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A Complete Analysis of Full Pareto Efficiency in Financial Markets for Arbitrary Preferences

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ABSTRACT

This paper provides a complete analysis of the necessary and sufficient conditions for financial markets to achieve fully Pareto-efficient allocation of aggregate wealth through trade in economies with arbitrary preferences. We show that full Pareto efficiency obtains only if the market structure of contingent claims spans the information partition of a minimal aggregate wealth statistic and a Halmos-Savage sufficient statistic for the beliefs of the traders. All the known allocation efficiency results in the literature due to Arrow, Hakansson, John, Ross, and others are unified by this result.

A SECURITY IS A certificate that entitles the holder to a stream of cash flows contingent on different events at different dates. In order that this be an enforceable contingent-claims contract between the issuer and buyer, the events must be publicly observable. Radner [24, 25] has shown that the structure of publicly observable information determines the structure of the financial markets. If there are no institutional barriers to entry, if opportunities for arbitrage are fully exploited and if markets are frictionless, then, in principle, every event revealed by the information structure produces a risk-sharing instrument that matures contingent on that event.[1]

In this sense, the "*complete*" Arrow-Debreu market structure (see [4] or [9]) is simply a financial market with risk-sharing securities on *every* "state of the world" so that trades in *all* state-contingent claims are feasible. The allocation of risk and aggregate income in the Arrow-Debreu market equilibrium is *fully Pareto-efficient* (FPE); i.e., no augmentation of the set of securities will produce Pareto improvements in the allocation of risk and income. Any other market structure that cannot *span* a full set of Arrow-Debreu securities is called an *incomplete market*, and the resulting equilibrium allocation is labelled *constrained Pareto-efficient* (CPE). Since incomplete markets are more the rule than the exception, considerable effort [9, 11, 12, 14, 17, 24, 26] has been directed at deriving conditions under which a CPE allocation is also FPE. The seminal

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[1] An example would be a security in a futures market based on the Dow-Jones Index. For instance, consider a security that pays $1 if the Index is greater than 1200 (event A) or 0 if it is less than or equal to 1200. This conceptualization of security markets is sometimes called the "state-preference" model to distinguish it from the more popular CAPM of Sharpe and Lintner. Rubinstein [28], Hakansson [11, 12], and Radner [24, 25] have also shown that insights connected to the role and value of information in financial markets and their welfare properties are more deeply investigated in the Arrow-Debreu-Radner framework (see [23] for a nice survey). Hence, we have adopted this framework in this paper.

result[2] of Hakansson [11, Theorem 4, p. 183] shows that a CPE allocation can be FPE if and only if the information structure on which allocation of income is based produces a partition of subsets of the state space such that the aggregate wealth in the economy and the probability ratios of the investors' beliefs remain constant across states in each subset.

An obvious conclusion (which is simply the first-order optimality conditions in words) from this result is that when investors' preferences depend only on wealth, and the probability ratios for two states are identical across individuals, then every individual must, in order for the markets to clear and to behave rationally, choose to acquire the same amount of income in both states.

A deeper question that arises, and whose answer reveals which publicly observable information structures and market structures would lead to FPE, is: What is the *common structural characteristic* of all those partitions of the state space over whose subsets the probability ratios are constant?

The purpose of this paper is to show that the answer to this question comes from the Halmos-Savage [13] theory of *sufficient statistics*. We show that FPE occurs *only* in those markets based on public information structures that span the partition produced by a *minimal aggregate wealth sufficient statistic* (*MAWSS*). A MAWSS is composed of a *minimal sufficient statistic* for the beliefs of the traders and the *aggregate wealth statistic* in the economy. Intuitively, the result shows that FPE can be attained in a securities market only if there is a statistic which is sufficient to "carry" the diversity of beliefs in its partition to the extent that side-betting and risk-sharing (see [6], [31]) can be consummated through trade in contingent claims contracts based on the partition's events.

