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Optimal Bank Behavior under Uncertain Inflation

YORAM LANDSKRONER and DAVID RUTHENBERG*

ABSTRACT

In this paper, we derive a model of the individual banking firm facing stochastic inflation. The bank is considered as a depository financial intermediary operating in the primary market as a multiproduct price discriminating firm. A secondary market is also considered where liquidity surpluses and deficits are traded. Two types of assets and liabilities are assumed: deposits and loans linked to a general price level and nonlinked instruments. Effects of changes in the parameters such as inflation rate and variability, reserve requirements are analyzed.

THE PURPOSE OF THIS paper is to derive a model for the optimal behavior of a commercial bank under uncertainty, where the major source of this uncertainty is inflation.

In the banking literature, two distinct approaches to the analysis of the banking firm can be observed. The first approach is based on modern portfolio theory. This approach was adopted by, among others, Kane and Malkiel [4], Pyle [9], and Hart and Jaffee [2]. The advantage of this approach is in its explicit consideration of uncertainty faced by the bank. On the other hand, however, it also has two important shortcomings: it assumes perfect competition in the deposit and loan markets and it does not consider operating costs of the bank.

A second approach is based on the traditional theory of the firm. This approach, which was adopted by Klein [6], Miller [8], and others, analyzes bank behavior under more realistic market conditions. It assumes imperfect competition, at least in part of the markets in which the bank operates, and also considers operating costs. On the other hand, this approach neglects to consider explicit uncertainty.

A number of recent papers have attempted to integrate the two approaches: Sealey [11][1] Tobin [12], and Elyasiani [1]. In this context, it is of importance to mention that in a number of papers, the traditional theory of the firm has been modified to consider the effects of uncertainty. The pioneering work in this area was done by Sandmo [10] and Leland [7]. Recently this approach was expanded for the international multiproduct firm by Katz, Paroush, and Kahana [5].

This paper combines the two theoretical approaches to the analysis of optimal bank behavior: portfolio theory and the theory of the firm under uncertainty. Our paper is the first to focus attention on the simultaneous effects of uncertain inflation on the decision variables of the bank and, via that, on the extent and composition of financial intermediation.

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[1] Sealey [11] was the first to make the explicit distinction between the two approaches.

In this paper we derive a single-period model of the individual banking firm under uncertainty. We consider here the bank in its traditional role as a depository financial intermediary. As such, the bank operates in an imperfect primary market where there exist two types of assets and liabilities: deposits and loans which are linked to a general price level (such as the CPI),[2] and deposits and loans which are nonlinked. In this market, the bank is a multiproduct price-discriminating firm. A secondary market is also considered where commercial banks and the central bank participate, and where liquidity surpluses and deficiencies are traded. We focus our attention on a single source of uncertainty that is stochastic inflation.[3]

The institutional constraint imposed on the bank is reflected by reserve requirements on the public deposits. Under the conditions described above, we derive a model for the optimal behavior of the bank and analyze the effects of exogenous variables such as the inflation rate and its variability, reserve requirements, and the degree of risk aversion on financial intermediation.

I. The Model

The banking firm, with a single-period horizon, operates in two markets: the primary market and the secondary market. In the primary market the bank acts in two separate segments: linked (real) and nonlinked (nominal). In the linked sector, the bank raises linked deposits and extends linked loans to the public. Similarly, in the nonlinked sector, the bank deals in nonlinked deposits and loans. We assume that, because of institutional constraints aimed at reducing the risk exposure of the bank, the bank will hold at any point in time a covered position in these two types of intermediation. Thus we do not allow the financing of linked uses from nonlinked sources and vice-versa.[4] In the primary market competition is imperfect and it is assumed that the bank sets the deposit rate and the quantity of loans it extends to the public, and faces uncertainty about the real cost of deposits and real returns from loans.

In the secondary market, the bank operates to cover liquidity gaps (positive or negative) that result from differences in the quantity of deposits raised from the public and the quantity of loans extended to the public. An important factor in determining the activity of the bank in this market will be the reserve requirements on deposits imposed by the central bank.

