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Source: *The Journal of Finance*, Vol. 40, No. 2 (Jun., 1985), pp. 577-581
Published by: Blackwell Publishing for the American Finance Association
Stable URL: http://www.jstor.org/stable/2327902
Accessed: 26/11/2009 08:39

THE JOURNAL OF FINANCE • VOL. XL, NO. 2 • JUNE 1985

# On the Theory of Rational Insurance Purchasing: A Note

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**ABSTRACT**

The authors consider the optimal amount of insurance purchased by an individual who behaves according to the Hurwicz criterion of choice under uncertainty. Their results are compared with earlier results obtained in alternative frameworks (expected utility maximization and Savage's regret criterion). It is shown that a positive amount deductible is often suboptimal.

IT IS A WELL-KNOWN proposition in insurance economics that a risk-averse expected utility maximizer will never purchase full insurance coverage if the exogenous insurance premium is based on expected insurance payments and includes a positive proportional loading to cover administrative expenses and the insurer's profit (see, e.g., Arrow [1], Mossin [7], and Smith [12]). Indeed, when the loading factor is zero, full coverage is optimal. But with a positive loading the insured prefers a policy with a positive amount deductible. Arrow [2] later extended this result to the case of state-dependent utility functions. Recently, Doherty and Schlesinger [5] proved that it strictly holds only if insurable and noninsurable losses are nonpositively correlated.

Using a binomial distribution for losses and Savage's minimax regret criterion as an alternative to the expected utility framework, Razin [8] obtained a stronger result: it is always optimal to purchase insurance with a positive amount deductible even if the loading factor is zero, i.e., if the insurance premium is actuarially fair.

However, these results seem to be contradicted by everyday observations; individuals rarely insist on positive deductibles in their insurance policies. As Borch [3] noted: "If a traveller insures his baggage at all, we will expect him to take insurance for its full value."

This paper shows that theory may be reconciled with observed behavior if we assume that individuals do not use all the available information. Individuals will be supposed to behave according to the Hurwicz [6] criterion; their choice under uncertainty will be based on a subjective weighting of the results obtained in only two states of nature, the best state and the worst state. The weighting factor

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($0 \leq \alpha \leq 1$) reflects their degree of pessimism:

—if $\alpha = 1$, the individual is extremely pessimistic and the Hurwicz criterion is identical to the minimax rule;
—if $\alpha = 0$, the individual is extremely optimistic and behaves as if he or she used the maximax rule.

The Hurwicz criterion is clearly not very attractive from a normative point of view, but it may have some intuitive appeal in a descriptive theory of decisions under conditions of bounded rationality (see Simon [11]). In the insurance purchasing context, it implies that deductibles are often suboptimal, which conforms to everyday observation. However, it is also interesting to note that the Hurwicz criterion does not rule out the optimality of deductibles; they may be optimal in certain situations, even if the insurance premium is actuarially fair.

Section I presents a general model by considering a continuous probability distribution for losses. Stronger results are obtained in Section II by using a binomial distribution. Section III provides a short synthesis of the results obtained under alternative decision-making frameworks. Finally, the conclusion summarizes the main implications of the paper.

## I. The General Model

Consider an individual owning a piece of property, the value of which is $L$. In addition, other assets are owned with a value of $A$. While other assets are nonrisky, the property in question may be subject to a random loss $X$, $0 < X \leq L$, with density $f(X)$ and cumulative probability $\pi = \int_0^L f(X)\, dX$. The no-loss state occurs with probability $1 - \pi$.

