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Asset Pricing, Higher Moments, and the Market Risk Premium: A Note

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FOLLOWING THE WORK OF Rubinstein [4], Kraus and Litzenberger (KL) [2] derived and tested a linear three-moment pricing model, finding the additional variable (co-skewness) to explain the empirical anomalies of the two-moment Capital Asset Pricing Model (CAPM). The three-moment model was reexamined by Friend and Westerfield (FW) [1] with mixed results.

The purpose of this note is to examine why the market risk premium ($\bar{R}_M - R_f$) may influence tests of asset pricing models with higher moments. When skewness is added to a pricing model developed within the usual two-fund separation assumptions, the market risk premium enters the pricing equation in a nonlinear fashion and is implicit in the estimation of *each* moment's coefficient. Unless this nonlinearity is recognized, incorrect conclusions regarding the tests of such models may result. Section I examines the implications of the risk premium nonlinearity while Section II contains a brief summary.

I. Skewness and the Market Risk Premium

Using the framework and notation developed in KL [2], the theoretical market equilibrium relationship between security excess returns ($\bar{R}_i - R_f$), the market risk premium ($\bar{R}_M - R_f$), systematic risk (β_i) and systematic skewness (γ_i) is:

$$\bar{R}_i - R_f = [(\bar{R}_M - R_f)/(1 + K_3)]\beta_i + [K_3(\bar{R}_M - R_f)/(1 + K_3)]\gamma_i \qquad (1)$$

where $K_3 = [(d\bar{W}/dm_W)/(d\bar{W}/d\sigma_W)](m_M/\sigma_M)$, the market's marginal rate of substitution between skewness and risk times the risk-adjusted skewness of the market portfolio.

The KL version of the model is given by (KL 3):

$$\bar{R}_i - R_f = [(d\bar{W}/d\sigma_W)\sigma_M]\beta_i + [(d\bar{W}/dm_W)m_M]\gamma_i. \qquad \text{(KL 3)}$$

Kraus and Litzenberger recognize that, in equilibrium, (KL 3) also implies the following condition:

$$\bar{R}_M - R_f = [(d\bar{W}/d\sigma_W)\sigma_M] + [(d\bar{W}/dm_W)m_M] \qquad (2)$$

since $\beta_M = \gamma_M = 1$. Thus, (2) produces one empirical hypothesis of the model—that the *sum* of the estimated coefficients should equal ($\bar{R}_M - R_f$). However, the (KL 3) specification does not reveal all of the information in the theoretical

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model since it does not specify the effects of the market risk premium on the individual coefficients of β_i and γ_i. That is, the development of (KL 3) and (2) also impose restrictions between $(\bar{R}_M - R_f)$ and each of the coefficients individually. These effects can be seen by dividing (KL 3) by (2) which produces Equation (1). Equation (1) indicates that $(\bar{R}_M - R_f)$ is implicit in each of the model's coefficients on β_i and γ_i. Although (KL 3) and (1) both come from the same theoretical model, the derivation of (1) is consistent with the risk premium formulation of the two-moment CAPM[1] and brings to light why empirical tests of the three-moment model may be sensitive to the market risk premium.

Consider the linear empirical version of (1):

$$\bar{R}_i - R_f = b_0 + b_1\beta_i + b_2\gamma_i \tag{3}$$

where

$b_0 =$ intercept, hypothesized to equal 0

$b_1 = [(\bar{R}_M - R_f)/(1 + K_3)]$

$b_2 = [K_3(\bar{R}_M - R_f)/(1 + K_3)].$

Previous studies have focused upon the entire coefficients of β_i and γ_i, b_1, and b_2, in the examination of risk and skewness. Such examinations, however, measure the joint effects of $(\bar{R}_M - R_f)$ and K_3. Failure to separate $(\bar{R}_M - R_f)$ from K_3 may result in incorrect inferences regarding the importance as well as the sign of risk and skewness.