In its financial intermediation activity, the bank incurs operating costs (on buildings, equipment, and manpower). We assume that these costs are defined over the elements in both segments of the primary market.

[2] A different conceivable linkage would be to the foreign exchange rate, or alternatively, assets and liabilities denominated in foreign exchange.

[3] Other sources of uncertainty, such as loan defaults and deposit changes were analyzed previously in the literature (see, e.g., Tobin [12], and Elyasiani [1]). Our paper is a first attempt at analyzing the impact of inflation uncertainty on bank behavior. A very reasonable way to proceed is to study inflation in isolation, enabling us to obtain explicit results concerning optimal bank behavior.

[4] A relaxation of this constraint may increase the risk undertaken by the bank, depending on market conditions and attitudes towards risk. However, the direction of the effects derived in our model will remain unchanged.

Following our assumptions, the bank is a multiproduct firm which markets its products in two segments, linked and nonlinked, using price discrimination. A number of papers have dealt with a multiproduct price-discriminating banking firm, e.g., Havrilesky and Pitts [3] and Elyasiani [1]. Our model, which goes beyond previous papers in its explicit examination of the simultaneous effects, is based in part on Katz, Paroush, and Kahana [5], who analyzed the optimal behavior of a price-discriminating firm which operates in local and international markets under conditions of uncertainty. In our model, the profit function of the bank is separable into two revenue functions of the two different segments. Thus, the revenue function, net of financial costs, of the nonlinked segment is given by:

$$\tilde{\Pi}_N = L_N r(L_N)\tilde{P} - i\tilde{P}D_N(i) - \tilde{P}s(L_N - (1 - \rho_N)D_N(i)) \tag{1}$$

where the decision variables are:

L_N = total quantity of nonlinked loans extended by the bank to the public.
i = nominal interest rate on nonlinked deposit.

The other variables are:

$\tilde{\Pi}_N$ = revenue in the nonlinked segment (net of financing costs) denominated in real terms
$r(L_N)$ = nominal interest rate on nonlinked loans, specified as a demand function for loans ($r' \equiv \partial r/\partial L_N < 0$)
$D_N(i)$ = supply function (by the public) of nonlinked deposits ($D_N' \equiv \partial D_N/\partial i > 0$)
s = exogenously determined nominal interest rate on funds traded in the secondary market. For simplicity we assume that borrowing and lending rates are equal.
ρ_N = reserve requirement ratio on nonlinked deposits
$\tilde{P}$ = uncertain inflation variable defined as an index of the purchasing power of money, i.e., $\tilde{P} = 1/(1 + g)$ where g is the inflation rate. We assume that $\tilde{P}$ can be written as:

$$P = E(P) + \gamma\tilde{\mu}$$

where $E(\mu) = 0$, $\sigma_\mu^2 = 1$, $E(P)$ is the expected value of P, and γ is its standard deviation. For simplicity we assume that $E(P) = 1$.

We can now rewrite the revenue Equation (1) as follows:[5]

$$\tilde{\Pi}_N = [L_N r(L_N) - iD_N(i) - s(L_N - (1 - \rho_N)D_N(i))](1 + \gamma\tilde{\mu}) \tag{1a}$$

where the term in brackets, denoted hereafter by K, is the revenue of the bank (net of financing costs) under certainty, where $K' > 0$ and $K'' < 0$.

Following similar logic, we may write the revenue equation in the linked

[5] Our assumption of additive uncertainty is common in the literature; see, e.g., Sandmo [10] and Leland [7]. This assumption enables one to obtain explicit and unambiguous relationships between uncertainty and variables of the objective function.

segment:

$$\Pi_L = L_L R(L_L) - I D_L(I) - S(L_L - (1 - \rho_L) D_L(I)). \tag{2}$$

The main difference between the two segments is that in the linked segment all variables are in real terms and therefore certain, whereas in the nonlinked segment the variables are denominated in nominal terms and therefore exposed to inflation uncertainty.

