portfolio holdings $[z_{ik}]$

Z_i vector of portfolio holdings of investor i

P^0 price vector $[P_k^0] = [t = 0$ price of security $k]$

P^1 payoff vector $[P_k^1] = [t = 1$ payoff of security $k]$

c_i $t = 1$ consumption of investor i; $c_i(\omega) = \Sigma_k z_{ik}(\omega) P_k^1(\omega)$

$c_i(\zeta, Z_i) = \Sigma_k z_{ik} P_k^1(\zeta)$ = payoff of portfolio Z_i in state ζ, at $t = 1$

$U_i(c_i)$ utility of consumption of investor i

U', U'' first and second derivatives of U

Ξ_i investor i's endowed information partition of Ω

Ξ^* the join (coarsest common refinement) of the $\{\Xi_i\}$: $\Xi^* = \bigvee_i \Xi_i$

Ξ^P the partition of Ω generated by $P^0(\omega)$

Ξ_i^E the join of Ξ_i and Ξ^P: $\Xi_i^E = \Xi_i \vee \Xi^P$ (this is investor i's information partition at equilibrium)

Ξ' an information-partition of Ω, with respect to which efficiency is being assessed

$S_i(\omega)$ the set in Ξ_i containing ω

$S^*(\omega)$, $S^P(\omega)$, $S_i^E(\omega)$, and $S'(\omega)$ definitions parallel that of $S_i(\omega)$

$\pi(\omega)$ unconditional probability for ω; homogeneously perceived

$\pi(S) = \Sigma_{\omega \epsilon S} \pi(\omega)$ = unconditional probability for set S

$\pi(\omega | S) = \pi(\omega)/\pi(S)$ = probability for ω conditional on S

$\hat{}$ indicates variable in an imaginary economy where each investor i receives Ξ' before trading, as well as Ξ_i

payoff at $t = 1$ is uncertain as of $t = 0$, but is known to depend on the "state of nature" at $t = 1$. There is a finite set Ω of possible states. A typical state is denoted ω; a typical set of states is $S \subset \Omega$.

Investors have homogeneous prior beliefs $\pi(\omega) > 0$ for the probability of state ω occurring.[2] Associated with each investor i is a partition Ξ_i of the state space Ω. Before trading at $t = 0$, investor i receives information about the true state, ω, by being told which set of her partition contains ω—that set is called $S_i(\omega)$. Ξ^* denotes the join (coarsest common refinement) of the $\{\Xi_i\}$, and thus represents all information known to anyone. $S^*(\omega)$ is the set in Ξ^* containing ω; so if investors pooled their information, they could narrow down the true state to being contained in $S^*(\omega)$.

The vector of $t = 0$ equilibrium security prices is $P^0(\omega)$, with typical element $P_k^0(\omega)$, written as a function of ω although it will only depend on ω through dependence on $S^*(\omega)$. In other words, $t = 0$ prices are determined not so much by the future (unknown) true state as by the information available today about the future state. The $t = 1$ payoff of security k is $P_k^1(\omega)$.

The characterization of equilibrium used here is one of rational expectations with learning-from-prices, as in Radner [11]. Let Ξ^P be the partition of Ω generated by $P^0(\omega)$, so that for any two states, ω and ζ,

$$P^0(\zeta) = P^0(\omega) \Leftrightarrow \zeta \in S^P(\omega).$$

Then, we can represent the information that investor i uses at equilibrium in the $t = 0$ trading round by the join of Ξ_i and Ξ^P, denoted $\Xi_i^E = \Xi_i \vee \Xi^P$. His or her posterior probabilities are then $\pi(\zeta \mid S_i^E(\omega))$ where:

$$\begin{aligned} S_i^E(\omega) &= S_i(\omega) \cap S^P(\omega) \\ \pi(\zeta \mid S_i^E(\omega)) &= \begin{cases} \dfrac{\pi(\zeta)}{\pi(S_i^E(\omega))} & \text{if } \zeta \in S_i^E(\omega) \\ 0 & \text{if } \zeta \notin S_i^E(\omega) \end{cases} \\ \pi(S_i^E(\omega)) &= \textstyle\sum_{\zeta \in S_i^E(\omega)} \pi(\zeta). \end{aligned}$$