Consistent with the two-moment model [a special case of (1)], the importance of risk is more properly measured by $(\bar{R}_M - R_f) = b_1 + b_2$, rather than b_1; likewise, the importance of skewness, K_3, is gauged by b_2/b_1, rather than b_2. Thus, skewness is evaluated on a relative basis as measured by the market's tradeoff between skewness and risk.[2] It is instructive to note that K_3 which equals b_2/b_1 is the ratio of two random variables. Therefore, the average value of the ratio over time will not necessarily equal the ratio of the two time series average values (its value can be approximated via a Taylor series expansion; see [3, p. 181]). Therefore, a

[1] In the two-moment version of the model, the investor's problem is to maximize: $E[U(W)] = U[\bar{W}, \sigma_W]$ subject to: $\sum_i q_i + q_f = W_0$. The equilibrium conditions are:

$$(\bar{R}_i - R_f) = [(d\bar{W}/d\sigma_W)\sigma_M]\beta_i \tag{a}$$

$$(\bar{R}_M - R_f) = [(d\bar{W}/d\sigma_W)\sigma_M]. \tag{b}$$

Dividing (a) by (b) produces the familiar two-moment CAPM in terms of the market risk premium, $\bar{R}_i - R_f = (\bar{R}_M - R_f)\beta_i$. In both the two-moment model and Equation (1), the market's marginal rate of substitution between return and risk times market risk, $(d\bar{W}/d\sigma_W]\sigma_M$, is not present in the final equation. However, the contribution of skewness to the model is evaluated by the relative importance of the third moment vis-à-vis the second moment (K_3).

[2] When $|K_3| < 1$ (>1), risk is more (less) important than skewness in the pricing of assets. When $|K_3| = 1$, the market views risk and skewness as equally important. When $K_3 = 1$, Equation (1) becomes:

$$\bar{R}_i - R_f = ½(\bar{R}_M - R_f)(\beta_i + \gamma_i).$$

However, when $K_3 = -1$, the theoretical model specification becomes ambiguous since as $K_3 \rightarrow -1$

reexamination of the statistical results found in FW [1] and KL [2] requires a specification of the sampling properties of K_3.

In addition, because of the interaction between $(\bar{R}_M - R_f)$ and K_3 in the determination of b_1 and b_2, under certain conditions b_1 and b_2 may also give misleading signals regarding the signs of risk and skewness in the model. When the empirical model estimates a negative market premium $(b_1 + b_2 < 0)$, b_2 may attach the wrong sign to skewness. When $(\bar{R}_M - R_f) < 0$ and $K_3 < 0$, $b_2 > 0$. Similarly, when $(\bar{R}_M - R_f) < 0$ and $K_3 > 0$, $b_2 < 0$.[3] Since the linear model focuses on b_2 rather than K_3, its use may lead to incorrect conclusions regarding the sign of skewness when $(\bar{R}_M - R_f) < 0$. In similar fashion, when $K_3 < -1$, the use of b_1, rather than $b_1 + b_2$, may result in an incorrect conclusion regarding the sign of risk. Friend and Westerfield [1] discovered this interaction when they divided the periods into cases where $\bar{R}_M > R_f$ and where $\bar{R}_M < R_f$ (see FW [1], Tables VI–X). Equation (1) explicitly shows how this will occur.

II. Conclusions

Recent research has examined the importance of skewness in the pricing of risky assets, finding the results of such tests to be influenced by the market risk premium. The purpose of this note has been to explore a not so obvious theoretical relationship within such models—namely, that such models are intrinsically nonlinear in the market risk premium. Failure to account for this interaction may lead to erroneous conclusions regarding the empirical results of such models.

$\Rightarrow (\bar{R}_M - R_f) \rightarrow 0$ and:

$$\begin{aligned}\bar{R}_i - R_f &= \lim_{\substack{K_3 \to -1 \\ \{\bar{R}_M - R_f \to 0\}}} [(\bar{R}_M - R_f)/(1+K_3)]\beta_i + [K_3(\bar{R}_M - R_f)/(1+K_3)]\gamma_i \\ &= [d\bar{W}/d\sigma_W)\sigma_M](\beta_i - \gamma_i) \qquad \text{or} \\ &= [d\bar{W}/dm_W)m_M][\gamma_i - \beta_i).\end{aligned}$$

It is only in this case, when $K_3 = -1$, where $(\bar{R}_M - \mathrm{R}_f)$ is not theoretically implicit in both coefficients on β_i and γ_i.

[3] This assumes that $|K_3| < 1$ when $K_3 < 0$. Exceptions to this in FW [1] corresponds to Table IV: 1972–1976 and Table VII: 1952–1976.

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