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Expected Inflation and Interest Rates in a Multi-asset Model: A Note

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ABSTRACT

This paper analyzes the effect of expected inflation on nominal interest rates, in a theoretical model with money and two different bond types. The inclusion of three assets instead of the usual two causes the effect of expected inflation on the interest rates to deviate from unity. Depending on the sizes of the wealth and interest rate effects on the various asset demands, the effect of expected inflation could even be negative. Several special cases are also considered, and the implications for the interpretation of empirical results are discussed.

THE PROFESSION HAS SHOWN a continuing strong interest in the effect of expected inflation on interest rates, both from a theoretical and an empirical perspective. Fisher's [6] original hypothesis was that there should be a one-for-one effect, so that expected inflation has no effect on the ex ante real interest rate. Numerous researchers have failed to find that strong an empirical effect on nominal rates, however, and some have expressed puzzlement over this empirical result. (The empirical literature includes [1, 2, 3, 5, 6, 10, 13, 14] and numerous others.)

Theoretical models generally consider a short-run situation in which the real rate of return on financial assets is free to deviate from the marginal product of capital. The simplest possible such macro model, with *IS*, *LM*, and vertical aggregate supply curve equations, generates the Fisher result that $dr/d\pi$—the effect of expected inflation π on the nominal interest rate r—equals unity. But there are several possible modifications to this model which can drive the value of $dr/d\pi$ either up or down.

First, the Mundell [12] effect states that the presence of a real balance effect on consumption depresses the value of $dr/d\pi$. Second, the Phillips effect [9, 10] states that a nonvertical aggregate supply curve depresses $dr/d\pi$. Third, Levi and Makin [10] showed that if the level of expected inflation influences inflation uncertainty, and if inflation uncertainty in turn affects the parameters of any behavioral function, then this channel alters the value of $dr/d\pi$. And finally, the Darby [4] effect states that the presence of a tax on nominal interest income magnifies $dr/d\pi$.

Recently, Carmichael and Stebbing [3] argued that bonds might be closer substitutes for money than for anything else, so expected inflation might have a zero impact on the nominal interest rate on bonds. However, they did not present

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a formal theortical model of this idea. (See [4, 8, 9, 10, 12] for a sample of the theoretical literature.)

The purpose of this paper is to analyze the Fisher effect in the context of a theoretical macro model with three financial assets instead of the usual two (money and bonds). The three assets are money and two different types of bonds. The model deliberately omits the Mundell, Phillips, uncertainty, and Darby effects, because the implications of these are already well-known. Thus, the model is kept as simple as possible in order to focus on the implication of the existence of more than two financial assets. Since the real world obviously consists of more than two financial assets, any results of the conventional model which break down in this model also must not hold in the real world.

The most basic conclusion is that even without the Mundell, Darby, Phillips, and uncertainty effects, the presence of several financial assets can cause the magnitude of the effect of expected inflation on nominal interest rates to deviate from unity. In fact, this effect could even be negative for one bond, or it could be zero as hypothesized by Carmichael and Stebbing. The wealth and interest sensitivities of the various asset demands influence the magnitude of the effect of expected inflation.

I. The Model

The model consists of a vertical aggregate supply curve, an *IS* equation, and supply-demand equilibrium equations for each of money, bond A, and bond B. Any one of the latter three equations is redundant. The fact that the aggregate supply curve is vertical serves to exclude the Phillips effect from the model.

The vertical aggregate supply curve pins down income (y) at a fixed value, so y is exogenous to the rest of the model. The *IS* equation equates output (identically equal to income y) with expenditure. Expenditure is affected by income, and both the consumption and investment component of expenditure may be affected by one or both real interest rates. Thus the *IS* equation, with income solved out, can be written as (1):

$$IS\colon y = f(r_A - \pi, r_B - \pi) \tag{1}$$

where r_A and r_B are the nominal interest rates on the two bonds, and π is expected inflation. The two derivatives, denoted f_A and f_B, are both negative.

Equations (2) to (4) are the equilibrium equations of the markets for bond A, bond B, and money, respectively:

$$\text{Bond A: } a^d(y, r_A - \pi, r_B - \pi, r_M - \pi, W/P) - A/P = 0 \tag{2}$$

$$\text{Bond B: } b^d(y, r_A - \pi, r_B - \pi, r_M - \pi, W/P) - B/P = 0 \tag{3}$$

$$\text{Money: } m^d(y, r_A - \pi, r_B - \pi, r_M - \pi, W/P) - M/P = 0 \tag{4}$$

where a^d, b^d, and m^d are the three demand functions; A, B, and M are the predetermined nominal quantities of the assets; $W = A + B + M$ is predetermined nominal wealth; P is the endogenous price level; and r_M is the fixed nominal return on money. The nominal supplies A, B, and M are predetermined because this is a standard beginning-of-period model in the sense of Foley [7]. Thus, for instance A is determined by last period's decisions by suppliers of bond A; during this period, the endogenous variables adjust to equate the endogenous demand with this period's supply.[1]

