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Joint Effects of Interest Rate Deregulation and Capital Requirements on Optimal Bank Portfolio Adjustments

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ABSTRACT

The 1980 Depository Institution Deregulation and Monetary Control Act (DIDMCA) mandates that Regulation Q be phased out by 1986. With deregulation of interest rate ceilings, the cost of raising capital funds for commercial banks would become more volatile and more closely related with interest rates in the money and capital markets. Thus, value-maximizing bank managers would need to be concerned not only with the internal risk, but also with the external risk in bank portfolio management decisions. Based upon the cash flow version of the capital asset pricing model, this paper analyzes the joint impact of interest rate deregulation and capital requirements on the portfolio behavior of a banking firm.

THE 1980 DEPOSITORY INSTITUTION DEREGULATION and Monetary Control Act (DIDMCA) mandates that Regulation Q be phased out by 1986. With deregulation of interest rate ceilings, the cost of raising capital funds for commercial banks would become more volatile and more correlated with interest rates in the money and capital markets. The bank's cash profit would be more dependent on the interest rate fluctuation, *ceteris paribus*. Thus, the value of a banking firm would also be more dependent on the correlation of the banking firm's cash profit with that of the external market, *ceteris paribus*. Therefore, as deregulation continues, a value-maximizing bank manager would need to be concerned not only with variance and covariances among the returns on bank assets, but also with covariances between rates on bank assets and liabilities as well as that in the external markets. Study of portfolio behavior of a banking firm should distinguish between the impact of internal risk and external risk on firm value. Furthermore, with interest rate deregulation and the ensuing interactions among various risks, banks may also react differently to capital regulation imposed by the regulatory agency. Despite numerous studies on regulation of bank capital, the joint effects of interest rate deregulation and capital requirements on bank portfolio adjustment have not been addressed explicitly[1] [1, 2, 3, 4, 5, 7, 9, 10,

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[1] Koehn and Santomero [5] consider the capital regulation problem when the deposit rate is fixed. However, they did not address the issue when Regulation Q is phased out. Furthermore, since they use a mean-variance portfolio model, the approach does not explicitly recognize the distinction between internal risks and external risks.

11]. We provide herein a model of banking firms which

(1) analyzes and contrasts the joint impact of interest rate regulation and capital requirements on the portfolio behaviors of a bank before and after the deregulation of interest rate ceilings;
(2) explicitly recognizes the interrelationship between portfolio risks internal to the bank and those related to the market; and
(3) is consistent with wealth maximization in modern financial economics.

The rest of the paper proceeds as follows. The first section formulates the model and explicitly recognizes the distinction between a bank's internal risk and its external risk. The second section derives and discusses a bank's portfolio choice when the interest rate ceiling on its deposits is binding. The third section discusses the portfolio risk adjustment when Regulation Q is phased out as the banking industry continues to be deregulated. The fourth section discusses and contrasts the effect of capital requirements on the probability of bank failure before and after interest rate deregulation. The final section summarizes the findings and concludes with some policy implications.

I. Bank Model and Portfolio Risks

Consider a bank which invests in N risky assets or projects each in the amount of A_j, with uncertain return $\tilde{r}_j$, j = 1, 2, $\cdots$ N. The bank issues one aggregate deposit account in the amount of D with interest cost $\tilde{d}$.[2] The interest rate on the deposit is uncertain after Regulation Q is phased out. However, when Regulation Q is in effect and binding, the deposit cost is usually fixed at the rate ceiling. The bank has also issued capital in the amount of K. This amount of capital is assumed to be fixed and not part of the decision of the bank.[3] Thus, the random cash profit at the end of the decision period is

$$\tilde{\pi} = \textstyle\sum_{j=1}^{N} \tilde{r}_j A_j - \tilde{d}D,$$

and the expected cash profit is

$$E(\tilde{\pi}) = \textstyle\sum_{j=1}^{N} \bar{r}_j A_j - \bar{d}D,$$

where

$\bar{r}_j = E(\tilde{r}_j)$ is the expected rate of return on asset or project j;

$\bar{d} = E(\tilde{d})$ is the expected cost of deposits.

Instead of maximizing certain preference functions of cash profit as in Koehn and Santomero [5], the bank is assumed to maximize its market value,

[2] One can interpret this aggregate account as the weighted average of all deposit accounts. Thus, the cost (d) is a weighted average of all deposit costs. This simplification merely allows for easy exposition and derivation, and the approach is general.

[3] As will be shown in later sections, the assumption of a fixed amount of capital is immaterial when Regulation Q is binding. In that case, as correctly assumed by Koehn and Santomero in their analysis, the capital decision is merely a scale decision [5]. However, when Regulation Q is phased out, portfolio investment and adjustment can be a function of the amount of capital.

which is the present value of the certainty-equivalent of the end-of-the-period cash profit within the capital asset pricings model (CAPM). The use of this objective function is appropriate since the stock of the banking firm is traded in perfect securities markets. Thus, the value of a banking firm is given as follows:

$$V = \frac{1}{R}\{E(\tilde{\pi}) - \lambda\, CV(\tilde{\pi}, \tilde{M})\}, \tag{1}$$

where

R = one plus the risk-free rate;
$\tilde{M}$ = aggregate cash flow of all firms in the market;
$CV(\tilde{\pi}, \tilde{M})$ = covariance between the cash profit of the bank and the aggregate cash flow of all firms; the systematic risk within the CAPM framework;
λ = market price of risk bearing.

