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The Impact of Inflation on the Aggregate Debt-Asset Ratio

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ABSTRACT

This paper demonstrates that, contrary to the results of previous studies, the impact of inflation on the aggregate debt-asset ratio cannot be determined theoretically. However, it is shown that inflation is likely to increase this ratio when personal income tax schedules are indexed to the price level and/or when leverage-related costs are relatively high and the personal tax rate on income from holding common stocks is relatively low.

STUDIES CONCERNING THE EFFECT of inflation on firms' capital structure conclude that inflation enhances debt financing. Jaffe [9], Modigliani and Cohn [16], and Modigliani [15] argue that nominal interest payments consist, in part, of true interest payments and, in part, of a compensation for the reduction in the real value of the principal. The fact that firms are allowed to deduct their entire interest expense, including the portion that is a return of principal, reduces their cost of debt capital. Gordon [6] concludes that the equilibrium debt-capital ratio rises when the inflation rate rises. He argues that an increase in the nominal interest rate induced by inflation increases the tax advantage of debt financing. Thus, debt financing becomes more attractive and a rise in leverage-related costs is necessary to restore equilibrium. Gordon asserts that the increase in leverage-related costs is brought about by an increase in the debt-equity ratio. All these studies, however, focus primarily on the tax benefits of debt financing to the corporation and implicitly assume that each firm has a unique optimal structure which may change with inflation. In contrast, Miller [14], in a world without inflation, concludes that firms are indifferent to their capital structure and that the aggregate debt-equity ratio is determined by the ratio of the wealth of investors who, for tax reasons, choose to hold bonds to the wealth of those who choose to hold stocks.

This paper explicitly incorporates tax considerations of both investors and firms into the analysis of the effect of inflation on the aggregate corporate debt-asset ratio. It is demonstrated that the effect of inflation on the aggregate debt-asset ratio cannot be determined theoretically. However, inflation is likely to increase this ratio when personal income tax schedules are indexed to the price level and/or when leverage-related costs are relatively high and the tax on income from holding common stocks is relatively low. We consider an economy in which investors are subject to a progressive tax system. Income from corporate bonds

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is taxed at the ordinary personal income tax rate, while the effective tax rate on income from common stock is lower. To facilitate the analysis, we follow Miller [14] and Kim, Lewellen and McConnell [12] in treating investors' portfolio-diversification considerations as exogenous to the model. Investors are assumed to be identical in all respects, except for their personal tax rates, and thus they choose between holding bonds and holding stocks according to their personal marginal tax rates. Investors whose personal marginal tax rates exceed some critical tax rate, t_p^*, hold stocks only, while investors whose tax rates are below t_p^* hold bonds only. Thus, the aggregate debt-asset ratio is determined by the wealth of investors whose personal tax rate is below t_p^* relative to the wealth of all investors. Since the focus of this paper is on the impact of inflation on the aggregate debt-asset ratio, it is assumed that firms are identical and, therefore, that the debt-asset ratio is the same for all firms. This economy is referred to in the paper as a "Miller-type" world.[1]

Inflation affects the aggregate debt-asset ratio in two ways: first, by changing investors' marginal tax rates and second by changing the critical tax rate, t_p^*. These effects are termed, in this paper, the "bracket creep" effect and the "critical rate" effect. In the absence of tax schedule indexation, inflation pushes investors into higher tax brackets. Thus, for a given critical tax rate, t_p^*, some investors whose personal tax rates increase due to inflation and exceed t_p^* are induced to replace their bond holding with stocks. The "bracket creep" effect therefore tends to decrease the aggregate debt-asset ratio as the share of the wealth of investors whose marginal tax rate is below that critical tax rate decreases. Inflation also affects the aggregate debt-asset ratio by its impact on the critical tax rate. This "critical rate" effect reinforces the "bracket creep" effect if inflation reduces t_p^*, and it has the opposite effect on the aggregate debt-asset ratio if inflation increases t_p^*.