ω is the state that is going to occur at $t = 1$, but at $t = 0$ investor i only has it narrowed down to $S_i^E(\omega)$, and so perceives probabilities $\pi(\zeta \mid S_i^E(\omega))$ for the various states indexed by ζ. She wants to maximize her expected value of her state-independent strictly concave utility function $U_i(c_i)$.[3] She considers various possible portfolio vectors, Z_i, before choosing the best one denoted $Z_i(\omega)$. Being ignorant of ω, she anticipates various possible $t = 1$ states ζ, in which her consumption would be $c_i(\zeta, Z_i) = \Sigma_k z_{ik} P_k^1(\zeta)$; but God knows investor i's consumption will be $c_i(\omega, Z_i(\omega))$, or simply $c_i(\omega)$.[4]

[2] The results are easily generalized to heterogeneous beliefs, but keeping them homogeneous makes for simpler notation.

[3] State-independent utility is assumed for simplicity. The results generalize to state-dependence by replacing $U_i(c_i(\zeta, Z_i))$ with $U_{i\zeta}(c_i(\zeta, Z_i))$.

[4] Readers who are atheists may simply assume the existence of God in this model.

An equilibrium is a pair of functions $\{Z(\cdot), P^0(\cdot)\}$ on Ω such that

(a) markets clear: $\forall k, \omega$: $\Sigma_i z_{ik}(\omega) = \Sigma_i \bar{z}_{ik}$;
(b) portfolios are affordably optimal: $\forall i, \omega$: $Z_i(\omega)$ maximizes $\Sigma_\zeta \pi(\zeta \mid S_i^E(\omega)) U_i(c_i(\zeta, Z_i))$ subject to the budget constraint $\Sigma_k z_{ik} P_k^0(\omega) \leqq \Sigma_k \bar{z}_{ik} P_k^0(\omega)$; and
(c) prices reveal no more than the aggregate of investors' information: Ξ^* refines Ξ^P.

Market efficiency with respect to an information-partition Ξ' will be defined by comparing the actual equilibrium with equilibrium in an otherwise identical economy where each investor receives Ξ' as well as Ξ_i. This equilibrium is denoted $\{\hat{Z}(\cdot), \hat{P}^0(\cdot)\}$. The partition generated by $P^0(\cdot)$ is $\hat{\Xi}P$, and $\hat{\Xi}_i^E = \Xi_i \vee \Xi' \vee \hat{\Xi}^P$. So in the above equilibrium conditions, a caret (ˆ) is added to z_{ik}, Z_i, S_i^E, c_i, and P_k^0.

II. Critique of Beaver's [1] Definition

Beaver raised the important question of whether to define efficiency with respect to a *given outcome* ("signal") of an information variable, or to define efficiency with respect to the *ex-ante variable* ("information system"). In his notation, at time t, investor i receives y_{it}, which is one realization of the many possible signals from his information system, η_{it}. So, y_{it} is like our S_i, and η_{it} is like our Ξ_i. Efficiency of the market at time t will be assessed with respect to either an information system, η_t', or a signal, y_t', from η_t. So, y_t' is like our S', and η_t' is like our Ξ'.

Beaver's [1, p. 28] definitions are:

"y-efficiency—

A securities market is efficient with respect to a signal y_t' if and only if the configuration of security prices $\{P_{jt}\}$ is the same as it would be in an otherwise identical economy (i.e., with an identical configuration of preferences and endowments) except that every individual receives y_t' as well as y_{it}.

η-efficiency—

A securities market is efficient with respect to η_t', if and only if y-efficiency holds for every signal (y_t') from η_t'."

