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Taxes, Default Risk, and Yield Spreads

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ABSTRACT

This paper develops a model of bond prices and yield spreads that incorporates the effect of both taxes and differences in default probabilities. The tax loss consequences of default are recognized. Traditionally, tax-free (municipal) bond yields have been viewed as linearly related to taxable yields with a slope coefficient equal to one minus the tax rate and the intercept representing differences in default risk. While our model supports the linearity assumption, it implies that the slope and intercept are both functions of both the break-even tax rate and the default probability(ies). Clientele effects among both municipal and taxable bonds are demonstrated. Finally, the implied marginal tax rates and the implied default probabilities are estimated for different categories of municipal bonds.

STUDENTS OF FINANCIAL MARKETS cannot help but be impressed by the number of new financial instruments that have been introduced successfully in recent years. The fixed income market has been particularly innovative, as "bonds" carrying a whole host of specific features have been successfully marketed. Bonds with small or zero coupons, variable coupon rates, or detachable warrants are three examples from among an expanding list of such features. Not surprisingly, there has developed a large and expanding literature attempting to explain the rate spreads between different fixed income securities. This literature has evolved much as one would expect. Early empirical work was in general not well founded in theory, and was not based on an underlying expected return (price) model. Later empirical research has tended to be more careful in its specification of factors affecting yield spreads and, as a consequence, better founded in valuation theory.[1]

The present paper represents an extension and integration of recent empirical and theoretical research on default risk and taxability. The purpose of the paper

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[1] An example of such an evolution begins with the important paper by Cook and Hendershott (C-H) [5] which estimates a comprehensive empirical model of yield spreads taking account of the effects of taxes, default risk, relative security supply, and through a simple proxy, for callability. Building on C-H, Yawitz and Marshall [14] reestimate their basic model, substituting a more sophisticated measure of the effect of call that is consistent with the general approach to option valuation. This more refined proxy for call enhances the overall explanatory power of the C-H equation and revises their conclusions about the effects of other factors on yield spreads.

is to develop and test a model of interest rate spreads which incorporates both the effect of taxes and differences in default probabilities in a theoretically correct manner. The theoretical model is an extension of earlier work on default by Bierman and Hass [2] and Yawitz [13], altered appropriately to take explicit account of tax effects. While there is a considerable literature that analyzes the effect of taxability on rate spreads, we are unaware of any previous study that considers tax consequences in the event of default, a rather surprising omission.

There is an important fundamental difference between our approach to explaining yield spreads and the approach most commonly taken in the literature. Unlike nearly all of the previous work, we do not begin with a yield spread model, i.e., one which begins by examining differences in yields, but rather begin with an expected return or pricing model, which can then be expressed in the yield spread format. This is a fundamental difference in approaches which we feel leads to a superior theoretical formulation which can then be tested empirically without many of the problems inherent in the alternative approach. For example, later in the paper we develop and then test a model which relates the municipal bond yield to three factors: the tax rate; the probability of default on the municipal bond; and the yield on an equal maturity government bond. Beginning with the pricing model, it is evident that these three factors enter the regression equation in a specific multiplicative fashion which differs from that usually assumed and which would have been very difficult to ascertain if one had instead begin with a yield spread model. This distinction will be highlighted at various points throughout our paper.

Of particular interest for our purposes are two recent papers, one by Trzcinka [11] and one by Kidwell and Trzcinka [8], which are typical of the most commonly used approach to modelling yield spreads between taxable and nontaxable bonds. These analyses provide a useful introduction to our paper. As Trzcinka correctly states. "If two bonds are identical except for tax status, an investor will be indifferent between them if the return on the tax-exempt bond, R_m, is equal to the aftertax return on the taxable bond:

$$R_m = (1 - \tau)R_T \tag{1}$$

where τ is the tax rate paid on the taxable return, R_T."