In Section I, we derive a valuation formula for a firm in a Miller-type world (with no leverage-related costs). It is demonstrated that the "irrelevance" theorem holds in the presence of inflation and therefore the aggregate debt-asset ratio is determined by investors' preferences only. We show that, in such an economy, t_p^* equals or exceeds the corporate tax rate and does not increase with inflation. Thus, the "critical rate" effect may reinforce the "bracket creep" effect, implying that, in a Miller-type world, inflation decreases the aggregate debt-asset ratio. In Section II, we assume positive leverage-related costs such that the tax rate that sorts investors into bondholders and stockholders (denoted there by t_p^{**}) is lower than the corporate tax rate. It is shown that, under this assumption, t_p^{**} increases with inflation. Thus, the total effect of inflation on the aggregate debt-asset ratio is ambiguous: the "bracket creep" effect tends to reduce the debt-asset ratio, where the "critical rate" effect increases the ratio. Section III summarizes and concludes the paper.

[1] Tax-free (municipal) bonds are issued by government agencies which do not issue common stocks. Since the focus of the paper is on the effect of inflation on corporate capital structure, the effect of inflation on the supply of tax-free bonds (which depends primarily on the financing needs of these agencies) is ignored in the analysis. Also, no assumption is made regarding the relationship between the rate of return on tax-free bonds and the rate of return on corporate bonds.

I. Firm Valuation, the "Bracket Creep," and the "Critical Rate" Effects

In this section, we derive the value of a firm to the marginal investor. We then demonstrate that, in a Miller-type world, firms are indifferent to their capital structure under any inflation rate and, therefore, the aggregate debt-asset ratio is determined by investors' preferences. We further show that, in such a world, the aggregate debt-asset ratio declines with inflation.

Auerbach and King [1] have demonstrated that in a world in which investors face different tax rates, no equilibrium exists unless some constraints are imposed on the securities available in the market. In particular they argue that, "It is difficult to reconcile value maximization with the existence of portfolios which contain both debt and equity." They have shown that, under certain assumptions, equilibrium consists of investors who, for tax reasons, hold bonds only or stocks only. This equilibrium implies that each firm is indifferent to its capital structure and that the aggregate debt-asset ratio depends on the wealth of investors who hold bonds relative to the wealth of all investors. Since the focus of this paper is on the effect of inflation on the aggregate debt-asset ratio (in the presence of corporate tax and progressive personal taxes), the economy described in Auerbach and King provides a convenient framework for the analysis and will be assumed throughout the paper.

Specifically, our main assumptions are:

(A1) Investors may hold only two types of assets, corporate bonds and common stocks.

(A2) All firms are identical and, therefore, no diversification of common stock is possible.

(A3) Investors differ only with respect to their personal tax rates on income from bonds, t_p. Income from equity investments is subject to a lower tax rate, t_s, which is the same for all investors. The corporate tax rate, t_c, and t_s do not vary with inflation.[2]

(A4) Most investors hold bonds only or stocks only. There exists a marginal tax rate, t_p^*, that makes an investor indifferent to the bond/stock composition of his portfolio; this investor is referred to in the paper as the "marginal investor."

(A5) There are no leverage-related costs.

Assumption (A3) implies that stockholders are in fact the same because they differ only with respect to their personal tax rates on income from bonds which, according to assumption (A4), they do not hold (except for the marginal investor). Thus, the valuation of a firm is the same for all stockholders. The market value of the firm (V^*) corresponds to the valuation of the marginal investor since he is the only investor who changes his holdings following (small) changes in the stock price. Equilibrium is achieved when all investors whose personal tax rates exceed t_p^* hold stocks only, while those whose personal tax rates are below t_p^* hold bonds only. Equilibrium exists because the returns from holding stocks to

[2] Qualitatively, our results still hold if the inflation-induced variations in t_s are small compared to the variations in t_p.

investors whose personal tax rates exceed t_p^* equal the returns from holding stocks to the marginal investor, while the returns they receive from holding bonds are less than the returns from holding bonds to the marginal investor. Since the marginal investor is, by definition, indifferent between holding stocks and bonds, investors whose personal tax rates exceed t_p^* prefer holding stocks to holding bonds. The reverse holds for investors whose personal tax rate is below t_p^*.