For example, suppose we are looking at efficiency of the market on September 2, with respect to a firm's announcement on September 1 of the amount of an upcoming dividend. Many (random) events before September 1 have helped determine what amount is announced. If one is ignorant of those events (e.g., take the view from the preceding January 1), then there are many possible amounts that may be announced. But from the viewpoint of an ex-post event study, only one amount was announced—say \$2. y-efficiency asks whether security prices on September 2 fully reflected the fact that \$2 was announced. η-efficiency asks not only that but also whether security prices on September 2 *would* have fully reflected the amount that would have been announced if preceding events had been different.

Beaver leaves open the question of which definition is preferable, but η-efficiency seems closer to conventional notions. y-efficiency does not fit well with the idea of no economic profits on the information, in the sense that that would imply no incentive to gather or purchase the information. The decision to acquire information must necessarily be made ex ante, but y-efficiency is an ex-post concept. Also, η-efficiency seems to fit better with empirical tests of informational efficiency, which assume some stationarity in the underlying structure, and then compute statistics from a large sample. Such procedures are designed to support conclusions about efficiency with respect to an information variable or system (η) rather than merely about an individual outcome of that variable (y). And finally, y-efficiency does not have the subset property, since one can imagine a piece of good news and a piece of bad news that, together, cancel each other out but, separately, would affect prices. For these reasons, y-efficiency is here set aside in favor of a closer look at η-efficiency.

While the definition of y-efficiency is clear, that of η-efficiency is ambiguous. Using the present paper's notation, in y-efficiency, the phrase "a securities market" clearly refers to one particular realization of everything at $t = 0$: $P^0(\omega)$, $Z(\omega)$, and especially their determinants $\{S_i(\omega)\}$ across i, are being thought of as specific realizations. What is not explicit in η-efficiency is how those random variables are being varied when different signals from η_t' are being considered. Equilibrium prices are being determined by other information besides η_t' (namely, by $\{\eta_{it}'\}$); is the other information being held constant as we look across the possible signals from η_t'?

It is difficult to imagine how to hold "all other" information constant while varying one specified piece of information. In reality, information signals are mixed together in many ways. In theory, you could set up a parameter system to describe the space of possible information outcomes. This would give unambiguous meaning to the idea of varying one parameter while holding the others constant. But then assessments of informational efficiency with respect to the same variable could give different answers depending on the choice of parameter system. Such a property would not fit conventional notions.

So, we seem forced to allow all information to vary, interpreting η-efficiency thus:

> A securities market is efficient with respect to η_t' if and only if y-efficiency holds with respect to the signal from η_t', in every possible realization of that securities market at time t.

Or, in our notation:

> A securities market is η-efficient with respect to Ξ' if and only if $\hat{P}^0(\omega) = P^0(\omega)$ for all $\omega \in \Omega$.

Furthermore, since existence and uniqueness of equilibrium in such models as this is by no means guaranteed, the definition is given more precisely as:

> An *equilibrium* is η-efficient with respect to Ξ' if and only if there exists an

equilibrium in the benchmark economy (Ξ' published) such that $\hat{P}^0(\omega) = P^0(\omega)$ for all $\omega \in \Omega$.

Let us call this "*S-B* efficiency" (for Sharpe-Beaver).

It turns out that *S-B* efficiency does not have the feature that efficiency with respect to an information set implies efficiency with respect to each of its subsets. That is proved by the counterexample below. (Ξ_1 is an "information subset" of Ξ_2 if and only if Ξ_2 refines Ξ_1.)

The full counterexample is based on a less complex example in which the release of information causes trading but no price changes. Beliefs and information are homogeneous (across investors) throughout. There are two investors, one more risk-averse than the other and two securities. Each investor is endowed with half a share of each security. One way to describe the securities and the information is that one security is riskier than the other, but at first, no one knows which is which, so both investors simply hold their endowments. The information then released tells which is riskier, and the investors trade accordingly, each increasing her or his welfare, while the numbers are chosen so that the price ratio is always 1.

