



American Finance Association

Towards a Semigroup Pricing Theory: Discussion

Author(s): Chi-Fu Huang

Source: *The Journal of Finance*, Vol. 40, No. 3, Papers and Proceedings of the Forty-Third Annual Meeting American Finance Association, Dallas, Texas, December 28-30, 1984 (Jul., 1985), pp. 861-862

Published by: Blackwell Publishing for the American Finance Association

Stable URL: <http://www.jstor.org/stable/2327811>

Accessed: 26/11/2009 08:43

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at <http://www.jstor.org/page/info/about/policies/terms.jsp>. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at <http://www.jstor.org/action/showPublisher?publisherCode=black>.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

JSTOR is a not-for-profit service that helps scholars, researchers, and students discover, use, and build upon a wide range of content in a trusted digital archive. We use information technology and tools to increase productivity and facilitate new forms of scholarship. For more information about JSTOR, please contact support@jstor.org.



Blackwell Publishing and American Finance Association are collaborating with JSTOR to digitize, preserve and extend access to *The Journal of Finance*.

<http://www.jstor.org>

REFERENCES

1. L. Bachelier. "Theory of Speculation (1900)." In P. Cootner (ed.), *The Random Character of Stock Market Prices*. Cambridge: MIT Press, 1964.
2. A. Beja, "Capital Markets with Delayed Learning." Ph.D. dissertation, Engineering Dept., Stanford, 1967.
3. R. Bellman. *Introduction to Matrix Algebra*. New York: McGraw-Hill, 1960.
4. F. Black and M. Scholes. "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy* (May/June 1973), 637-54.
5. J. Cox and S. Ross. "The Valuation of Options for Alternative Stochastic Processes." *Journal of Financial Economics* 3 (January/March 1976), 145-66.
6. U. Dothan and J. Williams. "Valuation of Assets on a Markov State Space." *Economic Letters* 1 (1978), 163-66.
7. D. Duffie. "Stochastic Equilibria: Existence, Spanning Number, and the 'No Expected Gain from Trade' Hypothesis." Working paper, Stanford, 1984.
8. ———. and M. Garman. "Intertemporal Arbitrage and the Semigroup Valuation of Markov Securities." Notes, Berkeley, 1984.
9. N. Dunford and J. Schwartz. *Linear Operators*. New York: Interscience Publishers, 1958.
10. J. Farkas, "Ober die Theorie der Einfachen Ungleichungen." *Journal fur die Reine und Angewandte Mathematik* 124 (1902), 1-27.
11. W. Feller. *An Introduction to Probability Theory and Its Applications*, vol. 2. New York: Wiley, 1966.
12. M. Garman. "Market Microstructure." *Journal of Financial Economics* 3 (June 1976), 257-75.
13. ———. "A General Theory of Asset Valuation under Diffusion State Processes." Working paper no. 50, School of Business, University of California, Berkeley, 1976.
14. M. Harrison and D. Kreps. "Martingales and Arbitrage in Multiperiod Securities Markets." *Journal of Economic Theory* 20 (1979), 381-408.
15. E. Hille and R. Phillips. *Functional Analysis and Semigroups*. Providence, RI: American Mathematical Society, 1957.
16. D. Kreps. "Three Essays on Capital Markets." Institute for Mathematical Studies in the Social Sciences, Technical Report No. 261, Stanford, 1979.
17. H. Kunita and S. Watanabe. "On Square-Integrable Martingales." *Nagoya Mathematics Journal* 30 (1967), 209-45.
18. R. Merton. "Theory of Rational Option Pricing." *Bell Journal of Economics and Management Science* 4 (Spring 1973), 573-615.
19. S. Ross. "Return, Risk, and Arbitrage." In I. Friend and J. Bicksler (ed.) *Risk and Return in Finance*. Cambridge: Ballinger, 1976.
20. M. Rubinstein. "An Aggregation Theorem for Security Markets." *Journal of Financial Economics* 1 (October 1974), 225-44.
21. ———. "The Valuation of Uncertain Income Streams and the Pricing of Options." *Bell Journal of Economics* 7 (Autumn 1976), 407-25.

DISCUSSION

CHI-FU HUANG*: This paper is a typical example of how a seemingly simple observation can be pushed far to show interesting results. It is well known that in an Arrow-Debreu equilibrium with more than one date, the implicit prices for Arrow-Debreu securities over time are linked in a special way to prevent arbitrage. Professor Garman goes a step further to recognize that those implicit prices form an evolution semigroup. In the continuous time case, given that the infinitesimal generator of the implicit prices semigroup exists, he demonstrates that an arbitrage-free price system must satisfy a differential relation. This results in continuous time is the interesting part of this paper. Professor Garman fails,

* Massachusetts Institute of Technology.

however, to provide conditions under which the infinitesimal generator exists. Without those conditions the contribution of this paper cannot be properly accessed. My own feeling is that those conditions will be very restrictive.

Many people working in the area of arbitrage and contingent claims pricing believe that the theory is preference free. That this is not the case has been made clear by Harrison and Kreps [2]. This paper suffers from that conventional view too.

In what follows, I shall give some specific comments.

1. In the discrete time finite states case, the existence of nonnegative implicit prices follows from the no simple free lunch condition. In the continuous time case, however, the no simple free lunch condition is not sufficient for that. The underlying commodity space and the sense of continuity of agents' preferences are not specified in the paper. Once they are properly specified, then no free lunches insures the existence of nonnegative implicit prices; cf. Kreps [3, Lemma 1]

It seems that the nonnegativity of implicit prices is not sufficient for them to form a semigroup. Strict positivity will be needed. The conditions for the existence of strictly positive implicit prices are that the underlying commodity space be normable and separable; cf. Duffie and Huang [1, Proposition A.1] (For the definition of simple free lunches and free lunches see Kreps [3]).

2. One sufficient condition for the infinitesimal generator to exist is that the semigroup be uniformly continuous on some Banach space. So we first take the commodity space to be a separable Banach space. Then one sufficient condition for the semigroup to be uniformly continuous is that the implicit prices be continuous processes and be bounded above. (The former makes economic sense. It simply says that agents' preferences are such that consumption at adjacent dates are almost perfect substitutes.) There is no Banach space, however, such that its normed-dual is continuous bounded functions. Thus, rather than talking about no free lunches we may have to resort to the concept of viability, cf. Kreps [3].
3. The conditions under which a finite dimension Markov structure can be identified will be severe in general. Without imposing equilibrium, the identification of such a structure seems formidable.

To sum up, I think this paper introduces a new methodology into the theory of arbitrage. How fruitful the methodology is will depend in large part on how restrictive its underlying assumptions are.

REFERENCES

1. D. Duffie and C. Huang. "Multiperiod Securities Markets with Differential Information: Martingales and Resolution Times." MIT mimeo, 1984.
2. J. M. Harrison and D. Kreps. "Martingales and Arbitrage in Multiperiod Securities Markets." *Journal of Economic Theory* 20 (1979), 381-408.
3. D. Kreps. "Arbitrage and Equilibrium in Economies with Infinitely Many Commodities." *Journal of Mathematical Economics* 8 (1981), 15-35.