The important derivatives of the demand functions are denoted and signed as follows. a_A^d, and b_B^d, and m_M^d are own-interest-rate derivatives and are positive. a_B^d, a_M^d, b_A^d, b_M^d, m_A^d, and m_B^d are cross-rate derivatives and are negative. The real wealth derivatives a_W^d, b_W^d, and m_W^d are nonnegative. The adding up constraint on the formation of asset demands ensures that any two of equations (2) to (4) imply the third, so one equation can be omitted. Henceforth, I will omit (2). Further, I will assume that the three interest derivatives of each demand function sum to zero, so that equal rises in all interest rates do not cause a reshuffling of the public's portfolio. The effect of this assumption is that all asset demands could be written as functions of just the nominal rates. Relaxing this assumption significantly complicates the results without altering the most basic result that a wide range of values of the effect of π on r_A or r_B is possible.

Thus, the model consists of (1), (3), and (4) in the endogenous variables r_A, r_B, and P. The key exogenous variable is π. Except for the inclusion of more than one bond, the model is formulated in a very standard fashion.

II. Results

The model gives the following comparative static results:

$$\frac{dr_A}{d\pi} = \frac{(f_A + f_B)b_B^d\eta_M - (f_A + f_B)m_B^d\eta_B}{\Delta} \tag{5}$$

$$\frac{dr_B}{d\pi} = \frac{(f_A + f_B)m_A^d\eta_B - (f_A + f_B)b_A^d\eta_M}{\Delta} \tag{6}$$

where

$$\Delta \equiv f_A b_B^d \eta_M + f_B m_A^d \eta_B - f_A m_B^d \eta_B - f_B b_A^d \eta_M \tag{7}$$

and where $\eta_M \equiv (1 - e_{mW})(M/P^2)$ and $\eta_B \equiv (1 - e_{bW})(B/P^2)$. Here, e_{mW} and e_{bW} are the wealth elasticities of money demand and bond B demand, respectively.

My working paper underlying this paper analyzes in detail the effects of various specific parameters on the comparative static derivatives. The most important observations about (5) and (6) appear below.

First, it can be shown that the benchmark value of unity is bracketed by $dr_A/d\pi$ and $dr_B/d\pi$. That is, one of these derivatives must be greater than unity, and the other less than unity. Note however, that the two derivatives need not average to unity.

Further, either $dr_A/d\pi$ or $dr_B/d\pi$ *could* be less than zero. This can be seen

[1] Both bonds are assumed to be of the fixed principal/variable interest type, so there are no interest-induced wealth effects. In other words, bonds are assumed to be extremely short-term in nature, so there are no capital gains or losses. This assumption is frequently, but not universally, used in the government budget constraint literature. Zwick [15] and Meyer and Yawitz [11] recognized that the presence of fixed coupon bonds would depress the effect of expected inflation on nominal interest rates.

because in each of (5) and (6), both the numerator and denominator are ambiguous in sign.

Next, consider the hypothesis of Carmichael and Stebbing [3] that if a bond (say bond B) is a perfect substitute for money, expected inflation has no effect on the nominal interest rate of that bond. This is the case in which m_B^d approaches negative infinity. In this case, (5) and (6) imply that $dr_A/d\pi > 1$ and $dr_B/d\pi = 0$ in the limit; the latter result confirms the Carmichael-Stebbing hypothesis. The intuition here is simply that the nominal interest rate on bond B is tied to that on money, and the latter is institutionally fixed and thus is not influenced by expected inflation.

Finally, consider the special case in which only one interest rate affects the IS curve. Suppose that $f_B = 0$ so only $r_A - \pi$ affects expenditure. (A symmetric result holds if f_A instead of f_B is zero.) Then in (5), $dr_A/d\pi$ simplifies to unity; so in this limiting case, the one-for-one Fisher effect does hold for one of the bonds.

III. Conclusions

This paper has shown some implications for the effects of expected inflation on interest rates which arise in a model with more than two financial assets. The basic result is that the multiasset framework causes the effect of expected inflation to diverge from unity. In fact, it could even be negative for one of the assets. Some special cases of the model were also examined.

Since the real world is certainly characterized by multiasset financial markets, with imperfect substitutability between assets, this model provides an important perspective for interpreting otherwise surprising empirical results. The model is consistent with a broad range of possible empirical results concerning the interest rate effect of expected inflation.

Finally, a further avenue of empirical research is suggested. Assets with similar risk and liquidity features should have similar effects of expected inflation on their nominal returns. If groups of similar assets can be identified, perhaps cross-group patterns can be found in the effects of expected inflation. For instance, perhaps it could be shown that assets judged to be more liquid (i.e., closer substitutes for money) tend to have a smaller effect of expected inflation. Identification of such patterns could go a long way toward systematizing the large number of seemingly unrelated empirical results already reported in the literature.

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