Table II is a simple but technically incorrect tableau of payoffs and probabilities. It is incorrect because a and b cannot be represented by a partition of Ω, as our model requires. Table III gives the correct version. Utility functions for $t = 1$ consumption are:

$$U_1(c) = -(8 - c)^2, \qquad \text{for } c < 7;$$

$$U_2(c) = -(12 - c)^2, \qquad \text{for } c < 7.$$

$[c_i(\omega) < 7$ for all i, ω in this example.] The coefficient of absolute risk aversion

Table II

Payoffs and Probabilities

		ω_1	ω_2	ω_3	ω_4
Security-1 payoff	$P_1^1(\omega)$	1	3	4	8
Security-2 payoff	$P_2^1(\omega)$	3	1	8	4
No-information probabilities	$\pi(\omega)$	0.25	0.25	0.25	0.25
Probabilities given signal a	$\pi(\omega \mid a)$	0.4	0.1	0.1	0.4
Probabilities given signal b	$\pi(\omega \mid b)$	0.1	0.4	0.4	0.1
$[\pi(a) = \pi(b) = 0.5]$					

Table III

Payoffs and Probabilities—Partitionable Version

	ω_{1a}	ω_{1b}	ω_{2a}	ω_{2b}	ω_{3a}	ω_{3b}	ω_{4a}	ω_{4b}
$P_1^1(\omega)$	1	1	3	3	4	4	8	8
$P_2^1(\omega)$	3	3	1	1	8	8	4	4
$\pi(\omega)$	0.2	0.05	0.05	0.2	0.05	0.2	0.2	0.05
$\pi(\omega \mid a)$	0.4	0	0.1	0	0.1	0	0.4	0
$\pi(\omega \mid b)$	0	0.1	0	0.4	0	0.4	0	0.1

$(-U''/U')$ is $1/(8 - c)$ for investor 1 and $1/(12 - c)$ for investor 2, so investor 1 is more risk-averse.

Under no-information, it is clear by symmetry that no trading occurs and that the price ratio $P_2^0/P_1^0 = 1$. If signal a is announced, the equilibrium equations are easily solved to give

$$P_2^0/P_1^0 = 1$$

$$z_{11} = z_{22} = 0.38$$

$$z_{12} = z_{21} = 0.62.$$

Here, security 1 is riskier, and is held more heavily by the less risk-averse investor (no. 2).

If b is announced, then security 2 is riskier, and

$$P_2^0/P_1^0 = 1$$

$$z_{11} = z_{22} = 0.62$$

$$z_{12} = z_{21} = 0.38.$$

Thus, the no-information economy is S-B-efficient with respect to the $\{a, b\}$ information partition, even though no one knows the information. Prices (and conditional expected returns) are "right," although portfolio holdings do not reflect the information.

To examine the subset property, we need two information partitions, one of which refines the other. We modify the above example by introducing another partition, denoted $\{c, d\}$, which has as refinement a partition $\{a, a', b, b'\}$ that is in effect equivalent to $\{a, b\}$ above. Recall that under no-information, $\pi(a) = \pi(b) = 0.5$; signal c can be thought of as indicating that $\pi(a \mid c) = 2/3$ and $\pi(b \mid c) = 1/3$; similarly $\pi(a \mid d) = 1/3$ and $\pi(b \mid d) = 2/3$. The simple but incorrect version of the tableau is shown in Table IV. To represent it correctly, with $\{a, b\}$ refining $\{c, d\}$, the state space is expanded as in Table V. A partition $\{a, a', b, b'\}$ is defined, where a and a' are the same except that a is associated with c while a' is associated with d. Likewise, b goes with d, and b' with c.