The definitions of the variables in Equation (2) are similar to those of Equation (1) with the above difference. The decision variables of the bank in the linked segment are:

I, L_L = *real interest rate on deposits and quantity of linked loans, respectively.*

The other variables are:

Π_L = revenue in the linked segment (net of financing costs) denominated in real terms

$R(L_L)$ = the public's demand function for linked loans $(R' = (\partial R/\partial L_L) < 0)$

$D_L(I)$ = supply function of linked deposits $(D_L' = (\partial D_L/\partial I) > 0)$

S = real interest rate on funds traded in the secondary market

ρ = reserve requirement ratio on linked deposits.

The loans and deposits of the bank in both the linked and nonlinked segments are assumed to be homogenous with respect to operating costs. Thus the total operating costs of the bank may be defined over the sum of deposits and loans in the two segments:

$$C(L_L + D_L(I) + L_N + D_N(i)) \qquad \text{where } C' > 0 \text{ and } C'' > 0. \tag{3}$$

Define the total profit of the bank, Π, to be:

$$\Pi = \Pi_N + \Pi_L - C.$$

We assume that the bank's objective function is the expected utility of profits:

$$EU(\Pi) \tag{4}$$

where U is an increasing and concave function, i.e., $U' > 0$ and $U'' < 0$.

It is assumed that the bank decides on the value of loans extended to the public (L_L, L_N) and the interest rates paid on deposits (I, i) before the state of the world is revealed. Its activity in the primary market determines, in turn, its operations in the secondary market which are aimed at bridging the gap between loans extended and deposits raised by the bank. This gap for the nonlinked and linked segments can be written, respectively, as

$$Y_N = L_N - (1 - \rho_N) D_N(i) \gtreqless 0$$

and

$$Y_L = L_L - (1 - \rho_L) D_L(I) \gtreqless 0.$$

When Y is positive, the bank has a deficit of sources over uses (considering also the reserve requirements) in the primary market. This deficit is financed in the secondary market (by borrowing from other banks or from the central bank).

If Y is negative, the bank has a surplus of sources and lends these in the secondary market. For simplicity, it is assumed that in either one of the segments, the interest rate earned on surpluses (the lending rate) is equal to the interest rate paid on funds raised to cover reserve deficiencies (the borrowing rate), and that these common rates (s, S) are exogenously determined.

II. Solutions of the Model

A. Static Equilibrium

To obtain the optimal solutions for the model, we derive the first-order conditions for the maximization of expected utility. This condition for the nonlinked segment and the decision variable, i, is:

$$\frac{\partial E(U(\Pi))}{\partial i} = E\left(U'\left[\frac{\partial K}{\partial i}(1+\gamma\tilde{\mu}) - C'\frac{\partial D_N}{\partial i}\right]\right)$$
$$= \left(\frac{\partial K}{\partial i} - C'\frac{\partial D_N}{\partial i}\right)E(U') + \frac{\partial K}{\partial i}\gamma E(U', \tilde{\mu}) = 0 \tag{5}$$

where

$$\frac{\partial K}{\partial i} = -D_N(i) - i\frac{\partial D_N}{\partial i} + s(1-\rho_N)\frac{\partial D_N}{\partial i}$$

and $E(U'\tilde{\mu}) = \text{cov}(U', \tilde{\mu}) < 0$.

The negative covariance between the marginal utility (U') and the stochastic component of inflation (μ) stems from the assumption of risk aversion.[6] Equation (5) consists of two parts: the first is the first-order condition under certainty (or risk neutrality). The second term is the inflation-associated risk. The first-order condition under certainty is:

$$\frac{\partial E(\Pi)}{\partial i} = \frac{\partial K}{\partial i} - C'\frac{\partial D_N}{\partial i} = 0.$$

Since, by assumption, $C' > 0$ and $\partial D_N/\partial i > 0$, it follows that $\partial K/\partial i > 0$. We assume that this result also holds under uncertainty.[7] Under uncertainty, for Equation (5) to hold, we must have $\partial E(\Pi)/\partial i > 0$ since $(\partial K/\partial i)\ \gamma E(U'\mu) < 0$. Thus, comparing the optimal solutions of the interest rate paid on nonlinked deposits, we find that i^* under certainty is greater than the optimal solution, i^{**}, under uncertainty, given that the second-order condition is negative, $\partial^2 E(\Pi)/\partial i^2 < 0$ (see Figure 1).

