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On the Relevance of Debt Maturity Structure

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ABSTRACT

In this paper, we present a tax-induced framework to analyze debt maturity problems. We show that under some modifications of the existing U.S. tax code, debt maturity is irrelevant even in the presence of taxes and bankruptcy costs that yield an optimal capital structure. If this restrictive structure is relaxed, and assuming the Miller [15] equilibrium does not prevail, tax reasons would usually imply the existence of an optimal debt maturity structure. If there exists a gain from leverage, then an increasing term structure of interest rates, adjusted for default risk, results in long-term debt being optimal. A decreasing term structure, under similar circumstances, renders short-term debt optimal. In the absence of agency costs, a Miller [15]-type result emerges at equilibrium and irrelevance prevails. We also argue that agency costs could again reverse the irrelevance and imply a firm-specific optimal debt maturity structure.

THERE HAS BEEN EXTENSIVE discussion in the literature concerning the existence of an optimal debt maturity structure. Kraus [13] states, without proof, that efficient capital markets preclude the existence of an optimal debt maturity. Stiglitz [20] demonstrates the irrelevance of debt maturity, but in an economic environment in which the entire financing decision is irrelevant due to the absence of taxes and bankruptcy costs. In contrast, Morris [16] argues that the issuance of short-term debt can reduce the risk to stockholders and thereby increase equity value, if the covariance between the net operating income and future interest rates is positive. This result by Morris is obtained even in the absence of taxes and when there is no probability of default. The discrepancy between Morris and Stiglitz arises because Morris uses the Bogue and Roll [3] multiperiod Capital Asset Pricing Model to incorporate uncertain future interest rates, which implicitly assumes that investors cannot diversify away intertemporal risk (see Fama [10]). Stiglitz's complete market framework, on the other hand, allows intertemporal risk diversification.

Myers [18], and Barnea, Haugen, and Senbet [1] argue that agency costs associated with debt result in the existence of an optimal debt maturity structure. In particular, short-term debt resolves the conflict of interest between stockholders and bondholders that arises due to informational asymmetry and moral hazard. However, Barnea, Haugen, and Senbet [1] also demonstrate that agency

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costs can be equivalently reduced by offering callable and convertible bonds. This equivalence between a short-term debt maturity structure and long-term bonds with appropriate call option features implies that the debt maturity structure is irrelevant in the absence of agency costs.

Boyce and Kalotay [4] demonstrate that, if both personal and corporate tax rates differ, and if the term structure of interest rates is not flat, then debt maturity will affect the after-tax cost of debt. That is, the total tax benefit of debt is not independent of the maturity structure. In particular, they find that a rising term structure of interest rates implies that long-term debt is optimal. This result is obtained assuming riskless bonds.[1]

On the other hand, Brennan and Schwartz [5], who assume that the value of the unlevered firm follows a Gauss-Wiener process, argue that short-term debt is optimal because they find that the optimal leverage ratio decreases with maturity. They allow for both taxes and bankruptcy costs (e.g., tax benefits of debt are lost upon bankruptcy) and assume a constant risk-free interest rate per year, but they do no firm value comparisons while altering the debt maturity strategy.

Finally, Dammon [8] examines the optimal leverage decision assuming an incomplete markets framework. In this case, since firms do not have complete supply-side flexibility (i.e., complete flexibility in altering the portfolio mix of financing to take advantage of price premiums) there will be an optimal financing decision. He finds that there is an optimal bond *duration* (i.e., the optimal life of zero-coupon bonds issued at time 0) that is unrelated to the term structure of interest rates. However, he does not demonstrate, as we do here, the existence of an optimal debt maturity strategy.[2]

This paper extends the models of Boyce and Kalotay [4] and Brennan and Schwartz [5] by allowing for default, possible agency costs, and a nonflat term structure of interest rates. Specifically, we emphasize the tax consequences of debt maturity when default is a possibility. We also analyze the equilibrium implications of debt maturity in an environment where both personal and corporate taxes are levied. This goes beyond the Boyce and Kalotay [4] disequilibrium model. Furthermore, we extend the Brick et al. [6] discussion of optimal debt maturity in providing a detailed tax-based decision rule for the debt maturity question.

I. The Irrelevance Proposition

To examine the relationship between debt maturity structure and the value of the firm, we must hold leverage constant. Therefore, the first issue we must address is how one defines constant leverage. A common approach in the literature has been to assume that the market value of debt remains equal across

[1] Brick et al. [6] have demonstrated the existence of an optimal debt maturity strategy with bankruptcy costs, but they do not develop general conditions to determine which maturity (short or long) strategy is optimal. Such conditions are obtained in Section II. Moreover, Brick et al. do not consider the equilibrium implications of their model.

[2] That is, are the (tax) benefits of a t-period long-term debt equivalent to the benefits of short-term debt whose principal is rolled over until period t?

alternative maturity structures (see Kraus [13], Morris [16, 17], and Boyce and Kalotay [4]). While this approach has an obvious intuitive appeal, it suffers from a conceptual drawback. Under this approach, the net payments to bondholders are not kept constant and hence, the value of equity is affected.

A second approach is to keep the after-tax payments to bondholders constant. This is the equivalent loan approach. This approach ensures that the value of the firm changes only as a result of variations in the debt maturity strategy. A variant of the second approach is to assume that the net (before-tax) payments to bondholders are kept constant. Although the irrelevance proposition, established in this section, is based upon this latter approach, the theorem ensures that all definitions yield equivalent results.