Equation (1) has been modified by Trzcinka and others to allow for differences in default risk by including an intercept term λ that presumably captures any differential risk premium between the two securities.

$$R_m = \lambda + (1 - \tau)R_T. \tag{2}$$

When Equation (2) was estimated by Trzcinka, a shifting intercept technique was employed to allow this risk premium to change over time. The Trzcinka equation can be written as follows:

$$R_{m_t} = \lambda_t + \beta R_{T_t}. \tag{3}$$

A positive value of λ_t has been interpreted to imply that in period t the tax-free security includes a larger risk premium than the taxable security. The specific shifting parameter technique used by Trzcinka assumed that λ_t was a random walk through time. Trzcinka then interpreted the value of $\hat{\beta}$ as an estimate of $1 - \tau$ and used this estimate to test the Miller hypothesis.[2]

Several comments are in order regarding the proper interpretation of this relationship. Using what we feel is a reasonable theoretical model to capture the tax consequences of default, we are able to show that if the two bonds have different probabilities of default, then β depends upon the relative magnitudes of these probabilities and cannot be interpreted simply as an estimate of $1 - \tau$. It is evident, then, that an estimation technique which allows the risk premium for default to vary stochastically, as Trzcinka does, is theoretically inconsistent with a stationary value of β. Stated alternatively, if the relative risk of the two bonds varies over time, then both the constant term *and* slope coefficients in Equation (2) will vary.[3]

In this paper, we develop a simple theoretical model which allows us to express the relationship between tax-free (municipal) and taxable bonds using a general linear equation.

$$R_{M_t} = \alpha_t + \beta_t R_{T_t} + \varepsilon_t. \tag{4}$$

The key insight from this model is that the parameters α_t and β_t are specific nonlinear functions of the break-even tax rate and the default probabilities on the two securities. When Equation (4) is estimated using two risky bond yields, the nonlinear restrictions implied by the theory allow us to infer the ratio of the default probabilities on the two securities. When a risky and a riskless yield are compared, as in this paper, one obtains a direct estimate of the default probability on the risky bond. In both cases the nonlinear restrictions imposed by the theory allow one to infer the break-even tax rate.

The remainder of this paper is organized as follows. In Section I we develop the analytical model of yield spreads. Section II reports the results of a series of empirical tests of the basic model. The final section presents a discussion of the major implications of the paper and suggestions for future research in the area of rate spreads.

I. The Default Model

In this section we present our simple model of the default process which is then used to develop a yield spread relationship between various types of securities. We assume that the probability that a borrower makes the full contractual payment in the stated period, conditional on default having not previously occurred in an earlier period, is constant over time and equal to P. If default

[2] Miller [9] hypothesized that in equilibrium, the differential between default-free taxable and nontaxable bonds will reflect the product of one minus the marginal corporate tax rate and one minus the marginal tax rate on equity income. Since the latter tax rate is usually assumed to be zero in tests of the Miller hypothesis, yield spreads between taxable and nontaxable bonds are compared to the corporate tax rate.

[3] It has come to our attention that Buser and Hess [4] attempt to modify the results of Trzcinka by incorporating the costs of leverage into their analysis. Their results and ours may be viewed as complementary in this regard.

occurs, no payment is made.[4] This specification is consistent with a model in which the probability function is stationary with the conditional distribution of default uniform over time.

Using this model of the default process, Bierman and Hass [2] and Yawitz [13] have demonstrated that in a risk-neutral world without taxes, the yield on a risky bond (r) can be expressed as a function of the yield on a risk free bond (i).

$$r = \frac{(1 - P)}{P} + \frac{i}{P}. \tag{5}$$

While this result is obvious for single period bonds, it is less obvious for multiperiod securities. As demonstrated by Yawitz [13], the intuition is straightforward. The present value of a risky dollar in year t is $(1 + r)^{-t}$. Under risk neutrality, this will also equal the expected value of that \$1, discounted at the risk-free rate. Given our model, the latter is simply $P^t(1 + i)^{-t}$. This equivalence results in Equation (5), which indicates that the yield spread between risk-free bonds and risky bonds with a stationary conditional probability of default is *independent* of the maturity of the bonds.[5] Given this result, the remainder of the theoretical analysis employs one period bonds for ease of exposition.

In this paper we modify the above result by introducing taxes into the analysis of yield spreads between risky and risk-free bonds. As we indicated earlier, we were surprised to find that the tax implications of default have not previously been incorporated into analytical or empirical models of interest rate levels and spreads. In this paper the tax environment for fixed income securities is modelled as follows:

(1) the coupon payment is either taxable as ordinary income *or* it is tax-free, depending on the type of bond, and
(2) in the event of default, no payment is made and the foregone principal is immediately deductible from taxable income. Whether complete deduction is applicable (ordinary loss) or the deduction is treated as a capital loss depends upon the tax status of the investor and the length of time the bond was held.[6]

[4] This "zero-one" specification of the return outcomes represents a considerable simplification since partial payment or renegotiation are generally also possible outcomes. We do not feel, however, that this simplification introduces a problem within the context of our risk neutral framework since the zero-one outcome can be viewed simply as capturing the expected value of the more complex payment stream.