Several benefits accrue from this new unifying concept. First, because we use the general Borch [6] necessary conditions for Pareto-optimal risk-sharing, the result holds for both finite and infinite (continuum) state spaces (e.g., S = the positive part of the real line) and arbitrary beliefs (rather than only positive probabilities). Second, the link to the sufficient-statistics theory provides a powerful conceptual tool for further research because it is well-known in the statistics literature (see Schmetterer [29]) that the Halmos-Savage [13] criterion for discerning sufficient statistics is much more useful than the rather cumbersome "equality of probability ratios."[3] Third, we can show (see [1]) that *allocational efficiency* obtains in the various market settings proposed in the literature [4, 7, 11, 20, 26, 28] because, in each setting, the tradeable instruments span securities based on the partition of the state space by a MAWSS. This also shows that there is a variety of Hakansson [11] FPE "superfunds" with the "finest" being the Arrow-Debreu structure, and the "coarsest" or *minimal* one being based

[2] Actually, Rubinstein's [28] work on nonspeculative beliefs generates somewhat similar insights, and predates Hakansson's result in published form. However, Rubinstein's Pareto-optimality results were incidental to his other results, whereas in Hakansson's case, his Theorem 4 [11, p. 183] is geared to deriving precisely the necessary and sufficient conditions for Pareto-optimality of trading equilibrium allocations. Recently, Milgrom and Stokey [20] have extended Rubinstein-Hakansson results to a more general setting. Their concept of "statistical information" though, is a special case of our sufficient statistics result here (see [1] for analytical details).

[3] For instance, suppose the asset cash flows are random variables of the exponential density type on the positive part of the real line. Then the difficulties of checking the ratios point by point for deriving conclusions about FPE are quite obvious.

on a MAWSS (see example in Section II). Further, we may price out (see [7] and [10] among others) MAWSS supershares using a Black-Scholes pricing scheme. Similarly, Bhatacharya's [5] multiperiod valuation results can be readily extended to MAWSS-based economies.

Interesting additional benefits derive from an understanding of this *minimal statistical structure* underlying FPE. For example, in a setting where production is also involved, but production decisions are unobservable (i.e., "moral hazard" obtains), it has been shown ([15], [16]) that optimal incentives and risk-sharing requires a sufficient statistic. Similarly, in [2] it is shown that a fully revealing rational expectations equilibrium obtains in a market with a MAWSS if the heterogeneous beliefs are due to differences in information produced by partitions coarser than the MAWSS.

We shall not prove the results in this paper since the mathematics is somewhat complex. Detailed proofs and extensions can be found in [1]. Section I develops the basic concepts and provides numerical illustrations of the main ideas. Section II states the main result and its implications. Section III discusses extensions and concludes the paper.

I. Model, Concepts, and Illustrations

Consider an economy of two dates, $t = 0, 1$, and n traders with utility functions U_i, $i = 1, \cdots, n$ defined on date-1 wealth, $x \in R$. Assume that U_i are twice differentiable and strictly concave (i.e., the traders are strictly risk-averse). Let S be the set of "states of the world" that encodes all the uncertainty in the economy. For simplicity, we shall assume that every subset in S is an event.[4] Hence any probability measure defined on these events is a function $\emptyset$: $S \longrightarrow [0, 1]$ such that for any event A in S, $\emptyset(A) = \Sigma_A \emptyset(s)$. There are m firms whose production decisions[5] generate date-1 cash flows defined by functions X_k: $S \longrightarrow R$. Thus, $X_k(s)$ is the total liquidating dividend by firm $k \in \{1, \cdots, m\}$ in state s at time $t = 1$. Each trader i is endowed with a share, $\bar{\alpha}_{ik} \geqq 0$, of the k^{th} firm's dividend stream at time $t = 0$, where $\Sigma_i \bar{\alpha}_{ik} = 1$. $X_M = \Sigma X_k$ denotes the securities market aggregate wealth. $P_i(s)$ denotes trader $i = 1, \cdots, n$, beliefs on S.

A financial market structure M is defined to be the set of securities which are exchanged at time 0 by the traders to change their endowed portfolio, $\bar{Z}_i(s) = \Sigma_k \bar{\alpha}_{ik} X_k(s)$, to their final portfolio, $Z_i(s)$, which is composed of the securities from M held in equilibrium after trade. Obviously, trade takes place under the usual budget constraints and market-clearing conditions. Further, we assume that M includes at least all the primitive assets X_k as tradeable instruments.