To prove our irrelevance proposition, we must also assume that the opportunity loss of the bond is tax-deductible. The opportunity loss of the bond is defined as the price appreciation which accrues between period $t-1$ and period t. The rationale for the tax deductibility of the opportunity loss is best explained in the following manner. Assume initially that the firm only issues pure discount bonds. Price appreciation of the bonds in each period is treated as a deductible interest expense, and price decline of the bonds is treated as taxable interest income. The price decline is income because the firm not only has the use of the funds for the period, but can also repay the liability for fewer dollars than it originally borrowed. The tax deductibility of the opportunity loss of bonds whose maturity at issue is less than one year is consistent with current U.S. tax laws. However, for bonds whose maturity exceeds one year, price appreciation is not tax-deductible, unless the firm retires or calls the bond.[3] We later relax the assumption that the opportunity loss of the bond is tax-deductible and demonstrate that then the debt maturity structure decision is relevant.

To facilitate the analysis, we assume a two-period model ($t = 0, 1, 2$), although our irrelevance proposition can be extended to an n-period model. We further make the initial simplifying assumption of no personal taxes. However, as we demonstrate below, personal taxes do not affect the proposition. Further, as we show later, the equilibrium implication of the interaction between personal and corporate taxation is that the Miller-type irrelevance prevails for debt maturity even under the existing tax code.

To prove our proposition, we examine the cash flows to stockholders and bondholders in each period. We assume that investment is given and that there is no retention of earnings. Because stockholders' claims are junior to the claims of bondholders, stockholders will receive a positive cash flow if and only if the firm completely satisfies the current claims of bondholders. Thus, at the end of the first period ($t = 1$), stockholders receive, in addition to the cash flow from operations, proceeds from refinancing (the issuance of new debt or equity at the beginning of the second period), less the currently due finance charges and taxes. Taxable income equals the cash flow from operations less the interest payment, the depreciation expense, and the opportunity loss of the bond.

The firm will default on its bond obligations if the cash flow from operations net of taxes and including the proceeds of refinancing is insufficient to cover

[3] Internal Revenue Code, Sec. 163.

current finance charges.[4] Stockholders will not lose more than their original capital upon default because of stockholders' limited liability. The above is summarized in Equation (1):

$$\tilde{\gamma}_1^s = \begin{cases} (\tilde{X}_1 - R_1 - P_1 - R_L) \\ - t(\tilde{X}_1 - R_1 - R_L - (P_1 - D_1) \\ - (D_{L_2} - D_{L_1}) - DP_1) + D_2 & \text{if } \tilde{X}_1 \geq \alpha \\ 0 & \text{if } \tilde{X}_1 < \alpha \end{cases} \tag{1}$$

where:

$\tilde{\gamma}_i^s$ = the random cash flow available to stockholders at period i

$\tilde{X}_i$ = the random cash flow from operations of period i

DP_i = depreciation expense of period i

R_i = the promised short-term (one-period) interest payment of period i

P_i = the promised short-term principal payment of period i

R_L = the promised long-term (two-period) interest payment

t = the corporate tax rate

D_i = the market value at the beginning of period i of the short-term bond

Hence, D_1 is the market value of the first period short-term bond at $t = 0$.

D_{L_i} = the market value at the beginning of period i of the long-term bond

S_2 = the market value of equity at the beginning of the second period

α = the point of bankruptcy

where

$$\begin{aligned} \alpha &= R_1 + P_1 + R_L - \frac{t(D_{L_2} - D_1 - D_{L_1}) + S_2 + D_2 + tDP_1}{1-t} \\ &= R_1 + R_L + P_1 - D_2 - \frac{t(D_2 + D_{L_2} - D_1 - D_{L_1}) + S_2 + tDP_1}{1-t} \end{aligned} \tag{2}$$

The following observations are in order. First, Equation (1) shows no explicit expression for the proceeds from the sale of new equity because it includes the total cash flow available to all stockholders, both new and old. Thus, the sale of new equity simply transfers cash from one set of stockholders (old) to another (new). Second, if the firm faces a cash shortfall, the stockholders can avoid default by issuing new equity (which is equivalent to liquidating a portion of their interest in the firm). The maximum shortfall which can be covered by this method is obviously the total value of the stockholder's equity. This explains the

[4] Our bankruptcy condition is strictly analogous to that of Scott [19] and differs from the Bulow and Shoven [7] criterion in that it is consistent with Fama and Miller's [11] "Me-First" Rules. Bulow and Shoven's [7] criterion allows the bank and stockholders to "defraud" the bondholders.

presence of S_2 in Equation (2). Finally, because we are keeping leverage constant, the stockholders cannot alter the leverage decision.