The result obtained here indicates the direction of the effect of uncertainty on the activity of the bank in the nonlinked deposit market: the introduction of uncertainty (or an increase in it) will result in a reduction of the interest rate paid on deposits and therefore a decline in deposits raised. In the next section,

[6] $E(U'\mu) = \text{cov}(U', \mu)$ since $E(\mu) = 0$. $\text{Cov}(U', \mu) < 0$ by the assumption of risk aversion, and $\partial\Pi/\partial\mu > 0$ by the definition of the profit function (e.g., (1a)).

[7] The implied assumption in this condition is that $(s(1-\rho)/i) - 1 > 1/\varepsilon$ where ε is the elasticity of supply of deposits. This seems to be a sensible assumption under realistic conditions.

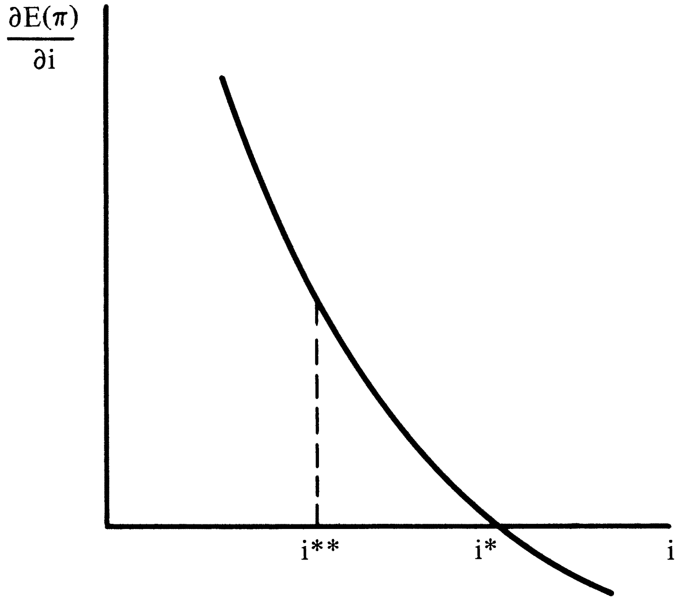

Figure 1

we analyze the simultaneous effects of uncertainty on both sides of the balance sheet.

The first-order condition for the decision variable in the loan market, L_N, is:

$$\frac{\partial E(U(\Pi))}{\partial L_N} = \left(\frac{\partial K}{\partial L_N} - C'\right) E(U') + \frac{\partial K}{\partial L_N} \gamma E(U'\mu) = 0 \tag{6}$$

where

$$\frac{\partial K}{\partial L_N} = r(L_N) + L_N \frac{\partial r}{\partial L_N} - s.$$

Again, as in the previous condition, here too there are two components: a certain and an uncertain one. Under certainty, the first-order condition is

$$\frac{\partial E(\Pi)}{\partial L_N} = \frac{\partial K}{\partial L_N} - C' = 0.$$

Thus, since $C' > 0$, we have $\partial K/\partial L_N > 0$ under certainty. Assuming that this inequality also holds under uncertainty, then, following similar reasoning to that of the first-order condition in the deposit market, we obtain $\partial E(\Pi)/\partial L_N > 0$. Thus, since $\partial^2 E(\Pi)/\partial L_N^2 < 0$, the optimal quantity of loans under uncertainty (L_N^{**}) will be smaller than under certainty (L_N^*) (see Figure 2).

Following are the solutions for the linked segment. They are similar to those under certainty because of neutrality of inflation with respect to this segment.