We begin the analysis with the derivation of the value of an unlevered firm to the marginal investor as a function of the expected inflation rate, π, the level of real investment, I, and the expected before-tax real operating (net of depreciation) cash flow,[3] $\bar{y}$. Assuming that inflation is expected to be constant over time and that it affects all prices at the same rate, the expected nominal after-tax cash flow, at the end of period t, is $(1 - t_c)\bar{y}(1 + \pi)^t$. Assume that in each period the firm, in order to minimize stockholders' tax bill, distributes all its earnings in cash by repurchasing stock from the stockholders.[4] Stockholders' income is, therefore, subject to a capital-gains tax at the rate t_s. This assumption is essential in the construction of the model, since otherwise income from stocks is taxed at the same rate as income from bonds, inducing firms to be all debt-financed (in the absence of leverage-related costs).

The real value of the firm's assets is assumed to be constant over time, implying that the nominal value of the asset, at the end of period t, equals $I(1 + \pi)^t$. It is further assumed that the firm defers the sale of the asset and therefore that the present value of the capital-gains tax on the appreciation of the asset's value is small and can be ignored. Let ρ denote the after-personal-tax nominal required rate of return on a pure equity stream.[5]

The value of the unlevered firm to the marginal investor, V_u^*, is

$$V_u^* = \sum_{t=1}^{\infty} \frac{(1 - t_s)(1 - t_c)\bar{y}(1 + \pi)^t}{(1 + \rho)^t} = \frac{(1 - t_s)(1 - t_c)\bar{y}(1 + \pi)}{\rho - \pi}. \quad (1)$$

Consider now a levered firm, and let α denote the proportion of the investment financed by debt. In order to maintain the same capital structure over time, the nominal increase in the value of the firm's assets induces the firm to issue additional debt and to use the cash received from the bond issue to repurchase stock.[6] The value of the firm's debt, in period t, B_t, equals $\alpha I\pi(1 + \pi)^t$. To maintain a constant debt to asset ratio, the firm issues bonds at nominal value

[3] Inflation affects the real tax shelter provided by depreciation deductions. This effect is ignored here since it influences the firm's investment decision but not its financing decision. The firm's investment decision is considered exogenous in this paper.

[4] Although systematic repurchase of stock is not allowed by the U.S. Internal Revenue Code, infrequent share repurchases and a possible acquisition of the company by another firm effectively permit earnings to be taxed as capital gains on a delayed basis (see Feldstein and Green [4]).

[5] The nominal rate ρ depends on inflation but is not labeled as $\rho(\pi)$ to conserve on notation.

[6] Alternatively, the firm may issue "indexed" bonds that will keep the real value of debt and the debt-asset ratio unchanged over time. Although we are not aware of the existence of indexed bonds in the U.S., indexed bonds exist in other countries (e.g., the Eurobond markets and Israel). These bonds have coupons which are denoted in real terms and hence the annual interest payments on these bonds are variable, rising roughly with the price level. The only difference between the policy of issuing indexed bonds and the issue of additional regular bonds each period (i.e., the policy assumed

of $\Delta B_t = B_t - B_{t-1} = \alpha I\pi(1+\pi)^{t-1}$ and distributes the cash to stockholders by purchasing stock. Let R denote the nominal market (pre-personal-tax) interest rate on corporate bonds. Assuming that R is constant over time (for a given expected inflation rate), nominal interest payments to bondholders vary over time and equal $RB_{t-1} = R\alpha I(1+\pi)^{t-1}$. Therefore, the after-personal-tax income to stockholders in each period is

$$(1-t_s)\{(1-t_c)[\bar{y}(1+\pi)^t - R\alpha I(1+\pi)^{t-1}] + \alpha I\pi(1+\pi)^{t-1}\}.$$