The question at hand is: *What structure M produces final allocations Z_i such that they are FPE?*

First, note that every tradeable security in M should be an enforceable contingent claims contract maturing at $t = 1$. Hence, the events in S on which the securities are predicated must be publicly observable. Further, if free entry is

[4] For rigour we should consider a sigma-algebra of events (see [19]) in S. The simplifying assumption is for expositional purposes, but is not necessary for any of the results.

[5] It is assumed that no moral hazard is present in these decisions. Further, the optimal production choices have been made and are exogenous data in the model (see, e.g., Mossin [22]).

allowed and no opportunities for arbitrage are left unexploited,[6] then M consists *of all those securities* (contingent claims) that can be based upon a *publicly observable information structure* F, namely the set of publicly observable events. Thus each information structure F produces a market structure M and each M identifies some F from which it is derived.

Now each information structure F can be identified with the *partition*[7] on S produced by some *information system*, η: $S \rightarrow Y$, which is simply a random variable (statistic) on S. Hence, the question that we posed above can be rephrased as: *What information structure (partition) produces a (contingent claims) market structure that produces a final allocation* Z_i *that is FPE?*[8]

Our main result (Theorem I, Section II) provides the general answer to this question. It states that Z_i is FPE *if and only if* the information structure F is at least as fine as the partition induced on S by a *minimal aggregate wealth sufficient statistic (MAWSS)*.

This MAWSS concept depends critically on the theory of sufficient statistics developed by Halmos and Savage [13] (which is *exactly* the familiar theory of sufficient statistics found in elementary textbooks—see footnote 10).

A statistic T: $S \rightarrow Y$ is called a *sufficient statistic for the beliefs* P_i, $i = 1, \cdots, n$ if $P(\cdot \mid \theta)$, the conditional probability functions given that $T(s) = \theta$, are *homogeneous* for each $\theta \in Y$ (Y is any well-defined set in which T takes its values). A sufficient statistic is called *minimal* if there is no other sufficient statistic producing a coarser partition on S.

THEOREM (HALMOS-SAVAGE [13]). *Let* $f_i(s)$ *be the densities*[9] *of* $P_i(s)$ *with respect*

[6] Objections could be raised that transactions costs could prevent arbitrage opportunities from being exploited, and consequently, many event-contingent securities may not exist for insurance purposes. While this argument has some merit, and has led to the study of Diamond [8] markets, it is not true that transactions costs invariably prevent the emergence of such markets. Some or many such instruments emerge depending on the demand (witness the current interest in market futures), and it is difficult to draw an arbitrary line *a priori* without specification of the transactions cost technology. If opportunities are not exploited despite a demand for trade, the restrictions must be assumed to be institutional, and so we are in institutionally incomplete markets rather than informationally incomplete markets. The former, as Stiglitz [30, p. 236, footnote 2] points out, are not suitable for the study of social value of information issues. This is also the reason why "spanning" arguments are so prevalent in those market frameworks.

[7] A collection of subsets $\Gamma = \{B_j \mid j \in J\}$ of S is called a partition if $B_j \cap B_r = \emptyset$, $j \neq r$, and the union of all the sets B is S. If Γ_1 and Γ_2 are two partitions, then Γ_1 is called *finer* than Γ_2 (equivalently Γ_2 is called *coarser* than Γ_1) of each element of Γ_1 is a subset of an element of Γ_2 and at least one is a proper subset. The terms *at least as fine (coarse) as* can be similarly defined. In general, as mentioned in footnote 4, F should be identified with a sigma-algebra of events in S.

[8] Care must be taken to distinguish the purposes and essence of Ross' [26] results (John's [17] "resolution" results are an elegant generalization of Ross' results) and the work of Hakansson [11, 12]. Our FPE results are in the spirit of the latter which originates with Radner [24], whereas the former are concerned with "spanning" the Arrow-Debreu market through augmentation by options. Although a "*totally resolving*" (see John [17]) options market attains FPE by virtue of the fact that it spans the Arrow-Debreu structure, that achievement is incidental to its purpose, which is proper equilibrium *valuation* of the primitive assets. In contrast, the main purpose of Hakansson's results is to derive conditions for attainment of *allocational efficiency* in equilibrium. This will become clear as we proceed.