[5] The result that risk premia for default are independent of the number of periods until receipt of the cash flow has also been shown to hold under risk aversion. In our model the term P^t serves to convert the promised or contractural payment due in t periods into its expected payment. Under risk aversion, a certainty equivalent factor, say K^t, plays a similar role by converting a cash flow into its risk-free equivalent. Further discussion of this point can be found in Brealey and Myers [3, pp. 176–182].

[6] For individuals, default results in a capital loss if the bond was purchased more than one year prior to default. However, banks are always allowed to treat default as an ordinary loss for tax purposes. The particular tax treatment of default that is impounded in the yields on risky bonds will depend upon the tax status of the marginal investor in those bonds. The empirical section of this paper attempts to identify whether default is treated as a capital loss or an ordinary loss at the margin.

In the theoretical model below, we allow for a distinction between the ordinary income tax rate, τ, and the rate applicable to this default loss, whether this is the ordinary or capital gains rate, by specifying that the latter is a fixed proportion, α, of the former, where $0 < \alpha \leq 1$.[7] Taking account of taxes, Equations (6) and (7) express the after-tax payment outcomes from investing a dollar in risky bonds that have either a taxable or a nontaxable coupon, respectively. Thus, Equation (6) would describe a corporate bond, while (7) would describe a municipal bond.

$$\theta_c = \begin{cases} [1 + r_c(1 - \tau)] & \text{with probability } P_c \\ \alpha\tau & \text{with probability } (1 - P_c) \end{cases} \tag{6}$$

$$\theta_M = \begin{cases} (1 + r_M) & \text{with probability } P_M \\ \alpha\tau & \text{with probability } (1 - P_M) \end{cases} \tag{7}$$

Throughout this discussion r's denote loan rates (yields) and P's denote the probability of payment.

The expected after-tax rate of return on the corporate bond can be expressed as

$$\bar{R}_C = E(\theta_c) - 1 = P_c r_c(1 - \tau) - (1 - P_c)(1 - \alpha\tau), \tag{8}$$

while the expected rate of return on the municipal bond is

$$\bar{R}_M = E(\theta_M) - 1 = P_M r_M - (1 - P_M)(1 - \alpha\tau). \tag{9}$$

These expressions for $\bar{R}_m$ and $\bar{R}_c$ can be interpreted as follows: the first term is the expected after tax coupon (interest) payment; the second term is the expected after tax *loss* due to default. The absolute value of the second term varies *inversely* with the tax rate, indicating that for a given market yield the possibility of default results in a larger reduction in expected after-tax return, the smaller the tax rate.

Yield spreads among the various securities can be derived easily if the assumption of risk neutrality is maintained. We consider two other bonds, one taxable, and one tax-free, and both riskless. The default-free, tax-free rate of return is denoted by i, while the default-free, taxable rate is denoted by r_g (government bonds). Note that for risk-free, tax-free security, its expected return, $\bar{R}_i$, equals i, while the after-tax return on the risk-free government bond, $\bar{R}_g$, equals $r_g(1 - \tau)$. One particularly interesting exercise is to derive the equilibrium relationship between the promised yields on government, municipal, and corporate bonds, and the risk-free, tax-free rate. Equations (10) to (12) express these relationships.

[7] This simple view is consistent with the treatment of capital gains under the U.S. tax systems, since investors will realize capital losses in the current tax year.

$$r_g = \frac{i}{1-\tau} \tag{10}$$

$$r_M = \frac{i}{P_M} + \frac{(1-P_M)}{P_M}(1-\alpha\tau) \tag{11}$$

$$r_c = \frac{i}{P_c(1-\tau)} + \frac{(1-P_c)}{P_c}\frac{(1-\alpha\tau)}{(1-\tau)}. \tag{12}$$

Consider first the municipal rate r_M. If $\alpha\tau = 1$ then $r_M = i/P_M$, indicating that principal is completely insured by the tax code in the event of default, but that the coupon payment is not insured. As $\alpha\tau$ falls toward zero, r_M must increase reflecting the additional compensation required for the possibility that a portion of the principal will not be recovered in the event of default. That is, the lower the tax rate that is applicable to a default, the lower the "insurance coverage" provided by the tax code and the greater the yield compensation required by investors.