Note that when $\tilde{X}_1 > \alpha$, the firm will be able to pay existing bondholders in full, and new debt worth, D_2, will be issued. On the other hand, if $\tilde{X}_1 < \alpha$, the firm will default on its loan obligations. Thus, bondholders will receive the total cash flow of the firm and become the new owners of the firm's assets less any accrued income taxes. The net cash flow to bondholders (as a whole) is given as follows:

$$\tilde{\gamma}_1^D = \begin{cases} R_1 + P_1 + R_L - D_2 & \text{if } \tilde{X}_1 \geq \alpha \\ (1-t)\tilde{X}_1 + A_1 + tb_1 + tDP_1 & \text{if } \tilde{X}_1 \leq \alpha \quad \text{and} \quad b_1 > 0 \\ (1-t)\tilde{X}_1 + tDP_1 + A_1 & \text{if } \alpha > \tilde{X}_1 \geq DP_1 \quad \text{and} \quad b_1 < 0 \\ \tilde{X}_1 + A_1 & \text{if } -A_1 < \tilde{X}_1 < DP_1 \end{cases} \tag{3}$$

where:

$\tilde{\gamma}_i^D$ = the random cash flow available to bondholders at period i

A_i = the market value of the firm's assets at the end of period i, in particular, $A_1 = D_2 + D_{L_2} + S_2$[5]

b_1 = $\tilde{X}_1 + (A_1 - D_1 - D_{L_1} + tDP_1)/(1-t)$; b_i is the "legal" interest paid upon default in period i.[6]

In the second period, provided that $\tilde{X}_1 > \alpha$, the stockholders will receive cash flows from operations, net of financing charges, and taxes. Furthermore, at the end of the second period, the firm will liquidate itself[7] and if solvent, bondholders will be paid in full, and the remaining proceeds will be distributed to the stockholders. The firm will default on its loan obligations if cash flow available

[5] We may perceive the cash flow of the firm in the second period given default in the first period in the following way. Assume that the bondholders are the new owners of the firm and sell their equity shares to outside investors. The firm will therefore continue its operations. We may equivalently assume that bondholders sell their shares of equity, their bonds, and any new bonds to outside investors, and therefore leave the firm. A_1 is the value of the continued operations.

[6] We are assuming that, in the event of default, the firm receives a tax benefit of debt only for interest paid. If the firm cannot fully pay the required coupon payment and principal payment upon default, the tax law [Newhouse 50 T.C. 783 (March 12, 1973)] defines interest as the amount paid to bondholders in excess of the principal payment. In our model, the interest payment upon default, b_i, is fully consistent with the above principal-first criterion. If bonds are not originally issued at par, the principal payment must reflect the amortization of the original premium or discount. Assuming the opportunity loss of the bond is tax-deductible implies that for period 1, the principal payment would simply equal the original value of the bond. Consequently, to determine the interest payment in the event of default, we set the total interest paid upon default, b_1, plus the principal payment (the original value of the bond) to the total cash flow of the firm. Thus, we solve for b_1 in the following equation:

$$\tilde{X}_1 + A_1 - t(\tilde{X}_1 - b_1 - DP_1) = b_1 + D_1 + D_{L_1} \qquad \text{if } \tilde{X}_1 < \alpha \quad \text{and} \quad b_1 > 0.$$

We thank Lemma Senbet for pointing out an error in our earlier draft.

[7] If the liquidating value of the firm differs from the book value, there will be additional tax consequences. However, these additional tax consequences are independent of the debt maturity policy. Therefore, our irrelevance proposition will remain intact even if we explicitly consider the taxability of salvage value.

to stockholders is negative. In this case, the bondholders will receive the entire proceeds. Thus, we have:

$$\tilde{\gamma}_2^s = \begin{cases} (\tilde{X}_2 - R_2 - P_2 - R_L - P_L) \\ - t(\tilde{X}_2 - R_2 - R_L - DP_2) + t((P_L - D_{L_2}) \\ + (P_2 - D_2)) + A_2 & \text{if } \tilde{X}_2 \geq \beta \\ 0 & \text{if } \tilde{X}_2 < \beta \end{cases} \tag{4}$$

$$\tilde{\gamma}_2^D = \begin{cases} R_2 + P_2 + R_L + P_L & \text{if } \tilde{X}_2 \geq \beta \\ (1-t)\tilde{X}_2 + A_2 + b_2 + tDP_2 & \text{if } \tilde{X}_2 \leq \beta \quad \text{and} \quad b_2 \geq 0 \\ (1-t)\tilde{X}_2 + tDP_2 + A_2 & \text{if } \beta > \tilde{X}_2 > DP_2 \quad \text{and} \quad b_2 < 0 \\ \tilde{X}_2 + A_2 & \text{if } -A_2 < \tilde{X}_2 \leq DP_2 \end{cases} \tag{5}$$

and

$$\beta = R_2 + P_2 + R_L + P_L + \frac{t(D_{L_2} + D_2) - A_2 - tDP_2}{1 - t} \tag{6}$$

where

P_L = the promised principal payment of long-term debt

$b_2 = \tilde{X}_2 + (A_2 - D_2 - D_{L_2} + tDP_2)/(1 - t)$.[8]

The theorem below sets the conditions for irrelevance of debt maturity structure under the additional assumption that net payments to bondholders are equal under all maturity structures in question.

IRRELEVANCE THEOREM. *Assuming stochastic cash flows with a known distribution function (all other variables are known with certainty), perfect capital markets (except possibly for taxes), temporal independence of cash flows and that the opportunity loss of bonds is tax-deductible, then changes in the debt maturity structure will not affect the current value of the firm.*

Proof: See Appendix.