[9] Technically, one measure Q has a (Radon-Nikodym) density with respect to another, L, if Q is *absolutely continuous* (see [19]) with respect to L (i.e., $L(A) = 0$ implies $Q(A) = 0$ for any such event A).

to the average pooled measure, $P(s) = \Sigma_i P_i(s)/n$. *Then a statistic* $\hat{T}\colon S \rightarrow R^n$, *the partition of which is identical to that of the vector statistic of densities* $[f_1(s), \cdots, f_n(s)]$ *is a minimal sufficient statistic*[10] *for the* P_i.

If $\hat{T}$ is a minimal sufficient statistic, and X_M is the aggregate market wealth, then the vector statistic Ψ given by $\Psi(s) = [\hat{T}(s), X_M(s)]$ is a MAWSS.

To illustrate these concepts, consider $n = 2$, $k = 2$, and $S = \{s_1, s_2, s_3, s_4, s_5, s_6\}$. Let X_1, X_2, and X_M be as shown below.

$$
\begin{array}{rcccccc}
 & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 \\
X_1 = & [1 & 2 & 6 & 4 & 5 & 5] \\
X_2 = & [2 & 1 & 3 & 5 & 6 & 6] \\
X_M = & [3 & 3 & 9 & 9 & 11 & 11]
\end{array}
$$

Assume that s is not observable and that no other information statistic is available except the cash flows from X_1 and X_2. Then the publicly available information structures are the partitions generated by X_1, X_2, the vector $[X_1, X_2]$, and X_M.

Consider the following sets of beliefs on S.

$$
\begin{array}{ll}
 & \begin{array}{lllllll} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 \end{array} \\
\textit{Example } 1 & \begin{cases} P_1 = [0.1 & 0.1 & 0.3 & 0.3 & 0.1 & 0.1] \\ P_2 = [0.25 & 0.25 & 0.15 & 0.15 & 0.1 & 0.1] \end{cases} \\
\textit{Example } 2 & \begin{cases} P_1 = [0.25 & 0.25 & 0.15 & 0.15 & 0.1 & 0.1] \\ P_2 = [0.05 & 0.05 & 0.1 & 0.4 & 0.2 & 0.2] \end{cases} \\
\textit{Example } 3 & \begin{cases} P_1 = [0.1 & 0.2 & 0.25 & 0.3 & 0.05 & 0.1] \\ P_2 = [0.2 & 0.15 & 0.05 & 0.1 & 0.4 & 0.1] \end{cases}
\end{array}
$$

For each set of beliefs, define the pooled measure, $P(s) = ½(P_1(s) + P_2(s))$. Then we can construct the densities $f_i(s) = P_i(s)/P(s)$ for each $s \in S$, $i = 1, 2$. The following facts obtain.

Example 1: For this example, any statistic producing the partition $\{A_1 = \{s_1, s_2\}, A_2 = \{s_3, s_4\}, A_3 = \{s_5, s_6\}\}$ is a MAWSS. For example, X_M is a MAWSS.

Example 2: Here, any statistic producing the partition $\{B_1 = \{s_1, s_2\}, B_2 = \{s_3\}, B_3 = \{s_4\}, B_4 = \{s_5, s_6\}\}$ is a MAWSS. Also, both X_1 and X_2 as well as the vector $[X_1, X_2]\colon S \rightarrow R^2$ produce partitions finer than a MAWSS. But X_M is *not* MAWSS since the densities f_1 and f_2 produce finer partitions than X_M.

Example 3: The MAWSS partition here is $\{\{s_j\} \mid j = 1, \cdots, 6\}$. None of the functions X_1, X_2, X_M, nor the vector $[X_1, X_2]$ is a MAWSS.

10. Observe that we have in this case that, if π_i denotes the i^{th} component projection function,

$$f_i(s) = 1 \cdot \pi_i(\hat{T}(s))$$

for each $i = 1, \cdots, n$. If we consider i as a parameter, then this is identical to the standard *decomposition* definition of sufficient statistics in elementary textbooks.