Figure 1 indicates how the tax rate affects the locus of yield-default probability

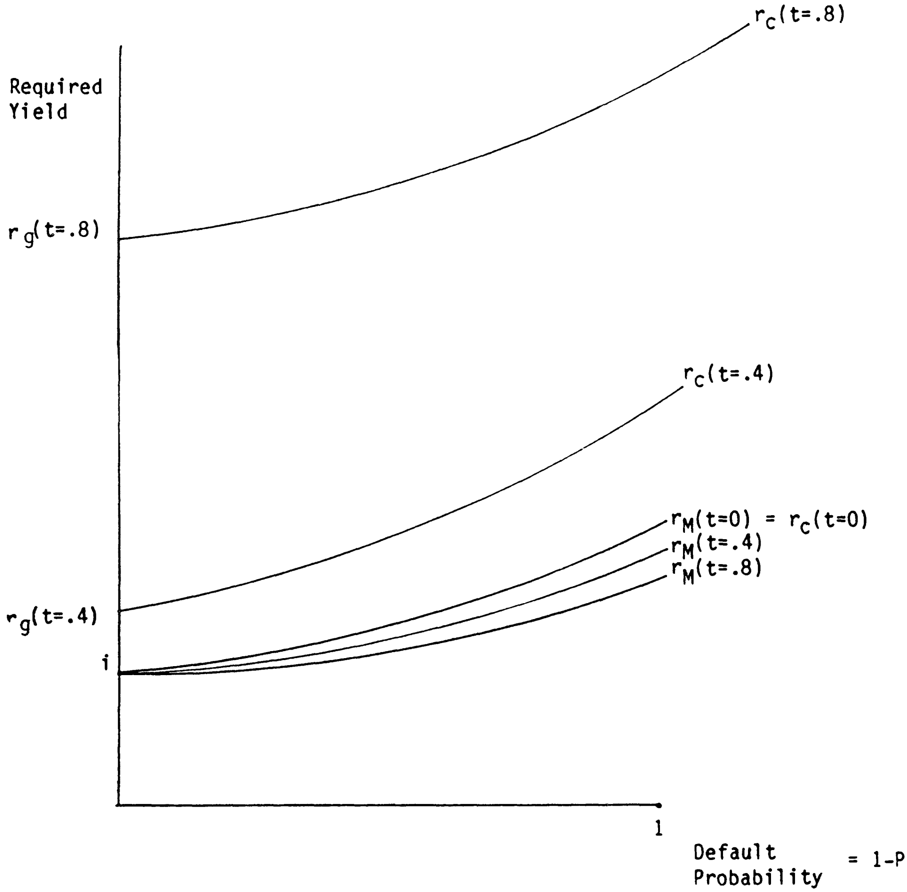

Figure 1. The Effect of Default and Tax Rates on Yields

combinations for which the expected after-tax return is a constant i. The figure is drawn assuming a constant value for α. Note that for a given probability of default, the yield required by an investor to be indifferent between a risky and a riskless *tax-free* bond varies *inversely* with the investor's tax rate. Stated alternatively, the expected after-tax rate of return on a given risky tax-free bond varies *directly* with the investor's tax rate. The intuition here is straightforward. If a tax-free bond makes its promised interest payment, the realized after-tax return is obviously independent of the investor's tax rate. On the other hand, if the bond defaults, the investor's loss is $(1 - \alpha\tau)$ per dollar of principal since the government bears $\alpha\tau$ of the loss. The larger the tax rate, the smaller the after-tax loss in the event of default, and hence, the lower the required yield on the bond. The fact that the investor's tax rate is relevant for making comparisons among tax-free bonds with different default probabilities is due to the asymmetric effect of taxes on the two return outcomes. If payment is made, the tax system is irrelevant. If default occurs, the capital gains rate determines the size of the loss to the investor.

The above discussion has important implications regarding the types of investors that will be attracted to risky, tax-free bonds. If equilibrium were to settle at the point where the relationship between rM and $(1 - P)$ reflects a break-even tax rate of τ^*, then investors with tax rates lower than τ^* would purchase risk-free, tax-free bonds, while investors with tax rates higher than τ^* would purchase risky tax-free bonds. This clientele effect can be demonstrated easily using Equations (9) and (11). If equilibrium in the municipal market settles at a point where the break-even tax rate is τ^*, then, from Equation (11) the equilibrium yield default relationship will be as follows:

$$r_M^* = \frac{i}{P_M} + \frac{1 - P_M}{P_M}(1 - \alpha t^*). \tag{13}$$

Substituting into Equation (9), one obtains the expected after-tax return from investing in municipal bonds to any investor

$$\bar{R}_M = P_M r_M^* + (1 - P_M)(\alpha\tau - 1) = i + \alpha(1 - P_M)(\tau - \tau^*). \tag{14}$$