Discussion

The irrelevance proposition is consistent with Stiglitz's [20] irrelevance theorem which argues that, if the state space is spanned by the securities markets, then changes in the maturity structure cannot add anything to the present set of investment opportunities available. However, Stiglitz's theorem was proven for a frictionless market with no taxes and no bankruptcy costs implying the irrelevance of the firm's financing policies altogether. Our proposition extends Stiglitz's results to an environment where the capital structure is relevant by virtue of taxes. We have shown that, if interest and price variations of the bond are perfect substitutes for tax purposes, then even in a non-Miller [15] equilibrium environment, each debt maturity structure has the same tax consequences (although capital structure still matters). Consequently, any debt maturity structure which yields the same equivalent net payments to bondholders will not alter

[8] See footnote 6.

the investment opportunities to investors. Moreover, under the assumptions set forth by the irrelevance proposition, the portion of the fixed payment which is treated as interest expense is independent of the assumed debt maturity structure. Since total before-personal-tax income to both stockholders and bondholders will also be independent of the debt maturity, the introduction of personal taxes will not alter our conclusion. In sum, as demonstrated below, it is the asymmetric tax treatment of bondholder cash flows (price variation and interest) that induces different valuation of different maturity strategies and not the tax liability itself. This is despite the fact that taxes may result in an optimal capital structure.

It is also easy to see that such maturity structure irrelevance is valid even in the context of the traditional tax-bankruptcy models of optimal capital structure if bankruptcy costs are independent of the debt maturity strategy. Such bankruptcy costs, as distinct from agency costs, include the loss of interest tax shields upon default, court fees, and indirect costs related to the disruption of normal customer-supplier relationships.[9]

Finally, given the set of conditions leading to the irrelevance proposition, the three approaches of defining constant leverage are equivalent. First, the value of debt in each period is independent of the debt maturity only if the payments to bondholders in each period are the same for alternative debt maturity strategies. Second, as we have already noted, the assumption that the opportunity loss of the bond is tax-deductible implies that the definition of interest and principal payments is independent of the debt maturity structure (i.e., the symmetric tax treatment of the cash flows paid to bondholders). Hence, if before-tax payments to bondholders are the same under alternative debt maturity strategies, then after-tax payments to bondholders are also equal under all maturity structures in question.[10]

[9] On the other hand, agency costs, such as those arising from moral hazard or informational asymmetry, can be minimized if the firm selects an appropriate maturity strategy (cf. [1, 14, 18]). Bankruptcy costs can also be minimized through informal reorganization in the capital markets (cf. [12]).

[10] Let F be the promised fixed payment in the second period and D be the present value of F at the beginning of the period. Assume that the two debt maturity strategies have the same after-tax payment, in the second period, i.e.,

$$F - t(F - D) = \hat{F} - t(\hat{F} - \hat{D}). \tag{F1}$$

We wish to prove that $F = \hat{F}$. Assume initially that $F > \hat{F}$, then from (F1) we have:

$$(1 - t)(F - \hat{F}) = t(\hat{D} - D). \tag{F2}$$

(F2) implies that if $F > \hat{F}$ then $\hat{D} > D$. By the assumption of identical after-tax payments, $\beta = \hat{\beta}$ and $b_2 > \hat{b}_2$ because $\hat{D} > D$. For simplicity, we will assume $DP = 0$ and $A = 0$. Hence $\beta = tD/(1 - t) + F$ and $b_2 = \tilde{X}_2 - D/(1 - t)$. Define d_2 as the wealth of the bondholders at the end of period 2. Therefore,

$$d_2 - \hat{d}_2 \geq \begin{cases} (F - \hat{F}) & \text{if } \tilde{X}_2 > \beta \\ t(\hat{D} - D) & \text{if } \tilde{X}_2 \leq \beta \quad \text{and } \hat{b}_2 > 0 \\ tb_2 & \text{if } \hat{b}_2 < 0 \quad \text{but} \quad b_2 > 0 \\ 0 & \text{if } b_2 < 0. \end{cases} \tag{F3}$$

Hence, $d_2 - d_2 > 0$, which implies that $\hat{D} < D$. A similar contradiction is obtained by assuming that $F < \hat{F}$. Thus, if the after-tax payments are the same in the second period then the before-tax payments, F, must also be equivalent. We can derive a similar proposition for payments in period 1.

II. When Does Optimal Maturity Exist?

In this section, we shall argue that violating the assumptions of our irrelevance proposition leads to the existence of optimal debt maturity. Specifically, we contend that if the opportunity loss of the bond is not tax-deductible, there is an optimal debt maturity structure.[11] A formal proof is difficult to construct, but we provide a simplified analytic model which illustrates our claim and highlights the important tax issues pertaining to debt maturity.

To simplify the presentation, we assume that upon default, the firm loses everything, debt is sold at par, and no depreciation tax shelters are allowed. Only interest income is taxed at the individual level, whereas, for simplicity sake, the personal tax on equity is assumed to be zero. Furthermore, we assume risk neutrality. Since the value of equity is the same for all maturity strategies,[12] we shall only examine the value of the debt. Retaining all the previous notation, the total value of debt, V_D, is simply:

$$V_D = [(1 + r_1(1 - t_b))D_1 + r_L(1 - t_b)D_{L1} + D_{L2}](1 - F_1)(1 + R_{F1})^{-1} \tag{7}$$

where

r_i = the coupon rate of short-term debt issued at period i

r_L = the coupon rate of a long-term par bond

t_b = the marginal tax rate applicable to bond income

R_{Fi} = the single-period, tax-exempt, risk-free interest rate of period i

F_i = the probability of default in period i.