This income to stockholders consists of after-corporate-and-personal-tax income from production (net of interest payments) and of cash distributed to stockholders from the issue of bonds. The marginal investor, indifferent between holding stocks and bonds, may hold both, implying that his or her discount rate for cash flows that have the same risk characteristics as income from corporate bonds is $(1-t_p^*)R$. Thus, the value of the levered firm, V_L^*, is:

$$\begin{aligned} V_L^* &= \sum_{t=1}^{\infty}(1-t_s)(1-t_c)\bar{y}\left[\frac{(1+\pi)}{(1+\rho)}\right]^t \\ &\quad - \frac{(1-t_s)\alpha I[(1-t_c)R-\pi]}{1+(1-t_p^*)R}\sum_{t=0}^{\infty}\left[\frac{(1+\pi)}{1+(1-t_p^*)R}\right]^t + \alpha I \\ &= \frac{(1-t_s)(1-t_c)\bar{y}(1+\pi)}{\rho-\pi} - \frac{(1-t_s)\alpha I[R(1-t_c)-\pi]}{(1-t_p^*)R-\pi} + \alpha I, \end{aligned} \tag{2}$$

where αI is the value of the debt issued in period zero. Using (1), we obtain[7]

$$V_L^* = V_u^* + \alpha I\left[1 - \frac{(1-t_s)[(1-t_c)R-\pi]}{(1-t_p^*)R-\pi}\right] \equiv V_u^* + \alpha IG^*. \tag{3}$$

Equation (3) implies that firms may be financed by combinations of debt and equity only if:

$$[(1-t_p^*) - (1-t_s)(1-t_c)]R = t_s\pi. \tag{4}$$

When t_s equals zero (as Miller assumes) this condition implies that $t_p^* = t_c$, which is identical to Miller's result with no inflation. However, the present result holds for all levels of inflation and interest rates, showing that if $t_s = 0$, $t_p^* = t_c$; hence, Miller's "irrelevance" theorem holds under inflation also. Because in this case t_p^* equals t_c and t_c does not vary with inflation, t_p^* is also unaffected by inflation. Consequently, the "critical rate" effect vanishes. There will be only the "bracket creep" effect and it will induce a reduction in the debt-asset ratio as inflation increases. This is brought about as investors whose marginal tax rates increase

in the text) is that under the latter policy an amount of $\alpha I\pi(1+\pi)^{t-1}$ in period t ($t = 1, 2, \cdots$) is tax deductible to the firm and is taxable to the investor, whereas with indexed bonds the tax deduction to the firm and the tax obligation of the investor vanish. Thus, indexed bonds should be preferred when $t_p^* > t_c$ and regular bonds should be preferred when $t_p^* < t_c$. Also, when indexed bonds are assumed, the "critical rate" effect vanishes and only the "bracket creep" effect is present.

[7] The value of leverage G^* is identical to l in Modigliani [15, p. 259, Equation (II-7)]. However, Modigliani derived the value of leverage in a single-period model with a uniform tax system.

and exceed t_c (due to inflation) are induced to replace their bond holdings by stocks.

In general, however, t_s need not equal zero. In the following, we show that in this case t_p^* exceeds t_c and that t_p^* declines with inflation[8] (i.e., $dt_p^*/d\pi < 0$). Thus, in the more general case (when $t_s > 0$), the "critical rate" effect reinforces the "bracket creep" effect in reducing the aggregate debt-asset ratio as inflation increases. Rewriting Equation (4) as $(t_s/R)[(1 - t_c)R - \pi] = t_p^* - t_c$ and assuming that the firm's real cost of debt capital $((1 - t_c)R - \pi)$ is positive implies that t_p^* exceeds t_c.[9] Totally differentiating Equation (4) with respect to the inflation rate yields:

$$-\frac{dt_p^*}{d\pi} R + [(1 - t_p^*) - (1 - t_s)(1 - t_c)] \frac{dR}{d\pi} = t_s.$$

Using (4) and rearranging results in:

$$\frac{dt_p^*}{d\pi} = \frac{t_s}{R}\left[\frac{\pi}{R}\frac{dR}{d\pi} - 1\right]. \tag{4a}$$