The equilibrium prices and portfolios are most conveniently calculated by applying the equilibrium equations to the numbers in Table IV. Equilibria under

Table IV

Payoffs and Probabilities with Information Subset

	ω_1	ω_2	ω_3	ω_4
$P_1^1(\omega)$	1	3	4	8
$P_2^1(\omega)$	3	1	8	4
$\pi(\omega)$	0.25	0.25	0.25	0.25
$\pi(\omega\mid a)$	0.4	0.1	0.1	0.4
$\pi(\omega\mid b)$	0.1	0.4	0.4	0.1
$\pi(\omega\mid c)$	0.3	0.2	0.2	0.3
$\pi(\omega\mid d)$	0.2	0.3	0.3	0.2

Note: $[\pi(a) = \pi(b) = \pi(c) = \pi(d) = 0.5]$.

Table V

Payoffs and Probabilities with Information Subset—Partitionable Version

	ω_{1ac}	$\omega_{1b'c}$	$\omega_{1a'd}$	ω_{1bd}	ω_{2ac}	$\omega_{2b'c}$	$\omega_{2a'd}$	ω_{2bd}
$P_1^1(\omega)$	1	1	1	1	3	3	3	3
$P_2^1(\omega)$	3	3	3	3	1	1	1	1
$\pi(\omega)$	0.1333	0.0167	0.0667	0.0333	0.0333	0.0667	0.0167	0.1333
$\pi(\omega\|a)$	0.4	0	0	0	0.1	0	0	0
$\pi(\omega\|b')$	0	0.1	0	0	0	0.4	0	0
$\pi(\omega\|a')$	0	0	0.4	0	0	0	0.1	0
$\pi(\omega\|b)$	0	0	0	0.1	0	0	0	0.4
$\pi(\omega\|c)$	0.2667	0.0333	0	0	0.0667	0.1333	0	0
$\pi(\omega\|d)$	0	0	0.1333	0.0667	0	0	0.0333	0.2667

	ω_{3ac}	$\omega_{3b'c}$	$\omega_{3a'd}$	ω_{3bd}	ω_{4ac}	$\omega_{4b'c}$	$\omega_{4a'd}$	ω_{4bd}
$P_1^1(\omega)$	4	4	4	4	8	8	8	8
$P_2^1(\omega)$	8	8	8	8	4	4	4	4
$\pi(\omega)$	0.0333	0.0667	0.0167	0.1333	0.1333	0.0167	0.0667	0.0333
$\pi(\omega\|a)$	0.1	0	0	0	0.4	0	0	0
$\pi(\omega\|b')$	0	0.4	0	0	0	0.1	0	0
$\pi(\omega\|a')$	0	0	0.1	0	0	0	0.4	0
$\pi(\omega\|b)$	0	0	0	0.4	0	0	0	0.1
$\pi(\omega\|c)$	0.0667	0.1333	0	0	0.2667	0.0333	0	0
$\pi(\omega\|d)$	0	0	0.0333	0.2667	0	0	0.1333	0.0667

Note: Here $\pi(c) = \pi(d) = 0.5$, while $\pi(a) = \pi(b) = 1/3$, $\pi(a') = \pi(b') = 1/6$, $\pi(a|c) = 2/3$, $\pi(a'|c) = 0$, $\pi(b|c) = 0$, $\pi(b'|c) = 1/3$, etc.

Table VI
Investor-1 Consumption

	ω_1	ω_2	ω_3	ω_4
No-information	2	2	6	6
Signal a	2.24	1.76	6.48	5.52
Signal b	1.76	2.24	5.52	6.48
Signal c	2.08	1.92	6.16	5.84
Signal d	1.92	2.08	5.84	6.16

no-information, signal a (and a') and signal b (and b'), are the same as before. If signal c is announced, we get