For the value of equity to be independent of the debt maturity strategy, the after-tax corporate payments to bondholders must be identical in each period for all debt maturity strategies. Let K_i be the total after-tax corporate payments to bondholders in period i. Thus,

$$[1 + (1 - t)r_1]D_1 + (1 - t)r_L D_{L1} - D_2 = K_1 \tag{8}$$

$$[1 + (1 - t)r_L]D_{L1} + [1 + (1 - t)r_2]D_2 = K_2. \tag{9}$$

The assumption that everything is lost upon bankruptcy enables us to write

$$(1 + r_1(1 - t_b)) = (1 + R_{F1})/(1 - F_1) \tag{10}$$

$$(1 + r_2(1 - t_b)) = (1 + R_{F2})/(1 - F_2) \tag{11}$$

and

$$r_L = \frac{(1 + R_{F1})(1 + R_{F2}) - (1 - F_1)(1 - F_2)}{[(1 + R_{F2})(1 - F_1) + (1 - F_1)(1 - F_2)](1 - t_b)}. \tag{12}$$

[11] Violation of other assumptions, such as certainty of interest rates and independence of cash flows over time, can also be shown to invalidate the conclusion.

[12] We are still keeping the after-tax payments of the different maturity strategies constant. Here, after-tax, as in the no personal tax case, means after corporate tax, not personal tax. Consequently, the net payments stockholders make to bondholders are not affected by the debt maturity strategy, and hence, the value of equity is held constant.

Substituting Equations (8) and (9) into Equation (7), one obtains a value expression in terms of the sole variable, D_{L1}. Furthermore, the restrictions set forth by Equations (8) and (9) imply that F_1 and F_2 are independent of the debt maturity strategy. This, in turn, implies that r_1, r_2, and r_L [as given by Equations (10) to (12)] are independent functions of exogenous variables. Taking the derivative of Equation (7) with respect to D_{L1}, and rearranging, we obtain:

$$\frac{\partial V_D}{\partial D_{L1}} = \frac{r_{Lb}(1 + r_{2b}) + (1 + r_{Lb})}{(1 + r_{1b})(1 + r_{2b})} - \frac{r_{Lc}(1 + r_{2c}) + (1 + r_{Lc})}{(1 + r_{1c})(1 + r_{2c})} \tag{13}$$

where

$$r_{ib} = r_i(1 - t_b) \qquad r_{ic} = r_i(1 - t).$$

If Equation (13) is positive (negative), then long-term (short-term) debt is optimal.[13] One can readily derive from Equation (13) that $\partial V/\partial D_{L1} = 0$, and the debt maturity is irrelevant if either (a) $r_1 = r_2$, or (b) $t = t_b$. The intuition behind the results is fairly straightforward. If $r_1 = r_2$, then $r_1 = r_2 = r_L$. (This can happen, for instance, when $R_{F1} = R_{F2}$ and $F_1 = F_2$.) In this case, we have a flat term structure of corporate interest rates. This implies (given the additional requirement of a constant after-tax payment to bondholders) that the total interest payment in each period is independent of the assumed maturity strategy. Consequently, irrelevance prevails. If $t = t_b$, we get a Miller-type result. In essence, interest payments for any maturity strategy offer no tax benefits because the corporate tax benefit of debt is offset completely by the personal tax advantage of equity.

In all other cases, the sign of Equation (13) is ambiguous. We performed several computations, letting R_{F1} and R_{F2} vary in increments of 1% from 5% to 26%. F_1 and F_2 were increased in increments of 1% from 1% to 21%. These assumed values of R_{F1}, R_{F2}, F_1 and F_2 determine r_1, r_2, and r_L. Note that Equation (13), and hence our simulation, is *distribution-free.* We found that for $t_b < t$, $\partial V/\partial D_{L1} > 0$ if $r_1 < r_L < r_2$, and $\partial V/\partial D_{L1} < 0$ if $r_1 > r_L > r_2$. The sign of the derivative in Equation (13) is reversed if $t_b > t$.

We could give some intuitive justification to our results. Recall that by construction, the after-tax payments to bondholders are identical for all maturity strategies. For $t_b < t$, if the term structure of *corporate coupon rates* is increasing, i.e., $r_2 > r_L > r_1$, then long-term debt maturity strategy becomes optimal because the tax benefit of debt is accelerated (i.e., the long-term debt pays more interest in the first period and less interest in the second period than does the short-term debt). On the other hand, if the term structure of corporate coupon rates is decreasing i.e., $r_1 > r_L > r_2$, a short-term debt maturity strategy is optimal. That is, if $t_b < t$, there is a net tax advantage to debt and hence, acceleration of (the tax benefits of) interest payments is beneficial. On the other hand, if $t_b > t$, the gain from leverage is negative. Consequently, the lower the present value of interest payments, the greater the value of the firm. This causes the reversal of the previous conclusions.

[13] We must interpret Equation (13) carefully. For example, if $\partial V/\partial D_{L1} > 0$, then the firm should issue long-term debt up until the point that D_1 and/or D_2 of Equations (8) and (9) become negative.