Assuming that there is at least one investor whose after-personal-tax real rate of return on bonds ($[(1 - t_p)R - \pi]/(1 + \pi)$) is not negative and that, holding his personal tax rate constant, this after-personal-tax real rate does not increase as inflation increases, implies that the nominal interest rate (R) is not elastic with respect to inflation[10] (i.e., $(\pi/R)(dR/d\pi) \leq 1$). These assumptions are not very restrictive and are consistent with either the "Fisher hypothesis" or the "Darby hypothesis".[11] Following these assumptions, Equation (4a) implies that the critical tax rate does not increase with inflation (i.e., $dt_p^*/d\pi \leq 0$), suggesting that when the tax rate on income from stocks, t_s, is assumed positive, the "critical rate" may reinforce (but not offset) the "bracket creep" effect.

[8] The inflation rate is an exogenous variable in the model. $dt_p^*/d\pi$ and $dR/d\pi$ are the total derivatives of the endogenous variables with respect to the inflation rate (i.e., $dt_p^*/d\pi = \partial t_p^*/\partial\pi + (\partial t_p^*/\partial\pi)(dR/d\pi)$ and $dR/d\pi = \partial R/\partial\pi + (\partial R/\partial t_p^*)(dt_p^*/d\pi)$).

[9] It is possible that, *ex post*, the real after-tax cost of debt could be negative, particularly for corporations paying the full statutory tax rate. It is, however, assumed that, *ex ante*, the real after-tax cost of debt is positive, otherwise all firms will be entirely debt-financed (see Equation (3)).

[10] Differentiating the real after-personal-tax interest rate with respect to the inflation rate, we have

$$d\left[\frac{(1 - t_p)R - \pi}{1 + \pi}\right] \Big/ d\pi = \frac{\left[(1 - t_p)\frac{dR}{d\pi} - 1\right](1 + \pi) - [(1 - t_p)R - \pi]}{(1 + \pi)^2}.$$

Assuming that this real after-personal-tax interest rate does not increase with inflation implies that $(1 + \pi)[(1 - t_p)dR/d\pi] - 1 - (1 - t_p)R \leq 0$, which, in turn, implies that

$$\frac{\pi}{R}\frac{dR}{d\pi} \leq \frac{\pi}{(1 + \pi)(1 - t_p)R} + \frac{\pi}{1 + \pi} \leq \frac{1}{1 + \pi} + \frac{\pi}{1 + \pi} = 1.$$

[11] If it is assumed that the pre-personal-tax real rate of interest, r, is independent of inflation (i.e., the "Fisher hypothesis" holds) then, $(1 + R) = (1 + r)(1 + \pi)$ and $dR/d\pi = 1 + r$, implying that

$$\frac{\pi}{R}\frac{dR}{d\pi} = \frac{\pi(1 + r)}{\pi(1 + r) + r} \leq 1, \quad \text{if} \quad r \geq 0.$$

The analysis so far suggests that, whether t_s is assumed to equal zero or not, the no leverage-related costs model implies that an increase in inflation induces the aggregate debt-asset ratio to decline. This result is contrary to the results implied by previous studies that suggest that the aggregate debt-asset ratio increases with inflation. These studies argue that debt financing becomes more attractive as the inflation rate increases because the real tax shelter due to interest deductions to the corporation increases as inflation increases. However, since our no leverage-related costs model suggests that the personal tax rate of the marginal investor equals or exceeds the corporate tax rate, the increase in corporate tax shelter is offset by the increased tax liability of the marginal investor. Thus, as inflation increases, debt financing does not become more attractive to the corporation and the marginal investor combined.