It is apparent from the above equation that the expected after-tax return from investing in a municipal bond will be greater (less) than i when τ is greater (less) than τ^*. In addition, since the default probability is multiplied by the difference between the two tax rates, it is clear that high tax investors gain more the higher the default probability. Since,

$$\frac{\partial^2 \bar{R}_M}{\partial(1 - P_M)\partial\tau} = \alpha \tag{15}$$

it is clear that if default were not accompanied by a tax deduction then this clientele effect would disappear.[8]

Figure 1 also portrays the effect of default on taxable bond yields. As indicated

[8] It is clear from Equation (14) that this clientele effect causes significant tax arbitrage opportunities if we allow short sales by individual investors. Therefore, we assume that these sales are not allowed. As indicated by the referee, this implies that the break-even tax rate may be sensitive to the supply of taxable and nontaxable securities. This question, however, is outside the scope of this paper.

by (10), the required yield on a riskless taxable bond varies directly with the investor's tax rate. This effect is straightforward and can be seen by moving upward along the vertical axis in Figure 1. The introduction of the possibility of default, however, complicates the analysis considerably. From Equation (12), it can be demonstrated that for given values of i and P_c, the required yield on a risky, taxable bond varies directly with the investor's tax rate, i.e.,

$$\frac{\partial r_c}{\partial \tau} = \frac{1}{P_c(1 - \tau)^2}\,[i + (1 - P_c)(1 - \alpha)]. \tag{16}$$

Two separate effects can be noted here. The higher the tax rate, the greater the yield required to make the expected tax coupon payments on a taxable bond equal to that on a risk-free, tax-free bond. This compensation is greater, the greater the probability of default since the compensation for default risk (higher coupon) is itself taxable in this case. The second effect of taxes on the yield of a risky bond occurs through the expected after-tax loss of principal in the event of default. The higher the investor's tax rate, the lower is the expected after-tax loss from default due to the "insurance" aspect of the tax code. However, since the former effect must dominate even if the write-off is at the ordinary income tax rate ($\alpha = 1$), the net effect on yields of a higher tax rate remains positive $(\partial r_c/\partial t) > 0$. Thus, in contrast to the case of a tax-free security, for taxable securities the required yield varies *directly* with the investor's tax rate.

As in the case of tax-free securities, the tax implications of default lead to a clientele effect for taxable securities as well. However, in this case the clientele effect interacts with the default probability in the opposite direction. To illustrate this, suppose the break-even tax rate for all probabilities of default is τ^*. The yield-default relationship for taxable securities will be as follows:

$$r_c^* = \frac{i}{P_c(1 - \tau^*)} + \frac{(1 - P_c)(1 - \alpha\tau^*)}{P_c(1 - \tau^*)}. \tag{17}$$

Substituting Equation (17) into Equation (8), one obtains an expression for the expected after-tax return for an investor with a tax rate τ.

$$\begin{aligned}\bar{R}_c &= P_c r_c^*(1 - \tau) + (1 - P_c)(\alpha\tau - 1)\\ &= i\,\frac{(1 - \tau)}{(1 - \tau^*)} + (1 - P_c)\left[\frac{(1 - \tau)(1 - \alpha\tau^*)}{(1 - \tau^*)} - (1 - \alpha\tau)\right]\\ &= \frac{[i(1 - \tau) + (1 - P_c)(1 - \alpha)(\tau^* - \tau)]}{(1 - \tau^*)}.\end{aligned} \tag{18}$$

Equation (18) implies that the expected after-tax return will be greater (less) than i when the investor's tax rate is less (greater) than the break-even tax rate. The interesting result, however, is the implication of different default probabilities. If we take the cross partial derivative of (18) with respect to τ and $(1 - P_c)$, we obtain the following expression

$$\frac{\partial^2 \bar{R}_c}{\partial(1 - P_c)\partial\tau} = -(1 - \alpha). \tag{19}$$

Note that this implies that the impact of a high tax rate on the expected after tax return on corporate bonds is lower, the greater the default probability. This clientele effect will disappear only if default is treated as an ordinary loss ($\alpha = 1$).