We also examine a more sophisticated model based upon Equations (1) to (6). We no longer assume that the opportunity loss of the bond is tax-deductible but retain the assumption that debt is sold at par. In particular, we set r_L so that it includes a default risk premium. We then equate the promised after-tax payments of the short-term debt to the promised after-tax payments of a long-term par bond with a coupon rate r_L. Since the expected value of payments to bondholders is a function of the market value of debt (i.e., there is no closed form valuation), we use an iterative numerical technique to compute the value of D_1, D_2, and D_{L_1}. Table 1 presents some of the results in the case where $t_b = 0$. We assume that both $\tilde{X}_1$ and $\tilde{X}_2$ are uniformly distributed where the range of $\tilde{X}_1$ is between \$5 and \$300 and the range of $\tilde{X}_2$ is between \$10 and \$875. We assume no depreciation expense and no salvage value at the end of the second period. (A more detailed description and set of results are available from the authors upon request.) Note that, as in our analytical model, we keep the after-tax payments constant. Hence, the optimal debt maturity policy is defined as the one maximizing the current value of debt.

Our simulation results are analogous to the results derived from the analytical model. In particular, where $r_1 < r_L < r_2$ (even if $R_{F1} = R_{F2}$), then $D_{L1} > D_1$, implying that long-term debt is optimal. When $r_1 > r_L > r_2$, short-term debt is optimal.[14] There is always a question about the robustness of a simulation analysis, and it certainly applies here. However, we have obtained qualitatively similar results when model parameters have been changed, and the general conclusions are consistent with our analytical model.

III. Equilibrium Implications

Our analysis so far has indicated that if we assume (as Miller did) that all firms are taxed at rate t and that the marginal investor has a tax rate of $t_b = t$, a Miller-type equilibrium will be obtained,[15] and firms will be indifferent between financ-

[14] Brick et al. [6] show that the improvement in the value of the firm obtained from an optimal debt maturity can be very substantial.

[15] There is one interesting implication from our assumption that upon default the firm receives tax benefits on interest paid (the principal first doctrine). If, additionally, we assume that tax shelters can only be lost upon default, then nondebt-related tax shields do not perturb the Miller-type equilibrium. This is in contrast to results obtained, for example, by DeAngelo and Masulis [9]. Under the principal first doctrine, interest benefits are always subordinated to the tax benefits derived from noninterest-related tax shields. In contrast, under the wealth tax assumption of DeAngelo and Masulis, the two types of tax credits are of the same seniority status.

More formally, let R be the promised interest payment, let B be the net principal payment to bondholders (i.e., $B < 0$ implies that the firm borrows additional funds), and assume that the firm issues only par bonds. Then, the total cash flow to the firm's stockholders and bondholders is given by (F4) and (F5) below:

$$\gamma^s = \begin{cases} (1-t)(\tilde{X} - R) - B + tDP & \text{if } \tilde{X} > \alpha \\ 0 & \text{if } \tilde{X} \le \alpha \end{cases} \tag{F4}$$

$$\gamma^D = \begin{cases} R(1-t_b) + B & \text{if } \tilde{X} > \alpha \\ (1-t)\tilde{X} + (t-t_b)b + A + tDP & \text{if } \tilde{X} < \alpha,\ b > 0 \\ (1-t)\tilde{X} + tDP + A & \text{if } \tilde{X} > DP, \text{ and } b < 0 \\ \tilde{X} + A & \text{if } -A < \tilde{X} < DP \end{cases} \tag{F5}$$

ing with short-term debt, long-term debt, and equity. It is not clear *a priori*, however, that firms will indeed reach such equilibrium in our framework. To investigate this question, we will invoke the Boyce and Kalotay [4] analysis which derives investors' aggregate demand conditions. (Note, we do not use the Boyce-Kalotay model to derive the firms' supply response function, since their analysis does not keep leverage constant.) Denote by t^* the tax rate implied by the yield differential between taxable corporate bonds and similar but tax-exempt securities. Boyce and Kalotay demonstrate that the wealth of investors with $t_b < t^*$ is maximized when such investors buy long- (short-)term debt if the term structure of interest rates is rising (falling). Although their analysis does not allow for equity financing, it is clear from Miller's analysis that investors with $t_b = t^*$ would be indifferent between debt (either short-term or long-term) and equity. By the same token, investors with $t_b > t^*$ would find that equity investment maximizes their after-tax return. [Because they do not consider equity financing, Boyce and Kalotay show that investors for whom $t_b > t^*$ would prefer to hold the short-term (long-term) maturity strategy if the term structure of interest rates is rising (falling).] Firms, in competition with each other, would raise the yield of debt to attract more investors away from the tax-exempt equity market. This would continue until $t^* = t$, where the firm's financing decision becomes irrelevant. However, there would be both an optimal level of debt and an optimal debt maturity composition for the economy.

Finally, we shall briefly discuss agency costs in our framework. Even if agency costs are independent of the debt maturity policy, there will be an optimal debt maturity structure. As demonstrated by DeAngelo and Masulis (9) and Barnea, Haugen, and Senbet (2), the supply curve of corporate debt, in this case, will slope downwards which in turn implies that $t^* < t$. Equation (13) shows that as long as t is not equal to t^*, there will be an optimal debt maturity structure for each firm. If, however, agency costs are related to the maturity policy, then the optimal debt maturity will be a result of agency cost minimization in addition to the tax considerations we have mentioned.

IV. Concluding Remarks

In this paper, we have presented a tax-induced framework to analyze debt maturity problems. We have shown that under some modifications of the existing U.S. tax code, debt maturity is irrelevant even in the presence of taxes and bankruptcy costs that render capital structure relevant. In that regard, we have extended Stiglitz's framework to an environment characterized by tax imperfec-

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where

$$b = X + (A - P + tDP)/(1 - t)$$

P = the promised principal payment.