Basically, the individual faces three alternative actions:

a) *No insurance.* In this case, final wealth is a random variable $Y$:

$$Y = \begin{cases} A + L, & \text{if no loss occurs} \\ A + L - X, & \text{if a loss occurs.} \end{cases}$$

b) *Full insurance.* In this case, the individual pays an insurance premium, $P$. Final wealth is $A + L - P$ with certainty.

c) *Partial insurance.* Assuming that partial insurance is provided through a deductible arrangement, the individual must in this case select the amount deductible, $D$. An insurance premium $P(D)$ is paid and final wealth is the random variable $Z$:

$$Z = \begin{cases} A + L - P(D), & \text{if no loss occurs} \\ A + L - P(D) - X, & \text{if a loss } X < D \text{ occurs} \\ A + L - P(D) - D, & \text{if a loss } X \geq D \text{ occurs.} \end{cases}$$

Suppose the individual behaves according to the Hurwicz criterion of choice under uncertainty; for each possible action he or she considers the best and the worst outcome and weights them by a degree of pessimism, $\alpha$. The individual then chooses the action yielding the maximum weighted result. Considering that no insurance means $D = L$ and full insurance $D = 0$, the individual's decision problem may thus be stated as

$$\operatorname*{Max}_{D} H = \alpha[A + L - P(D) - D] + (1 - \alpha)[A + L - P(D)]$$

$$\text{subject to} \quad 0 \leq D \leq L.$$

Assuming that the premium is based on expected insurance payments and incorporates a loading factor $\lambda$, we have

$$P(D) = (1 + \lambda) \int_D^L (X - D)f(X)\, dX.$$

The decision problem may thus be restated as

$$\operatorname*{Max}_{D} H = A + L - (1 + \lambda) \int_D^L (X - D)f(X)\, dX - \alpha D$$

$$\text{subject to} \quad 0 \leq D \leq L. \tag{1}$$

First- and second-order conditions for a maximum yield, respectively,

$$(1 + \lambda) \int_D^L f(X)\, dX = \alpha \tag{2}$$

$$-(1 + \lambda)f(D) < 0. \tag{3}$$

From (2) it can be seen that the optimizing individual will choose an optimal amount deductible $D^*$ which adjusts the probability of the insurer paying an indemnity—inflated by the loading factor—to the value of the Hurwicz coefficient. The no-insurance choice ($D = L$) is not optimal, except for the maximax individual. But the full insurance choice ($D = 0$) is optimal whenever $(1 + \lambda)\pi < \alpha$. Thus, in many instances, full insurance will be prefered by the decision-maker, in spite of the presence of a loading factor. On the other hand, if $\lambda = 0$, i.e., if the insurance premium is actuarially fair, the optimality of a positive deductible cannot be ruled out.

These results stand in sharp contrast to the traditional results obtained in the expected utility framework by Arrow [1], Mossin [7], etc., with similar models of a single insurable risk.

The preference for full insurance depends upon the relationship between three parameters: the pessimism, i.e., risk-aversion, of the decision-maker, on the one hand; the probability of a loss and the value of the loading factor on the other hand. To be more specific, two main implications emerge from our analysis:

1. The demand for deductibles is an increasing function of:
   - the optimism of the individual (risk-proneness),
   - the probability of a loss,
   - the value of the loading factor.
2. If the probability of an insurable loss is typically low, even moderately pessimistic individuals will require full insurance.

## II. The Two-State Case

In this section we consider the special case where only two states of nature matter. Either the property at risk is subject to a full loss, which occurs with probability $p$, or there is no loss, which occurs with probability $1 - p$. This case applies, for example, when the property in question is a work of art to be insured against theft.

In this case, the insurance premium reduces to

$$P(D) = (1 + \lambda)p(L - D),$$

and (1) may be restated as

$$\max_{D} H = A + L - (1 + \lambda)pL - [\alpha - (1 + \lambda)p]D$$

$$\text{subject to} \quad 0 \leq D \leq L. \tag{4}$$

Obviously, if $\alpha < (1 + \lambda)p$, $H$ is maximized for $D = L$, meaning that the no-insurance solution prevails. Inversely, if $\alpha > (1 + \lambda)p$, $H$ is maximized for $D = 0$, meaning that the full insurance solution prevails. Thus, using the Hurwicz rule in a two-state framework always yields corner solutions while, as shown by Razin [8], deductibles are always optimal when Savage's regret criterion is used.

The intuitive explanation for this result is that the degree of pessimism, $\alpha$, may be interpreted as a subjective probability of damage evaluated by the individual which is then compared to the "inflated" probability $(1 + \lambda)p$ computed by the insurance company. If the individual is more optimistic than the insurance company (i.e., if $\alpha < (1 + \lambda)p$), the individual does not take any insurance. If the converse holds, full insurance is optimal.