$$P_2^0/P_1^0 = 1$$

$$z_{11} = z_{22} = 0.46$$

$$z_{12} = z_{21} = 0.54.$$

If d then

$$P_2^0/P_1^0 = 1$$

$$z_{11} = z_{22} = 0.54$$

$$z_{12} = z_{21} = 0.46.$$

Here, the subset property is being obeyed: the no-information economy is S-B-efficient *both* with respect to the $\{a, a', b, b'\}$ partition *and* with respect to the (information subset) $\{c, d\}$ partition. But we can perturb a utility function to upset the latter while preserving the former. Consider the various possible levels of investor 1's consumption at $t = 1$, under the three different information settings (no-information, $\{a, b\}$ and $\{c, d\}$), as summarized in Table VI. His utility function is $U_1(c) = -(8 - c)^2$. If we change $U_1(c)$ in a small region around $c = 2.08$, we can change the equilibria under signals c and d while preserving the equilibria under a, b, and no-information. We must maintain $U_1' > 0$, but that still leaves plenty of scope for altering U_1 in the interval $2 < c < 2.24$. The old U_1 has $U_1'(2.08) = 11.84$. If the new U_1 has $U_1'(2.08) = 11.85$, then $P_2^0/P_1^0 = 1$ will no longer be the equilibrium price ratio under signal c: investor 1 would demand more of security 1 (and less of security 1) than before. So the new signal c equilibrium, P_2^0/P_1^0, will be > 1; likewise signal d will have $P_2^0/P_1^0 < 1$.

It is this final example, with the perturbed utility function, $U_1(c)$, which shows that S-B efficiency does not have the subset property.[5] The price of security 2 in terms of security 1, P_2^0/P_1^0, is 1 under no-information, and 1 under any of signals $\{a, a', b, b'\}$. So the no-information economy is S-B-efficient with respect to $\{a, a', b, b'\}$. But under signal c, $P_2^0/P_1^0 > 1$, and under d it is < 1, so the no-information economy is S-B-*in*efficient with respect to $\{c, d\}$. Finally, $\{c, d\}$ is an

[5] Another counterexample (in the given information and security regimes) is:

$$U_1(c) = \log(c)$$

$$U_2(c) = -(8.02613 - c)^2.$$

information subset of $\{a, a', b, b'\}$ because the latter refines the former, since $c = (a, b')$ and $d = (a', b)$. Therefore, *S-B* efficiency with respect to an information set does not imply *S-B* efficiency with respect to any subset.

There are several ways one could react to this shortcoming. One could forego the subset property, or one could change the efficiency definition in some fashion. The latter course is taken below.

III. Proposed Definition: "*E*-efficiency"

It turns out that *all* cases in which *S-B* efficiency violates the subset principle involve an information set that would affect portfolios but not prices, like $\{a, a', b, b'\}$ in the above example. So one way to salvage the subset property would be to deem it inefficiency whenever prices *or* portfolios change on publication of the information. That is the view of *E*-efficiency:

Definition: An equilibrium is *E*-efficient with respect to Ξ' if and only if there exists an equilibrium in the benchmark economy (Ξ' published) such that $\hat{P}^0(\omega) = P^0(\omega)$ and $\hat{Z}(\omega) = Z(\omega)$ for all $\omega \in \Omega$.[6]

We next examine the properties of *E*-efficiency, arguing that although it may seem a radical departure, it is nonetheless the most useful formal definition for understanding what is conventionally meant by market efficiency.

THEOREM 1 (SUBSET PROPERTY). *If an equilibrium is E-efficient with respect to Ξ', and Ξ' refines Ξ'', then it is E-efficient with respect to Ξ''.*

Proof: See Appendix.

Notice that Theorem 1 proves the claim that the only way to construct an example where *S-B* efficiency fails the subset test is with an information set whose publication changes portfolios but not prices. For this to happen, the new information must affect each investor's portfolio demand function so that at the previously prevailing prices, aggregate new purchase orders equal aggregate new sale orders for each security.[7] Such a perfect balance is not robust to small perturbations in utility functions, as illustrated by the counterexample in Section I. So, the set of instances where portfolios would change while prices hold constant is small and likely to have zero measure under any reasonable measure definition.