Adding (F4) to (F5), one can readily show that if $t = t_b$, the cash flow of the levered firm equals the cash flow of the unlevered firm. Consequently, if $t = t_b$, the firm's financing policies are irrelevant, even when $DP > 0$.

Table I

Value of Long-Term and Short-Term Debt (No Personal Taxes)

D_L	r_L	D_1	r_1	D_2	r_2	ALPHA[a]	BETA[b]	R_{F1}	R_{F2}
$82.45	0.1000	$82.21	0.05	$80.14	0.1605	1.0	0.8114	0.05	0.05
$82.34	0.1050	$82.06	0.05	$79.79	0.1722	1.0	0.8112	0.05	0.06
$82.26	0.1100	$81.95	0.05	$79.47	0.1839	1.0	0.8109	0.05	0.07
$83.44	0.1584	$82.67	0.05	$78.13	0.3049	1.0	0.8034	0.05	0.17
$83.73	0.1631	$82.91	0.05	$78.16	0.3174	1.0	0.8022	0.05	0.18
$84.06	0.1678	$83.18	0.05	$78.21	0.3300	1.0	0.8009	0.05	0.19
$68.95	0.1678	$69.03	0.19	$69.80	0.1416	1.0	0.8388	0.19	0.05
$68.86	0.1731	$68.92	0.19	$69.51	0.1530	1.0	0.8386	0.19	0.06
$68.81	0.1784	$68.85	0.19	$69.26	0.1644	1.0	0.8383	0.19	0.07
$68.80	0.1837	$68.82	0.19	$69.04	0.1760	1.0	0.8379	0.19	0.08
$68.81	0.1889	$68.82	0.19	$68.86	0.1876	1.0	0.8374	0.19	0.09
$68.87	0.1941	$68.85	0.19	$68.71	0.1992	1.0	0.8369	0.19	0.10
$68.95	0.1993	$68.92	0.19	$68.59	0.2109	1.0	0.8362	0.19	0.11
$70.13	0.2300	$69.93	0.19	$68.51	0.2828	1.0	0.8308	0.19	0.17
$70.43	0.2350	$70.20	0.19	$68.59	0.2950	1.0	0.8296	0.19	0.18
$70.76	0.2400	$70.49	0.19	$68.70	0.3073	1.0	0.8283	0.19	0.19

[a] ALPHA is the probability of solvency in period 1.

[b] BETA is the probability of solvency in period 2.

tions. It is these imperfections and not taxes per se that cause the relevance of debt maturity structure. If this restrictive structure is relaxed, tax reasons would usually imply the existence of an optimal debt maturity, assuming Miller equilibrium does not prevail. An increasing term structure of interest rates, adjusted for default risk, usually results in long-term debt being optimal, whereas a decreasing term structure renders short-term debt optimal. This reasoning provides an interesting argument as to why long-term debt may be issued even though from agency costs considerations, short-term debt is often optimal (cf. Barnea, Haugen, and Senbet [1]). It may be conjectured that, in the presence of a rising term structure of interest rates, long-term debt, with appropriate call and convertible features, may play a role in maximizing the tax benefits of debt and simultaneously resolving agency conflicts which arise due to informational asymmetry and moral hazard.

The introduction of personal taxes does not substantially alter our argument, except that a Miller [15]-type result may emerge at equilibrium and irrelevance will then prevail even in the absence of our restrictive conditions. However, we also argue that agency costs would again reverse the irrelevance and imply a firm-specific optimal debt maturity structure.

Appendix

In this Appendix, we provide the proof for the irrelevance proposition. Adding Equation (1) to (3) and (4) to (5) yields the total cash flows to bondholders and to stockholders for each period, respectively. More formally, we have:

$$\gamma_1 = \begin{cases} (1-t)\tilde{X}_1 \\ \quad + t(R_1 + R_L + P_1 - D_1 + D_{L_2} - D_{L_1} + DP_1) & \text{if } X_1 \geq \alpha \\ (1-t)\tilde{X}_1 + tb_1 + A_1 + tDP_1 & \text{if } \tilde{X}_1 < \alpha, b_1 \geq 0 \\ (1-t)\tilde{X}_1 + tDP_1 + A_1 & \text{if } \tilde{X}_1 > DP_1 \text{ and } b_1 \leq 0 \\ \tilde{X}_1 + A_1 & \text{if } -A_1 < \tilde{X}_1 \leq DP_1 \end{cases} \tag{A1}$$

$$(\gamma_2 \mid \tilde{X}_1 \geq \alpha) = \begin{cases} (1-t)\tilde{X}_2 + \\ \quad t(R_2 + R_L + P_2 + P_L - D_{L_2} - D_2 + DP_2) + A_2 & \text{if } \tilde{X}_2 > \beta \\ (1-t)\tilde{X}_2 + A_2 + tb_2 + tDP_2 & \text{if } \tilde{X}_2 < \beta, b_2 \geq 0 \\ (1-t)\tilde{X}_2 + A_2 + tDP_2 & \text{if } \tilde{X}_2 > DP_2 \text{ and } b_2 < 0 \\ \tilde{X}_2 + A_2 & \text{if } -A_2 < \tilde{X}_2 \leq DP_2. \end{cases} \tag{A2}$$