The first-order condition in the linked deposit market is:

$$\frac{\partial E(U(\Pi))}{\partial I} = \left(-D_L(I) - I \frac{\partial D_L}{\partial I} + S(1 - \rho_L) \frac{\partial D_L}{\partial I} - C' \frac{\partial D_L}{\partial I}\right) E(U') = 0 \tag{7}$$

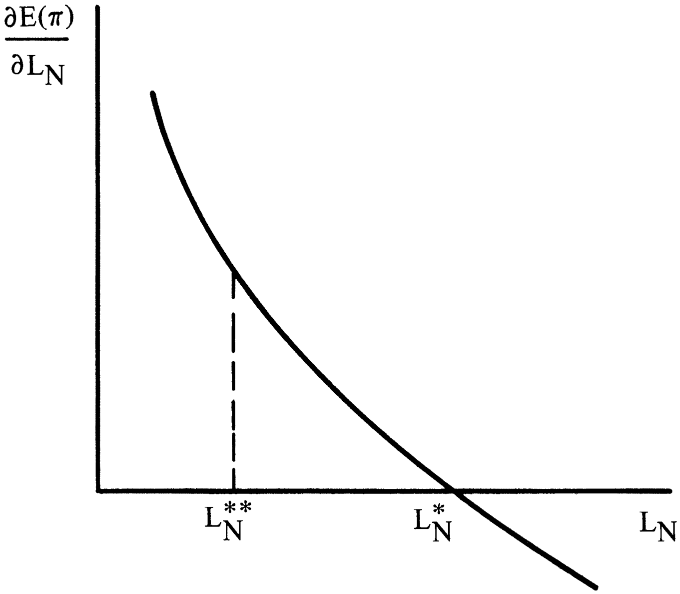

Figure 2

and the first-order condition in the linked loan market is:

$$\frac{\partial E(U(\Pi))}{\partial L_L} = (R(L_L) + L\frac{\partial R}{\partial L_L} - S - C')E(U') = 0. \tag{8}$$

B. Comparative Statics Analysis

In this section, we analyze the simultaneous effects of the parameters (exogenous variables) in our model on the behavior of the bank. These parameters are: uncertain inflation, risk aversion, reserve requirements, and the rate of return in the secondary market.

These simultaneous effects are derived by analyzing the first-order conditions of Equations (5) to (8) and results are presented here diagrammatically. Geometric analysis is our main analytic method in this section. This is a simple and powerful tool of analysis. This approach is based on the paper by Katz, Paroush, and Kahana [5] who analyzed the behavior of the business firm under uncertainty.

As outlined previously, the banking firm is assumed to market its products in two segmented markets (linked and nonlinked) at discriminating prices. This price discrimination follows from the assumptions that the bank has similar operating costs in both segments, and that mobility of inputs (physical capital and labor) is unlimited between the two segments.

B.1. Loan Market

The bank's optimal behavior in the loan market follows from the first-order conditions, (6) and (8). Since the operating cost function is defined over the total sum of the arguments in both segments, we may write Equation (8) as follows:

$$\left(R(L_L) + L_L \frac{\partial R}{\partial L_L} - S\right) = C'(L_L + L_N + D_N(i) + D_L(I)). \quad (9)$$

This equation is for the linked segment where the left-hand side is the marginal revenue (net of financial cost S) and the right-hand side is the operating marginal cost. The gross marginal revenue can be written as: $R(L_L) + L_L(\partial R/\partial L_L) = R_L(L_L)(1 + 1/\eta)$, where η is the elasticity of demand for loans. Simlarly, we derive an equation for the nonlinked segment based on equation (6):

$$(1 - \gamma A^*)\left(r(L_N) + L_N \frac{\partial r}{\partial L_N} - s\right) = C'(L_L + L_N + D_N(i) + D_L(I)). \quad (10)$$

The additional term here, $(1 - \gamma A^*)$, is due to uncertainty prevailing in this segment. γ represents uncertainty (dispersion) of inflation and $A^* = -E(U'\mu)/EU'$ represents risk aversion of the bank.[8] The product, γA^*, can be considered as a risk premium consisting of an objective and a subjective factor, respectively.