II. Analysis of FPE Market Structures

The general FPE result here depends critically on the necessary and sufficient conditions of Borch [6] for Pareto-optimal risk-sharing which state that an allocation Z_i (of the aggregate wealth X_M) is FPE *if and only if* for each $s \in S$, i, $j = 1, \cdots, n$,

$$U_i'(Z_i(s))f_i(s) = \lambda_{ij}U_j'(Z_j(s))f_j(s) \tag{1}$$

where U_i' denotes the first derivative of U_i and λ_{ij} is a positive constant. Intuitively, (1) implies that at each $s \in S$, the marginal rates of substitution between the utilities accruing to i and j are such that any further Pareto superior risk-sharing is not possible through a redistribution of the $X_M(s)$.

THEOREM 1. *(i) An allocation Z_i: $S \rightarrow R$ of the aggregate wealth X_M: $S \rightarrow R$ is FPE only if for any MAWSS $\Psi(s) = [\hat{T}(s), X_M(s)]$ from S to $R^n \times R$ (where $\hat{T}$ is a minimal sufficient statistic for the P_i), there exist functions g_i: $R^n \times R \rightarrow R$ such that for each $s \in S$, $Z_i = g_i(\Psi(s))$. That is, each $Z_i(\cdot)$ is a composite function of $\Psi(\cdot)$ and $g_i(\cdot)$.*

(ii) The vector of allocations $[Z_1, \cdots, Z_n]$ itself considered as a statistic from S to R^n is a MAWSS if Z_i are FPE.

Theorem 1 provides the necessary conditions for an FPE market structure. Note that the Rubinstein [28]—Milgrom-Stokey [20] result on "statistical information" is a special case of this result. Hakanson's [11, Theorem 4] result is generalized and extended by Theorem 1.

COROLLARY 1. *Let M be a market structure produced by an information structure F. If the initial endowments of the traders are portfolios of securities in M, then the equilibrium portfolios after trading at time 0 are FPE if and only if F contains the partition of a MAWSS.*

To illustrate these results, consider Example 1 of Section I. The market structure based on the partition produced by the vector $[X_1, X_2]$ contains the securities shown below:

$$\begin{array}{lcccccc} & s_1 & s_2 & s_3 & s_4 & s_5 & s_6 \\ H_1 = & [1 & 0 & 0 & 0 & 0 & 0] \\ H_2 = & [0 & 1 & 0 & 0 & 0 & 0] \\ H_3 = & [0 & 0 & 1 & 0 & 0 & 0] \\ H_4 = & [0 & 0 & 0 & 1 & 0 & 0] \\ H_5 = & [0 & 0 & 0 & 0 & 1 & 1] \end{array}$$

Clearly, $H_1, \cdots, H_5$ can generate X_M, which is a MAWSS. Hence equilibrium portfolios in the market structure of Example 1 are FPE and are simply linear combinations of the H_j securities. Note also that the market structure of the H_j and their linear combinations is not a complete Arrow-Debreu structure and yet it produces an FPE. However, it does *not* produce FPE portfolios in equilibrium *for all* belief configurations. This leads to the next result.

COROLLARY 2. *A market structure produces an FPE allocation for every group of traders with all possible belief configurations if and only if it is the Arrow-Debreu market structure.*

There is a body of results in the literature called the "spanning" results whose origins lie in the work of Diamond [8]. In the Diamond setting, there are institutional (see footnote 6) restrictions on trades other than in *linear multiples* of a specified class of basic securities (primitive assets). Thus, even though an event A may be publicly observable, a contingent claim based on A may not be a tradeable security in a Diamond market if it cannot be constructed as a linear sum of the basic securities. For example, in the examples in Section I, a Diamond market would restrict trade to linear multiples of X_1 and X_2 only. Hence, a security of the type H_1 (described above) is not a tradeable security. For a security to be tradeable in a Diamond market, the basic securities must *span* the desired security.[11] The span of a Diamond market may not encompass the contingent claims based on a MAWSS and thus FPE will not obtain.