II. Empirical Results

Before proceeding with the presentation and discussion of empirical results, it is useful to anticipate possible problems that could be introduced from the data. Certainly we are not proposing that the yield spread observed between different types of equal maturity bonds is determined entirely by taxes and ex ante probabilities. We do feel, however, that the model developed in the previous section has much to recommend it, both in terms of the simplicity with which it captures taxes and default differences, and its readily estimable form.

The empirical work uses government and municipal bond data. The government yield series should be fairly "clean" of the influence of factors such as callability or marketability. Governments are either noncallable, in the case of bills and intermediate maturity bonds, or callable only in the last few years, in the case of long-term issues. Marketability problems, even for the less widely traded issues, should be minimal for the government yield series. Municipal's, on the other hand, are readily subject to callability and to marketability problems. Most municipal bonds with an initial maturity of more than 10 years are callable; the industry norm on long-term issues is 10 years of call protection. Since the call provision reduces the price of a bond to which it is attached, the yield on a callable municipal bond will be higher than the yield on an otherwise identical, noncallable bond. It is reasonable to expect that marketability will also reduce municipal bond prices and hence cause stated yields to be higher.

Differences in callability and marketability between government and municipal bonds would have the effect of increasing observed municipal bond yields relative to Treasury yields. To the extent that municipal yields are higher, our estimates of the parameters in Equation (20) would tend to be biased downward. Consequently, we recommend that our estimates of the probabilities of default on municipal bonds be viewed as upper bounds of the true probabilities of default, recognizing that these estimates are likely to be influenced by differences in callability and marketability between government and municipal bonds.

Our use of government and municipal yield series minimizes the measurement error problem common to yield spread studies. As is well-known, random measurement error in the independent variable leads to coefficients biased toward zero in any least-squares estimation while measurement error in the dependent variable does not result in bias although it does lead to less efficient estimates. Yield spread studies, e.g., Trzcinka [11], often regress municipal and corporate rate series for equivalent ratings and terms to maturity on each other. Since both municipals and corporates are heterogeneous and lightly traded, these series undoubtedly contain measurement error and yield-biased parameter estimates. Since U.S. government issues are more homogeneous and more heavily traded, this measurement error is much smaller. Thus, the use of governments as the independent variable minimizes measurement error bias in the parameters.

Table I

Estimation Results: Constant Probabilities Capital Losses Partially Deductible ($\alpha < 1$)[a]

Dependent Variable	τ	P	ρ^*	R^2
Prime 20	0.500	0.979	0.86	0.947
	(0.058)	(0.0061)		
Good 20	0.514	0.976	0.87	0.944
	(0.061)	(0.0065)		
Medium 20	0.469	0.976	0.89	0.949
	(0.068)	(0.0072)		
Prime 10	0.517	0.985	0.76	0.928
	(0.041)	(0.0042)		
Good 10	0.509	0.984	0.79	0.929
	(0.045)	(0.0046)		
Medium 10	0.497	0.981	0.82	0.927
	(0.054)	(0.0055)		
Prime 5	0.557	0.985	0.67	0.913
	(0.032)	(0.0032)		
Good 5	0.556	0.984	0.71	0.917
	(0.034)	(0.0035)		
Medium 5	0.540	0.982	0.75	0.915
	(0.042)	(0.0043)		

[a] Asymptotic standard errors are in parentheses.
* Estimates of the autocorrelation coefficient (see footnote 11).

Using Equations (10) and (11) from the previous section, we can express the appropriate equilibrium relationship between the yield on a risky municipal bond and a default-free government bond as follows,

$$r_M = \frac{(1 - P)(1 - \alpha\tau)}{P} + \frac{(1 - \tau)}{P} r_g \tag{20}$$

where τ is the break-even tax rate for this probability of default, α is the proportion of default losses that are written off as ordinary income, and P is the probability of payment for the municipal. Note that although there is a linear relationship between r_M and r_g according to the theoretical model of the previous section, the slope and intercept of that relationship are both nonlinear functions of the probability of payment and the break-even tax rate. In particular, it should be noted that the slope coefficient does not equal one minus the break-even tax rate, as commonly assumed, unless the tax-free security is also default-free. Thus, previous studies that make this inference underestimate the tax rate when the tax-free security is subject to a nonzero probability of default.