Mussa and Kormendi [17], Hendershott and Koch [8], and Skelton [18] have found that the personal tax rate of the marginal investor is well below the corporate tax rate. These findings seem to be inconsistent with the implication of the Miller model. It seems that the counterfactual implications of the Miller-type model emerge due to two possible reasons: (1) the model ignores the benefits of portfolio diversification, and (2) the model ignores leverage-related costs. The portfolio diversification effect is not discussed in this paper since, as Auerbach and King have shown, its existence, in the presence of progressive personal taxes, is inconsistent with unanimity and complicates the derivation of an optimal policy for the firm. In addition, once the benefits of diversification are taken into account, all investors may hold both bonds and stocks and thus the dichotomy between bondholders and stockholders disappears (see Kim [11]). In the next section, we incorporate leverage-related costs into the model and derive the effect of inflation on the aggregate debt-asset ratio.

II. The Two Effects in the Presence of Leverage-Related Costs

In the preceding section, we assumed a Miller-type world (without leverage-related cost) in which firms are indifferent to their capital structure at any inflation level and showed that inflation reduces the aggregate debt-asset ratio. It was also shown that in such a model the personal income tax rate of the marginal investor (t_p^*) is not lower than the corporate tax rate. In this section, we incorporate leverage-related costs into the model, implying that each firm has a unique optimal capital structure. We assume that these leverage-related costs are high enough so that the personal income tax rate of the marginal investor (denoted here by t_p^{**}) is lower than the corporate tax rate.[12] We show that, under

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If it is assumed that, for an investor with personal tax rate t_p, the after-personal-tax interest rate, r^*, is independent of inflation (i.e., the "Darby hypothesis" holds ($dr^*/d\pi = 0$); see Darby [2]), then, $1 + r^* = [1 + (1 - t_p)R]/(1 + \pi)$ and $dR/d\pi = (1 + r^*)/(1 - t_p)$, implying that

$$\frac{\pi}{R}\frac{dR}{d\pi} = \frac{\pi(1 + r^*)}{\pi(1 + r^*) + r^*} \leq 1, \quad \text{if} \quad r^* \geq 0.$$

[12] This assumption is consistent with the results obtained by De Angelo and Masulis [3] and Kim [11]. They argue that bondholders as a group bear the ex ante leverage-related costs in the process of determining the equilibrium aggregate corporate borrowing. In the equilibrium described by these

these new assumptions, inflation increases the personal tax rate of the marginal investor. Consequently, as the inflation rate increases, some stockholders may become bondholders, inducing an increase in the aggregate debt-asset ratio. This effect, however, is countered by the "bracket creep" effect, and the net effect cannot be determined theoretically.

When leverage-related costs are considered, the value of the firm to the marginal investor can be rewritten as:

$$V_L(t_p^{**}) = V_u + \alpha I\left[1 - \frac{(1 - t_s)[(1 - t_c)R - \pi]}{(1 - t_p^{**})R - \pi}\right] - F(\alpha, I, \pi)$$

$$\equiv V_u + \alpha IG^{**} - F(\alpha, I, \pi), \tag{5}$$

where t_p^{**} is the critical tax rate that sorts investors into bondholders and stockholders in the presence of leverage-related costs, and $F(\alpha, I, \pi)$ is the leverage-related costs. These costs consist of three components (see Kim [11]): the present value of total bankruptcy costs (Kim [13]), the ex ante agency costs (Jensen and Meckling [10]), and the present value of the loss of nondebt tax shields in nonbankruptcy states (De Angelo and Masulis [3]). It is assumed that, for a given level of assets, increasing the debt-asset ratio increases the leverage-related costs ($\partial F/\partial\alpha > 0$) at an increasing rate ($\partial^2 F/\partial\alpha^2 > 0$)[13], and that the impact of a one-dollar increase in debt financing on real leverage-related cost is unaffected by inflation (i.e., $\partial^2 F/\partial\alpha\partial\pi = 0$).

The optimal capital structure for the firm is obtained by differentiating (5) with respect to α and setting it equal to zero:

$$\frac{\partial V_L(t_p^{**})}{\partial\alpha} = IG^{**} - \frac{\partial F}{\partial\alpha} = 0. \tag{6}$$

Equation (6) implies that since $\partial F/\partial\alpha$ is positive, G^{**} must be positive and that each firm will increase its leverage up to the point where the marginal tax benefits from debt equal the marginal ex ante leverage-related costs.