Therefore *E*-efficiency and *S-B* efficiency are in practise very similar. *E*-efficiency implies *S-B* efficiency, and the converse would also hold but for a few exceptional cases. If we grant that *S-B* efficiency, with its focus on prices (and hence on returns), is appropriate for interpreting the existing return-

[6] "*E*" is for equilibrium, since not only equilibrium prices but also equilibrium holdings are considered. Rubinstein [13, p. 823] suggested imposing such a condition on portfolios.

[7] Technically, the benchmark for both *S-B* and *E*-efficiency is an equilibrium resulting from publishing Ξ' and trading *from the original endowments* of the actual economy. But Latham [8] proves that equivalent definitions (to both *S-B* and *E*-efficiency) can be formed by letting the benchmark economy's endowments be the actual economy's *equilibrium holdings*. So you can think of assessing efficiency by publishing Ξ' immediately after the actual round of trading and allowing retrading.

oriented empirical literature, then we must grant that E-efficiency is likewise appropriate. Empirical tests never prove informational efficiency. They either reject it or fail to reject it. E-efficiency is (slightly) stronger than S-B efficiency, so any test that rejects the latter also rejects the former, and anything that tests the latter tests the former. The extra strength of E-efficiency merely opens opportunities for further tests, that might not have been relevant for testing S-B efficiency. These would be tests of portfolio revision and thus of trading volume.

IV. Conclusions and Open Questions

A new formal definition of capital-market informational efficiency is proposed: the market is "E-efficient" with respect to an information set if and only if revealing that information to all investors would change neither equilibrium prices nor portfolios. In addition to providing a framework suitable for interpreting the existing empirical informational-efficiency literature, E-efficiency captures the spirit of previous theoretical definitions, as in Fama [3] and [4], Rubinstein [13], Jensen [6], and Beaver [1], while avoiding some of their drawbacks. E-efficiency can be defined relative to any chosen information set, even when information and beliefs differ across investors. And it has the widely accepted subset property: E-efficiency with respect to an information set implies E-efficiency with respect to any subset.

E-efficiency may seem too exacting a standard to those used to thinking of informational efficiency in terms of prices, returns, and profit opportunities. If everyone were well informed except for a few small investors, then there would be E-inefficiency but no significant profit opportunities. This makes E-efficiency look very different from earlier, return-oriented notions. But we are just being confused by a question of degree. Except for a measure-zero set, if the economy's assets are allocated slightly "wrongly" (compared to the benchmark economy), then prices (and hence expected returns) will be slightly "wrong." Therefore E-efficiency is not significantly more exacting than previous definitions such as S-B efficiency.

It may be desirable to devise a notion of "informational near-efficiency," to measure how close the market is to being perfectly efficient with respect to some given information. The present paper does not attempt this; E-efficiency is an extreme ideal. While E-efficiency does not, in general, obviate private incentives to acquire information,[8] it will do so in many cases; if that information is costly, then for those cases, the equilibrium cannot become E-efficient given rational investors, as pointed out by Grossman and Stiglitz [6]. Near-efficiency might then be a useful concept.

Returning to E-efficiency, this precise explicit definition provides the basis necessary for the analysis of many important questions: Is informational effi-

[8] *No* conventional concept of market efficiency, including those of Fama [3] and [4], Jensen [6], Beaver [1], and the current paper, has the property that efficiency with respect to some information implies zero private incentive to acquire the information. This is shown in Latham [9].

ciency necessary or sufficient for productive or allocative efficiency? How should degrees of near-efficiency be defined when information is costly? Can trading volume be used to test informational efficiency? How should efficiency be defined in an intertemporal setting? How important is it to allow for heterogeneous information in security-return models used as benchmarks for empirical tests? Even partial answers would contribute significantly to our understanding of what is probably the most widely studied concept in finance.