Two alternative estimates of Equation (20) are presented in Tables I and II. Any estimation utilizing two variables can provide only two unique parameter estimates. Since Equation (20) has three parameters, some assumptions must be imposed regarding one of the three. Accordingly, Equation (20) is estimated under two alternative assumptions. In Table I, we assume that, for securities

Table II

Estimation Results: Constant Probabilities Capital Losses Fully Deductible ($\alpha = 1$)[a]

Dependent Variable	τ	P	ρ^*	R^2
Prime 20	0.493	0.969	0.87	0.946
	(0.059)	(0.0115)		
Good 20	0.508	0.964	0.87	0.944
	(0.062)	(0.0129)		
Medium 20	0.466	0.966	0.89	0.948
	(0.069)	(0.0133)		
Prime 10	0.512	0.977	0.76	0.928
	(0.041)	(0.0076)		
Good 10	0.506	0.975	0.79	0.929
	(0.045)	(0.0083)		
Medium 10	0.493	0.972	0.82	0.927
	(0.054)	(0.0101)		
Prime 5	0.544	0.976	0.66	0.914
	(0.031)	(0.0063)		
Good 5	0.554	0.973	0.70	0.918
	(0.034)	(0.0070)		
Medium 5	0.538	0.971	0.75	0.915
	(0.041)	(0.0084)		
Prime 1	0.620	0.970	0.70	0.944
	(0.029)	(0.0073)		
Good 1	0.633	0.964	0.74	0.924
	(0.030)	(0.0082)		
Medium 1	0.636	0.958	0.78	0.922
	(0.034)	(0.0101)		

[a] Asymptotic standard errors are in parentheses.

* Estimates of the autocorrelation coefficient (see footnote 11).

with more than one year to maturity, any default losses are treated as long-term capital losses so that α equals the percentage of capital gains (losses) that are included in taxable income.[9] In Table II, the estimates of P and τ are presented assuming that $\alpha = 1$. This implicity assumes that the marginal investor is allowed to fully deduct from income any default losses for tax purposes (e.g., banks). Since it is assumed that any short-term losses are fully deductible, the relationship is only estimated under the latter assumption for securities with a year or less until maturity. These two assumptions should bracket the true value, and a comparison of Tables I and II should shed some light on which is the more appropriate.

Before turning to the empirical results, some brief comments on the data and

[9] Throughout most of the sample period, that percentage was 0.4, but for some of the earlier periods it equalled 0.5. Note that if α had not varied over the sample period, the least-squares estimate of the slope and intercept and the sum of squares error would have been equal to those for $\alpha = 1$ (or any other constant α). However, estimates of P and τ would have varied with α in a known manner.

estimation technique are in order. The data used in this study consist of monthly observations of the yields required to sell new municipal and government securities at par.[10] All yields are annualized. Therefore, the estimated default probabilities are also annualized measures. The data sample begins in August 1965 and ends in March 1981, and includes observations with maturities of 20, 10, 5, and 1 years, respectively. In addition, the municipal bond data includes observations from three different grades: prime; good; and medium. Thus, there are twelve separate regressions that can be performed.

The parameter estimates in Tables I and II were obtained using the Gauss-Newton estimation procedure for nonlinear equations. Using the first derivative to indicate the effect on the sum of squared errors of any parameter change, this iterative procedure chooses estimates to minimize the sum of squared errors. These estimates are consistent and have all the maximum likelihood properties if the error terms are normally distributed. Consistent estimates of the variance-covariance matrix are also provided, and these are used to obtain the asymptotic t-statistics shown in Tables I and II. Since the initial runs revealed serious first-order autocorrelation, the equations were respecified and estimated in a nonautocorrelated form, and it is these results which are shown in Tables I and II.[11]

The results in Tables I and II provide strong evidence that this model of the relationship between a riskless taxable bond and a risky, tax-free bond is a reasonable one. The model explains over 90% of the variation in municipal bond rates, and the t-statistics are significant at the 1% level in every case.[12] The estimated tax rates are reasonable, and the estimated annual default probabilities $(1 - P)$ are surprisingly and encouragingly consistent.[13] All of the default probabilities are fairly low, as one would expect; yet no estimates are below zero. Nonetheless, the estimated default rates (most of which are between 1.5 and 2.5%) exceed recent default experience.[14] As explained previously, we feel that

[10] The government yield series were obtained from the Citibank Economic Database, while the municipal yield series were obtained from the *Analytical Record of Yields and Yield Spreads*, published by Salomon Brothers.