Now define Z_L and Z_N as the set of points in the (L_L, L_N) plane which yield the optimal solutions for Equations (9) and (10), respectively (see Figure 3). We focus our attention around the intersection of Z_L with Z_N. In this neighborhood we can, without loss of generality, approximate the curves of Z_L and Z_N by lines.

Let us determine first the slopes of the lines, Z_L and Z_N. The slope of Z_L is negative and greater in absolute terms than 45°, and the slope of Z_N is negative and smaller than 45° (see Figure 3). The proof is provided in the Appendix. The intersection point represents an equilibrium for both decision variables, L_N and L_L. We may now analyze the simultaneous effects of the parameters on the quantity of loans extended by the bank.

Inflation. Its effect is two-fold, that of the inflation level, $E(P)$, (assumed to equal unity for simplicity) and that of inflation uncertainty, as measured by the standard deviation, γ.

Since inflation does not affect directly the linked segment, we analyze its effects on the nonlinked segment and through it the indirect effects on the linked segment. An increase in γ brings about a downward parallel shift of Z_N($\bar{Z}_N$ to Z_N in Figure 3). It follows from Equation (10) that an increase in γ causes the left-hand side to decline. Thus, for given L_N, L_L on the right-hand side must be reduced, too. It can be seen in Figure 3 that such a shift of Z_N, resulting from increased uncertainty, causes a reduction of the investment in the risky asset, i.e., the nonlinked loans extended by the bank, and an increased investment in the riskless asset, i.e., the linked loans, such that total loans extended $(L_L + L_N)$ will be reduced in the new equilibrium (a move from A to B in Figure 3). Thus, as expected, the total effect of increased inflation uncertainty is to reduce the overall level of bank activity,[9] with substitution (between linked and nonlinked

[8] There is a direct relationship between A^* and the Pratt-Arrow measure of absolute risk aversion $A = -U''/U'$. Given $U'' < 0$, $E(U'\mu) < 0$, thus as (U'') increases so do A and A^*.

[9] This result, that increased uncertainty reduces the level of activity of the firm, is a familiar one in the literature. See, e.g., Sandmo [10], and specifically for the banking firm, see Tobin [12] and Elyasiani [1].

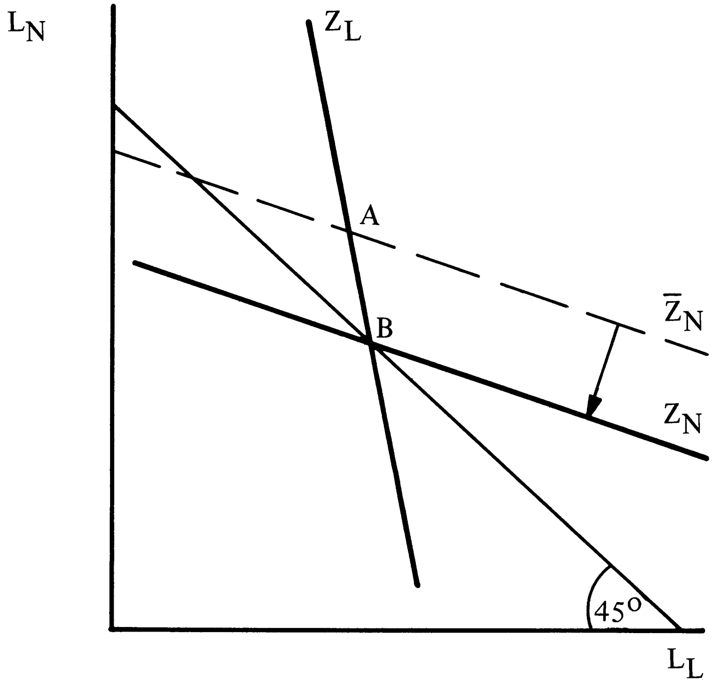

Figure 3

loans) being a partial offsetting factor. Similarly, an increase in the level of inflation, i.e., a decrease of $E(P)$, which is a purchasing-power-of-money index, will result in a downward shift in Z_N, with similar effects.