Appendix

Proof of Theorem 1: What follows is based on the fact that if an individual knows that his or her decision, after receiving some extra information, would be the same across all possible outcomes of that information (and if he or she knows what that decision would be), then he or she would make the same choice without that extra information.

Given that the actual-round equilibrium $\{Z(\cdot), P^0(\cdot)\}$ satisfies both the actual-round equilibrium conditions and the imaginary-round-with-Ξ' equilibrium conditions, we will show that it also satisfies the imaginary-round-with-Ξ'' equilibrium conditions.

For the benchmark economy with Ξ'' (and not Ξ') published:

(a) $Z(\cdot)$ clears all markets.

(b) Take any ω, i. Budgets and prices are the same in all three rounds being considered, so investor i chooses Z_i from the same affordable set in all three optimizations. We want to show that $Z_i(\omega)$ maximizes

$$\Sigma_\zeta \pi(\zeta \mid \tilde{S}_i^E(\omega))\, U_i(c_i(\zeta, Z_i))$$

where

$$\tilde{S}_i^E(\omega) = S_i(\omega) \cap S''(\omega) \cap S^P(\omega).$$

The partition $\{\tilde{S}_i^E(\omega)\}$ is denoted $\tilde{\Xi}_i^E$. Now Ξ' refines Ξ'', so $\hat{\Xi}_i^E$ refines $\tilde{\Xi}_i^E$. So $\tilde{S}_i^E(\omega)$ is the union of n sets in the partition $\hat{\Xi}^E$, where $n \geq 1$.

Suppose $n = 2$. Then there is some $\omega_2 \in \tilde{S}_i^E(\omega)$ such that

$$\tilde{S}_i^E(\omega) = \hat{S}_i^E(\omega) \cup \hat{S}_i^E(\omega_2).$$

From the actual-round equilibrium, we know that $Z_i(\omega)$ maximizes

$$\Sigma_\zeta \pi(\zeta \mid S_i^E(\omega))\, U_i(c_i(\zeta, Z_i))$$

and $Z_i(\omega_2)$ maximizes

$$\Sigma_\zeta \pi(\zeta \mid S_i^E(\omega_2))\, U_i(c_i(\zeta, Z_i)).$$

But $\omega_2 \in \tilde{S}_i^E(\omega)$ implies $\omega_2 \in S_i^E(\omega)$, so that $S_i^E(\omega_2) = S_i^E(\omega)$. Furthermore, budgets and prices are the same for ω_2 as for ω. Therefore, $Z_i(\omega) = Z_i(\omega_2)$ since they both solve the same maximization problem and U_i is strictly concave.

Now

$$\Sigma_{\zeta}\pi(\zeta \mid \tilde{S}_i^E(\omega))U_i(c_i(\zeta, Z_i) = \frac{\pi(\hat{S}_i^E(\omega))}{\pi(\tilde{S}_i^E(\omega))}\Sigma_{\zeta}\pi(\zeta \mid \hat{S}_i^E(\omega))U_i(c_i(\zeta, Z_i)) + \frac{\pi(\hat{S}_i^E(\omega_2))}{\pi(\tilde{S}_i^E(\omega))}\Sigma_{\zeta}\pi(\zeta \mid \hat{S}_i^E(\omega_2))U_i(c_i(\zeta, Z_i)).$$

$Z_i(\omega)$ maximizes each of two parts of the above sum separately, so it also maximizes the sum. The adaptations necessary if $n \neq 2$ are straightforward.

(c) Ξ^* refines Ξ^P, so clearly the corresponding condition holds for the with $-\Xi''$ economy: $\Xi^* \vee \Xi''$ refines Ξ^P.

Therefore, the equilibrium $\{Z(\cdot), P^0(\cdot)\}$ does not change on publication of Ξ'' and is thus E-efficient with respect to Ξ''. Q.E.D.

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