We now show that when t_p^{**} is less than the corporate tax rate, t_p^{**} increases with inflation. Totally differentiating Equation (6) with respect to π, and assuming that the level of the firm's investment is unaffected by inflation, we obtain:[14]

$$I\frac{dG^{**}}{d\pi} - \left(\frac{\partial^2 F}{\partial\alpha^2}\right)\left(\frac{d\alpha}{d\pi}\right) = 0. \tag{7}$$

Consider the first term in (7). Differentiating G^{**} with respect to the inflation

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authors, the relation between the taxable and the tax-exempt rates implies a marginal bondholder income tax rate that is less than the tax on corporate income.

[13] A positive $\partial^2 F/\partial\alpha^2$ is a sufficient condition for an internal solution for α.

[14] In fact, it is not necessary to assume that firms' investments are unaffected by inflation. The same results hold even when firms' investments increase with inflation as the cost of capital declines when the inflation rate increases.

rate we obtain:

$$\frac{dG^{**}}{d\pi} = \frac{\left\{(t_c - t_p^{**})\left(1 - \frac{\pi}{R}\frac{dR}{d\pi}\right) - \frac{dt_p^{**}}{d\pi}[(1 - t_c)R - \pi]\right\}R(1 - t_s)}{[(1 - t_p^{**})R - \pi]^2}. \quad (8)$$

As demonstrated in the previous section, it is reasonable to assume that the market (pre-personal-tax) nominal interest rate is inelastic with respect to the inflation rate. Recall that we have also assumed that the real after-tax cost of debt to the corporation ($[(1 - t_c)R - \pi]$) is positive and that $t_c > t_p^{**}$. Thus, it follows from (8) that $dG^{**}/d\pi$ is positive if $dt_p^{**}/d\pi$ is nonpositive. However, a negative $dt_p^{**}/d\pi$ implies that the "critical rate" effect reinforces the "bracket creep" effect in reducing the aggregate debt-asset ratio as inflation increases (i.e., $d\alpha/d\pi < 0$). Thus, a nonpositive $dt_p^{**}/d\pi$ implies both that $dG^{**}/d\pi$ is positive and that $d\alpha/d\pi$ is negative. These two implications of a nonpositive $\partial t_p^{**}/\partial\pi$ are mutually inconsistent according to Equation (7) since, when $\partial t_p^{**}/\partial\pi \leq 0$, the first term in (7) is positive and $[-(\partial^2 F/\partial\alpha^2)(d\alpha/d\pi)]$ is positive as well. Equations (7) and (8) imply, therefore, that $dt_p^{**}/d\pi$ is positive, proving our assertion that, when t_p^{**} is less than the corporate tax rate, t_p^{**} increases with inflation.

The intuitive explanation for our result is as follows. Replacing one dollar of equity by one dollar of debt, the firm increases its tax savings by t_cR. The tax liability of the marginal investor, who increases his debt holding by one dollar, increases by $[t_p^{**} - (1 - t_c)t_s]R$, which is less than t_cR. Thus, the net tax gain, from increasing debt financing by one dollar to the corporation and the investor combined, increases as the nominal interest rate rises with inflation. Assuming that the impact of a one-dollar increase in debt financing on real leverage-related costs is unaffected by inflation implies that inflation provides an incentive to increase debt financing. However, debt financing can be increased only if more investors prefer to hold bonds rather than stocks. This implies that the critical tax rate increases with inflation. The "bracket creep" effect, however, tends to decrease the aggregate debt-asset ratio and the net result depends, therefore, on the magnitudes of these two effects. Although it cannot be shown analytically, the "critical rate" effect is likely to dominate the "bracket creep" effect and hence the net effect is an increase in the debt-asset ratio.[15]

In summary, assuming a Miller-type world with no leverage-related costs and zero tax on income from holding stocks (t_s) implies that the "critical rate" effect vanishes and thus that inflation reduces the aggregate debt-asset ratio. When t_s is assumed to be positive (implying that $t_p^* > t_c$), the "critical rate" effect reinforces the "bracket creep" effect in reducing the aggregate debt-asset ratio.