Risk Aversion. The risk aversion variable, A^*, together with the risk factor, γ, determine the risk premium, which reflects the total effect of uncertainty. Thus, the effect of a change in A^* is similar to that of γ; an increase in A^* reduces the total volume of loans, where linked loans increase and nonlinked loans decline.

Rate of Return in the Secondary Market. An increase in S or s result in a downward shift of Z_L or Z_N, respectively. That is, an increase in S reduces total loans extended because of the higher alternative return (opportunity cost) in the second market. In the linked segment, linked loans are reduced and nonlinked loans, which do not experience interest rate changes in the secondary market, increase. Similarly, an increase in s will reduce total loans extended in the primary market, where linked loans are increased and nonlinked loans are reduced.

B.2. The Deposit Market

Consider now the simultaneous effects in the deposit market, both in the linked and nonlinked segments. Rewriting first-order conditions (5) and (7) yields the following relationship for the linked segment:

$$-\frac{D_L(I)}{D_L'(I)} - I + S(1 - \rho_L) = C'(L_L + L_N + D_N(i) + D_L(I)). \tag{11}$$

The left-hand side is the marginal revenue, and it can be expressed as $S(1 - \rho_L) - I(1 + 1/\varepsilon)$ where ε is the supply elasticity of deposits. The bank will raise deposits up to the point where marginal revenue equals marginal operating cost, C'. Similarly, for the nonlinked segment, we may derive:

$$(1 - \gamma A^*)\left(-\frac{D_N(i)}{D_N'(i)} - i + s(1 - \rho_N)\right) = C'(L_L + L_N + D_N(i) + D_L(I)). \quad (12)$$

Note that by assumption, the marginal operating costs in both segments are equal. The decision variables in this market are the interest rates paid on deposits. However, since the relationship between the interest rate and the quantity of deposits raised is known ($D' > 0$), we have chosen for convenience to analyze the effects of the exogenous variables on the quantities of deposits rather than on the interest rates.

Lines N_L and N_N in Figure 4 are the sets of points which yield the optimal solutions for Equations (11) and (12), respectively. The reasons for the linearity and slopes are similar to those presented for loans. As before, we are interested in examining the simultaneous effects of the parameters on the bank's optimal behavior.

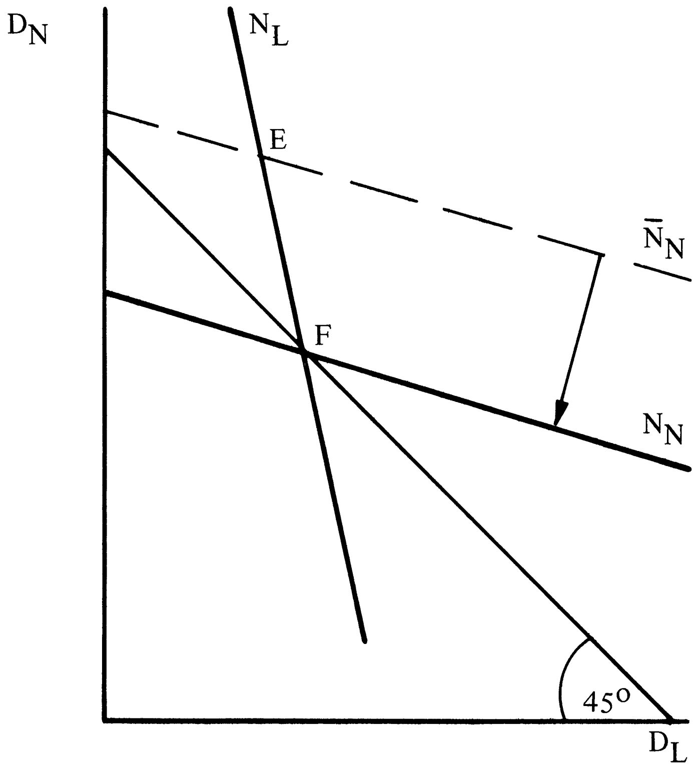

Figure 4

These effects are:

Inflation and Risk Aversion. As would be expected, the main findings here are similar to those obtained in the loan market. An increase in the risk premium, via an increase in inflation uncertainty or risk aversion, results in a reduction of nonlinked deposits coupled with an increase in linked deposits such that total deposits raised are reduced. An increase in the level of inflation would have a similar effect.

Rate of Return in the Secondary Market. The effect would be in the opposite direction to that found in the loan market. An increase in the linked rate of return, S, causes an upward shift of N_L. That is, an increase in the return on linked loans in the secondary market causes an increase in the total deposits raised by the bank in the primary market where the increase in D_L (linked deposits) due to the higher return in the linked secondary market is partly offset by a decrease in D_N. An increase in s would have a similar effect: an increase in D_N which is partly offset by a decline in D_L.

Reserve Requirements. An increase in the reserve requirement rate in the linked segment, ρ_L, causes a downward shift of N_L. As a result, linked deposits, with the higher reserve requirements, would be reduced; nonlinked deposits would be increased, such that total deposits are reduced. An increase in ρ_N would have a simlar effect. This time, however, nonlinked deposits would go down and linked deposits up. In general, the effect of reserve requirements is similar to that of taxation. An increase in these requirements (like an increase of a tax rate) causes a reduction in the interest paid to the public, which, in turn, results in a decline in the quantity supplied of deposits.

III. Summary and Concluding Remarks

In this paper, we derive a model of the banking firm under conditions of uncertain inflation. Using the model, the simultaneous effects of a number of parameters on the bank's optimal decisions are examined. These parameters can be divided into three categories:

(a) Policy variables of the central bank (or government). These include reserve requirements and rates of return in secondary markets (rates paid on excess reserves or penalty rates paid on reserve deficiencies).
(b) Macroeconomic variables which include inflation uncertainty and the level of inflation.
(c) A subjective characteristic of the bank—its risk aversion.

The main finding is that, as expected, an increase in inflation uncertainty reduces the extent of financial intermediation by the bank (in real terms). This is consistent with previous findings about the effects of uncertainty on the level of activity of the firm, e.g., Sandmo [10] and Elyasiani [1]. However, an additional finding, unique to our study, is that uncertainty will affect the composition of financial intermediation by the bank between the linked and nonlinked segments. Our results may be applied to other economic systems in which the bank is operating in a segmented market such as a bank which operates in local currency (raising deposits and making loans) and in a foreign currency, whereby the major source of uncertainty is due to exchange rate risk.

The results derived here are of importance from a policy standpoint as well as from the standpoint of the individual bank. Monetary policy makers, when changing policy, have to consider the direct effects of these changes as well as the indirect effect, through the effects of inflation, on the behavior of the banking firm operating in markets which are affected differently by inflation (in our paper linked and nonlinked segments).

Appendix

Taking the total derivative of (8) yields

$$(\Pi''_L - C'')\, dL_L - C''\, dL_N = 0$$

where Π''_L, the second derivative of the linked revenue, is assumed to be negative and $C'' > 0$. Thus, the slope of Z_L is

$$\frac{dL_N}{dL_L} = \frac{\Pi''_L - C''}{C''} < -1.$$

Similarly, taking the total derivative of (6) yields the slope of Z_N:

$$\frac{dL_N}{dL_L} = \frac{E(U')C''}{E(U''(PK' - C')^2) + E(U'(PK'' - C''))} < 0$$

where $K' > 0$ and $K'' < 0$. Since the denominator is negative and the numerator is positive, the slope of Z_N is negative. In absolute terms, the numerator is smaller than the denominator. Hence, the slope, in absolute terms, is smaller than unity. Therefore, the slope is greater than -1.

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