Ross' [26] brilliant insight shows that a Diamond market structure can be augmented[12] by options based on portfolios of the primitive assets. However, the condition for a market of stocks and options to produce an FPE is still the same as that spelled out in the preceding paragraph (see also footnote 8), namely that it must span the contingent claims on a MAWSS. A "totally resolving" (see John [17]) Ross options market spans the Arrow-Debreu securities and Corollary 2 shows that it is FPE under all circumstances.

The form of the FPE equilibrium portfolios Z_i can be determined explicitly for HARA class utilities U_i with identical cautiousness[13] (see [3] and [22]). In particular, if beliefs are homogenous, and all traders in the economy are HARA class traders with identical cautiousness, then they hold linear multiples of X_M in an FPE euilibrium. Consequently, a Diamond market that has X_1 and X_2 as basic securities can achieve FPE.

Finally, Hakansson [11] type "*supershares*" based on a MAWSS can easily be constructed. Consider, for instance, Example 2 of Section I. A supershare on the MAWSS partition would be a contingent claim that paid \$1 if event B_j occurred ($j = 1, \cdots, 4$) and \$0 elsewhere. However, suppose that trade is restricted to a Diamond type of market structure consisting of assets X_1, X_2, and the Hakansson supershares described above. The issue is whether FPE is attainable *with* or *without short selling* in this market structure. If the supershares are issued by individuals, then they are in zero net supply. In this case FPE is attainable if unlimited short selling is permissible. But if there are market imperfections (limited liability and bankruptcy), then FPE may not be attainable even though trading in supershares is permissible.

[11] That is, the linear span (the linear subspace of arbitrary linear combinations) of the basic securities must contain the desired security.

[12] The augmentation is obviously nonlinear (otherwise the basic securities can span it (see Hokansson [12])).

[13] The risk-tolerance function (see Wilson [31]) of a utility function U is the ratio $\tau(x) = U'(x)/U''(x)$ (first over second derivative). If $\tau(x) = ax + b$ for some constants a, b, U is said to belong to the HARA class and a is called the *risk-cautiousness* of the utility. Examples of the HARA class are log, exponential, and power utility functions.

Two ways to overcome this problem are now explained. Hakansson ([11], Theorem 2, pp. 176–179 and [12], Theorem 3, pp. 765–766) shows that if there is a "*superfund*" that actually holds a portion of the real assets in the economy against which it issues supershares, then individual traders need not go short on any security in the marketplace in order for the economy to attain FPE. Alternatively, if the market structure encompasses the securities $H_1, \cdots, H_5$ (described above), then in this market no trader need go short on any security since transfer of title to an event-contingent slice of a real asset in his possession can be accomplished through $H_1, \cdots, H_5$. Further, notice that the number of securities with the superfund or $H_1, \cdots, H_5$ is much less than those in a totally resolving Ross options market. Therefore, if transactions costs of producing new instruments are large and FPE with the given assets instead of valuation of nontradeable assets is the objective, one would expect a superfund, instead of options, to emerge. In a sense, *the superfund acts as a central planner* who efficiently allocates the aggregate income. Indeed, trade in market futures that has been recently implemented supports this conjecture.

III. Conclusions

All the results here can be generalized to multiperiod settings for both time- and nontime-additive utilities (see [1]). They also hold for markets with an infinite number of securities (see [18]), and hence in a "Black-Scholes" setting.

It is evident that the results on market aggregate wealth sufficient statistics (MAWSS) and their role in the attainment of FPE through trade in securities markets provide a complete analysis of allocational efficiency. The key result here that was not known in the literature is that there can be no FPE without public observability of a MAWSS and allocation based on it. That is, whether one approaches the problem through spanning by augmenting a primitive asset Diamond market through options (as in [17], [26]), or through a superfund (as in [11]), in equilibrium an FPE allocation is possible only if there are securities that span a MAWSS. Further, the FPE allocation itself is a MAWSS. This linkage of FPE to the powerful Halmos-Savage theory of sufficient statistics provides a unifying link for the results here to the case where production and moral hazard are present because sufficient statistics play a critical role in attainment of Pareto-optimal risk-sharing (see [15, 16]) in such situations.

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