Assume a firm chooses another maturity policy, $\hat{R}_1$, $\hat{P}_1$, $\hat{R}_L$, $\hat{P}_L$, $\hat{R}_2$, and $\hat{P}_2$ where

$$\text{Condition (A):} \quad R_2 + R_L + P_2 + P_L = \hat{R}_2 + \hat{P}_2 + \hat{R}_L + \hat{P}_L$$

and

$$\text{Condition (B):}\quad R_1 + P_1 + R_L - D_2 = \hat{R}_1 + \hat{P}_1 + \hat{R}_L - \hat{D}_2.^{16}$$

hold. Conditions (A) and (B) ensure that, in each period, the net payments to bondholders are identical for all debt maturity strategies. We see that if $\hat{D}_1 + \hat{D}_{L_1} = D_1 + D_{L_1}$ and $D_2 + D_{L_2} = \hat{D}_2 + \hat{D}_{L_2}$, then $\alpha = \hat{\alpha}$, $\beta = \hat{\beta}$, and $\hat{b}_i = b_i$. This, in turn, implies that $\gamma_i = \hat{\gamma}_i$. Hence, we will demonstrate that in each period, the total value of debt is the same for any debt maturity policy.

By assumption, the promised net payments to the bondholders of the second period are the same for each maturity policy [Condition (A)]. Assume that $\hat{D}_{L_2} + \hat{D}_2 > D_{L_2} + D_2$. Then, $\hat{\beta} > \beta$ and $\hat{b}_2 < b_2$. Define $\Delta = \hat{\gamma}_2^D - \gamma_2^D$. Then

$$\Delta \leq \begin{cases} 0 & \text{if } \tilde{X}_2 \geq \hat{\beta} \\ 0 & \text{if } \hat{\beta} > \tilde{X}_2 \geq \beta \quad \text{and} \quad \hat{b}_2 > 0 \\ t(\hat{D}_2 + \hat{D}_{L_2} - \hat{R}_L - \hat{R}_2 - \hat{P}_L - \hat{P}_2) & \text{if } \hat{\beta} > \tilde{X}_2 \geq \beta \quad \text{and} \quad \hat{b}_2 < 0 \\ t(\hat{b}_2 - b_2) & \text{if } \tilde{X}_2 < \beta \quad \text{and} \quad \hat{b}_2 > 0 \\ -tb_2 & \text{if } b_2 \geq 0 > \hat{b}_2 \\ 0 & \text{if } b_2 < 0 \end{cases}$$

which implies $\Delta \leq 0$ and is negative in at least one case. Thus, at equilibrium, we would expect $\hat{D}_{L_2} + \hat{D}_2 < D_{L_2} + D_2$ and, hence, we would have a contradiction. If we assume that $\hat{D}_{L_2} + \hat{D}_2 < D_{L_2} + D_2$, we derive a similar contradiction. Consequently, it must be that $\hat{D}_{L_2} + \hat{D}_2 = D_{L_2} + D_2$. Hence, all we need to show is that $\hat{D}_1 + \hat{D}_{L_1} = D_1 + D_{L_1}$.

If $D_1 + D_{L_1} > \hat{D}_1 + \hat{D}_{L_1}$, then $\alpha > \hat{\alpha}$, and $b_1 < \hat{b}_1$. Define d_1 as the wealth of existing bondholders at the end of period 1; that is, d_1 is the cash flow to existing bondholders plus the value of outstanding debt. Hence,[17] we have:

$$d_1 - \hat{d}_1 \leq \begin{cases} 0 & \text{if } \tilde{X}_1 \geq \alpha \\ 0 & \text{if } \alpha \geq \tilde{X}_1 > \hat{\alpha}, b_1 > 0 \\ t(D_{L_1} + D_1 - R_1 - R_L - P_1 - D_{L_2}) & \text{if } \alpha > \tilde{X}_1 > \hat{\alpha}, b_1 < 0 \\ -t(D_1 + D_{L_1}) + t(\hat{D}_1 + \hat{D}_{L_1}) & \text{if } \tilde{X}_1 < \hat{\alpha}, b_1 > 0 \\ -t\hat{b}_1 & \text{if } b_1 < 0, \hat{b}_1 > 0 \\ 0 & \text{if } \hat{b}_1 < 0 \end{cases} \tag{A3}$$

which implies $\gamma_1^D - \hat{\gamma}_1^D \leq 0$. Thus, at equilibrium, we would expect $D_1 + D_{L_1} \leq \hat{D}_1 + \hat{D}_{L_1}$ and, hence, we have a contradiction. If we assume $D_1 + D_{L_1} < \hat{D}_1 + \hat{D}_{L_1}$, we arrive at a similar contradiction. Consequently, $D_1 + D_{L_1} = \hat{D}_1 + \hat{D}_{L_1}$ and hence $\alpha = \hat{\alpha}$, and $b_1 = \hat{b}_1$. This, in turn, implies $\gamma_1 - \hat{\gamma}_1 = 0$. Q.E.D.

[16] Note that condition B does *not* imply $\hat{R}_1 + \hat{P}_1 + \hat{R}_L + \hat{D}_{L_2} = R_1 + P_1 + R_L + D_{L_2}$ (i.e., that the total promised wealth is the same for the two maturity strategies).

[17] We have shown that $D_{L_2} + D_2 = \hat{D}_{L_2} + \hat{D}_2$ and $\beta = \hat{\beta}$. Thus, $\gamma_2 = \hat{\gamma}_2$, and therefore the value of the firm at the beginning of the second period is the same for both maturity policies. Hence, $S_2 = \hat{S}_2$.

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