[11] This autocorrelation likely arises from or is exaggerated by two factors: (1) our yields are monthly averages, and (2) the municipal figures are Salomon Brothers specialists' estimates of the yields required to sell new issues and any measurement error may be serially correlated. With time subscripts and an error term attached, Equation (20) can be expressed as

$$r_{M_t} = \frac{(1 - P)(1 - \alpha\tau)}{P} + \frac{(1 - \tau)}{P} r_{g_t} + u_t.$$

If the error term is first-order autocorrelated, i.e., if $u_t = \rho u_{t-1} + \varepsilon_t$, then the nonautocorrelated form

$$r_{M_t} = \frac{(1 - P)(1 - \alpha\tau)}{P}(1 - \rho) + \frac{(1 - \tau)}{P}[r_{gt} - \rho r_{gt-1}] + \rho r_{M_{t-1}} + \varepsilon_t \qquad \text{(a)}$$

may be derived. It is the nonlinear estimates of Equation (a) which are shown in Tables I and II.

[12] The R^2 reported in the tables are for r_{M_t}, not quasi-differences as are sometimes reported for linear-autocorrelated models.

[13] Least satisfactory are the results for one year securities where the tax rate appears too high and the probabilities too low. Examination of the variance covariance matrix revealed that the covariance between these two estimates was negative so that if one was overestimated the other would tend to be underestimated. That appears to be the case here.

[14] See [1, 6, 12] for data on actual default experience.

these default probabilities are most likely biased upward due to differences in callability and marketability between municipal and government bonds. Given our results, it appears that there was such a bias but that it was slight. Inspection of the results also yields the following observations and conclusions:

(1) With only one exception, the estimated probability of payment for equal maturity bonds is lower, the lower the municipal bond's rating. The estimated probabilities for good and medium 20-year bonds are the sole exception. This result certainly provides support for Equation (20) as a model of the yield relationship.
(2) There appears to be little systematic relationship between estimated tax rates and bond ratings. There was a tendency for the estimated tax rate to decrease as one moved from good to medium bonds but the relationship between prime and good was very inconsistent.
(3) In most cases, the estimated tax rate increased as maturity decreased for bonds of a given grade. This result, which is consistent with a market segmentation hypothesis in the municipal market,[15] could also be due to the effect of callability which would tend to increase the yields on longer term municipal bonds, ceteris paribus.
(4) It is difficult to draw any strong conclusions regarding the appropriate assumption concerning α from a comparison of Tables I and II.[16] The R^2's are virtually identical, and the parameter estimates are similar—although the assumption of fully deductible losses ($\alpha = 1$) generally resulted in slightly lower estimates of both the tax rate and the probability of payment. Basically it appears that the results are not very sensitive to this choice.

III. Implications and Conclusions

The results of this study have a number of interesting implications. One particularly interesting implication of the empirical results is that the yield spreads among different grade municipal bonds are much smaller than the spread between a prime grade municipal bond and a government bond, even after adjustment for taxes. To see this, consider the implications of the results in Table I for 5-year bonds. Suppose for simplicity that the break-even tax rate is constant across probabilities of default at 0.5, that the government rate r_g is 5%, and that α is 0.4. Using the estimated P values in Table I, the following yields are implied.

	Pre-tax Yield	After-tax Yield
Government Bond	5%	2.5%
Prime Grade Municipal	3.76%	3.76%
Good Grade Municipal	3.84%	3.84%
Medium Grade Municipal	4.01%	4.01%

[15] See Hendershott and Kidwell [7] for a discussion of this hypothesis.

[16] The fact that the explanatory power is virtually identical is probably due to the fact that the α series used in Table II exhibited very little variation (see footnote 10) over the period and, therefore, had little influence on the overall explanatory power of the equations.

Note that the spread between the after-tax yield on a government bond and a prime grade municipal is approximately 4 times as large as the spread between the yields on the prime and medium grade municipal bonds. Since the after-tax yield on the government bond in this example corresponds to the rate on a hypothetical risk-free municipal security, this exercise implies that prime grade municipal bonds do have significant risk premiums embodied in their yields.

In summary, in this paper we have developed and tested a model of the interaction of default and taxation on yield spreads. The theoretical model, although quite simple, seems to have a high degree of explanatory power. The relationship between government yields and municipal yields was investigated empirically. The estimated parameters representing default risk and tax rates from the empirical analysis are quite reasonable. This strong evidence implies that this model is potentially useful for the determination of the interaction of default risk and taxation in studies of the yield spreads between other types of securities. In addition, further tests of the Miller hypothesis could be developed using this framework. In particular, some attempt to relate break-even tax rates to the relative supplies of taxable and tax-exempt debt using this framework would be both useful and enlightening.

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