It may also be noted that if $\alpha = 1$ (minimax rule), the individual takes full insurance, except when the loading factor exceeds $(1 - p)/p$, where it is optimal to remain noninsured. If $\alpha = 0$ (maximax rule), the noninsurance choice is always optimal.

## III. Rational Insurance Purchasing under Alternative Decision-Making Frameworks

Before concluding, it may be interesting to compare the results obtained by using different decision-making frameworks. This is done in Table I.

In the more realistic hypothesis of a positive loading factor, the Hurwicz rule and its special case, the minimax rule, are the only frameworks which can explain the observed preference for full insurance. However, all these results refer to models where the decision-maker faces a single insurable risk. When the portfolio context to the insurance decision is considered, maximization of expected utility may also lead to corner solutions (see Doherty [4] and Schulenburg [9]).

## IV. Conclusion

This note has shown that individuals taking their decisions under conditions of bounded rationality will not automatically purchase insurance policies with a positive amount deductible when these policies are actuarially unfair. These individuals will prefer full insurance coverage in a number of cases, particularly

**Table I**

Optimal Insurance Coverage under Alternative Decision Frameworks

| | Value of the loading factor | |
|---|---|---|
| | $\lambda = 0$ | $\lambda > 0$ |
| Expected Utility | Full insurance | Deductible |
| Savage Regret (Two-State Model) | Deductible | Deductible |
| Hurwicz Rule | Full insurance or deductible | Full insurance or deductible |
| Minimax Rule | Full insurance | Full insurance or deductible |
| Maximax Rule | No insurance | No insurance |

when the probability of a loss is low. If there are only two states of nature—no loss or full loss—they will never even ask for a deductible and choose either full insurance or no insurance at all.

These results seem to conform more to the observed behaviors in the markets for personal insurance lines (automobile, theft, etc.) than the traditional results obtained in the expected utility framework. In these markets, deductibles do exist, but the reason for their existence cannot be traced back to the insureds' preferences. They are imposed by the insurers for different reasons, such as their effort to mitigate moral hazard when the level of individual care cannot be observed (see Shavell [10]).

## REFERENCES

1. Kenneth J. Arrow. "Uncertainty and the Welfare Economics of Medical Care." *American Economic Review* 53 (December 1963), 941–73.
2. ———. "Optimal Insurance and Generalized Deductibles." *Scandinavian Actuarial Journal* (1974), 1-42.
3. Karl H. Borch. "The Optimal Insurance Contract in a Competitive Market." *Economics Letters* 11 (1983), 327–30.
4. Neil A. Doherty. "Portfolio Efficient Insurance Buying Strategies." *Journal of Risk and Insurance* 51 (June 1984), 205–24.
5. ——— and Harris Schlesinger. "Optimal Insurance in Incomplete Markets." *Journal of Political Economy* 91 (December 1983), 1045–54.
6. Leonid Hurwicz. "Optimality Criteria for Decision Making Under Ignorance." Cowles Commission Paper 370, 1951.
7. Jan Mossin. "Aspects of Rational Insurance Purchasing." *Journal of Political Economy* 76 (July/August 1968), 553–68.
8. Assaf Razin. "Rational Insurance Purchasing." *Journal of Finance* 31 (March 1976), 133–37.
9. Matthias Graf v.d. Schulenburg. "Optimal Insurance Purchasing in the Presence of Compulsory Insurance and Uninsurable Risks." Paper presented at the 11th Seminar of the European Group of Risk and Insurance Economists, Geneva, September 1984.
10. Steven Shavell. "On Moral Hazard and Insurance." *Quarterly Journal of Economics* 91 (November 1979), 541–62.
11. Herbert A. Simon. "A Behavioral Model of Rational Choice." *Quarterly Journal of Economics* 69 (February 1955), 99–118.
12. Vernon L. Smith. "Optimal Insurance Coverage." *Journal of Political Economy* 76 (January/February 1968), 68–77.