Assuming that leverage-related costs are high enough such that $t_p^{**} < t_c$ implies that the critical tax rate increases with inflation, and thus that the "critical rate" effect offsets the "bracket creep" effect. Although the net effect cannot be

[15] Note that $d\alpha/d\pi$ is negative only if $dG^{**}/d\pi$ is negative. This is possible only if $dt_p^{**}/d\pi$ is positive and very large (i.e., there is a strong "critical rate" effect). But a negative $d\alpha/d\pi$ implies that "bracket creep" effect dominates the (strong) "critical rate" effect. We believe that this case is unlikely.

determined theoretically, it is likely that the "critical rate" effect dominates the "bracket creep" effect resulting in an increase in the aggregate debt-asset ratio. This dominance is more likely to occur the higher are leverage-related costs and the lower is t_s (yielding a strong "critical rate" effect), and/or the weaker is the "bracket creep" due to, for example, indexation of personal tax schedules to the price level.[16]

III. Summary and Concluding Remarks

In this paper, we considered an economy similar to the one described in Miller [14] and in Auerbach and King [1], and examined the effect of inflation on the aggregate debt-asset ratio. Equilibrium in this economy is characterized by investors who, for tax reasons, are specialized: investors whose personal tax rates are below some critical tax rate hold bonds only, and those whose tax rates are above the critical rate hold stocks only. The aggregate debt-asset ratio is, therefore, determined by the wealth of investors holding bonds relative to the wealth of all investors. Within this framework, we presented a model in which inflation has two effects on the aggregate debt-asset ratio. First, assuming that the critical tax rate is not affected by inflation, since inflation pushes investors into higher tax brackets, it reduces the wealth of investors who prefer to hold bonds relative to the total wealth of all investors. Hence, this "bracket creep" effect lowers the aggregate debt-asset ratio. Second, we demonstrated that, when the personal tax rate of the marginal investor is below the corporate tax rate, inflation increases the critical tax rate that sorts out investors into bondholders and stockholders. This suggests that the "critical rate" effect tends to increase the aggregate debt-asset ratio. The net impact of these two effects cannot be determined theoretically. The inconclusiveness of this theoretical result is reflected also in the empirical evidence concerning the impact of inflation on the aggregate debt-asset ratio. While Gordon and Malkiel [7] reported that the aggregate debt-asset ratio has grown from 0.168 to 0.325 between 1960 and 1978, Modigliani [15, p. 266, footnote 7] argued that when more suitable measures of total assets are employed, the association between leverage and inflation disappears.

The results in this paper are based upon a few assumptions that may, at first glance, seem to be restrictive. We assumed that all firms are identical, implying that no diversification of common stock is possible. Relaxing this assumption should not, however, affect the conclusions. Investors who, for tax reasons, prefer holding common stock will hold portfolios of stocks. This will only reduce the risk premium required by stockholders since the risk contribution of each stock to the portfolio is lower when diversification exists. Once the stockholders' discount rate is redefined to consider this effect, no other change is necessary. It was also assumed that most investors hold bonds only or stocks only, ignoring

[16] The "bracket creep" effect may weaken also due to investments in pension funds. As inflation increases, investments in pension funds become more attractive when investors are pushed into higher tax brackets. Since pension funds' assets consist mainly of fixed-income securities, the "bracket creep" effect is weakened.

the benefits of bond/stock portfolios. Relaxing this assumption will violate the dichotomy between bondholders and stockholders, complicating the identification of the aggregate holdings of bonds and stocks. However, we believe that the existence of bond/stock portfolios does not impede the forces that bring about the two (inflationary) effects. The "bracket creep" effect induces each investor to increase the proportion of common stocks in his portfolio, whereas the "critical rate" effect induces investors to increase the proportion of bonds in their portfolios since inflation increases the real tax gains of debt financing.

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