In this form, the market value of the banking firm's common stock is the present value of the expected cash profit at the end of the period adjusted by a risk premium, $\lambda CV(\tilde{\pi}, \tilde{M})$. It should be noted that the aggregate cash flows of all firms include that of the bank and other firms in the economy, and thus

$$\tilde{M} = \tilde{\pi} + \tilde{W},$$

where $\tilde{W}$ = the aggregate cash flow in the market excluding that of the bank.

The covariance term in (1) above can, therefore, be rewritten as

$$CV(\tilde{\pi}, \tilde{M}) = CV(\tilde{\pi}, \tilde{\pi}) + CV(\tilde{\pi}, \tilde{W}).$$

Explicit recognition of the two components of a firm's systematic risk yields interesting insight in the study of a bank's portfolio adjustments, especially when Regulation Q is phased out, as we will see later. The first term, $CV(\tilde{\pi}, \tilde{\pi})$, is the variance of the cash profit of the bank and is a weighted average of the variances and covariances of returns on the assets and cost of the deposit internal to the bank and is therefore called "*internal risk.*" The second term, $CV(\tilde{\pi}, \tilde{W})$ is a weighted average of the covariances of the bank's cash profit with that of all other firms in the economy and is called "*external risk.*" Thus, the value of the firm in (1) can be written as

$$V = \frac{1}{R}\{[E(\tilde{\pi}) - \lambda CV(\tilde{\pi}, \tilde{W})] - \lambda CV(\tilde{\pi}, \tilde{\pi})\},$$

with

$$CV(\tilde{\pi}, \tilde{W}) = \textstyle\sum_{j=1}^{N} A_j \sigma_{jw} - D\sigma_{dw},$$

$$CV(\tilde{\pi}, \tilde{\pi}) = \textstyle\sum_{i=1}^{N} \sum_{j=1}^{N} A_i A_j \sigma_{ij} + D^2\sigma_d^2 - 2D \sum_{j=1}^{N} A_j \sigma_{jd},$$

where

σ_{ij} = covariance between rates of return on asset i and asset j;
σ_{jw} = covariance between return on asset j and cash flows of all other firms;
σ_{dw} = covariance between the cost of deposit and cash flows of all other firms;
σ_{jd} = covariance between return on asset j and cost of deposit;
${\sigma_d}^2$ = variance of cost of deposit.

Assuming that the regulator imposes a capital requirement represented by a maximum deposit to capital ratio $C \equiv D/K$ on the bank, the decision problem facing the bank becomes:[4]

$$\max_{\langle A_j \rangle, D} V = \frac{1}{R} \{[E(\tilde{\pi}) - \lambda CV(\tilde{\pi}, \tilde{W})] - \lambda CV(\tilde{\pi}, \tilde{\pi})\} \tag{2}$$

subject to

$$\textstyle\sum_{j=1}^{N} A_j = D + K \tag{3}$$

and

$$D \le CK \tag{4}$$

where (3) represents the balance sheet constraint and (4) represents the bank capital requirement.

Let L represent the Lagrange function, and let γ and γ_1 denote the Lagrange multipliers associated with (3) and (4), respectively, the first-order necessary conditions are:

$$\frac{\partial L}{\partial A_k} = \frac{1}{R} \left\{ \bar{r}_k - \lambda \frac{\partial CV(\tilde{\pi}, \tilde{W})}{\partial A_k} - \lambda \frac{\partial CV(\tilde{\pi}, \tilde{\pi})}{\partial A_k} \right\} - \gamma$$
$$= 0, \quad k = 1, 2, \cdots N \tag{5}$$

$$\frac{\partial L}{\partial D} = \frac{1}{R} \left\{ - \bar{d} - \lambda \frac{\partial CV(\tilde{\pi}, \tilde{W})}{\partial D} - \lambda \frac{\partial CV(\tilde{\pi}, \tilde{\pi})}{\partial D} \right\}$$
$$+ \gamma - \gamma_1 = 0 \tag{6}$$

$$\frac{\partial L}{\partial \gamma} = \textstyle\sum_{j=1}^{N} A_j - D - K = 0 \tag{7}$$

$$\frac{\partial L}{\partial \gamma_1} = D - CK \le 0 \tag{8}$$

$$\gamma_1 \frac{\partial L}{\partial \gamma_1} = 0 \tag{8a}$$

Solving (5) and (6) simultaneously, we obtain

$$\gamma_1 = \frac{1}{R} \left\{ (\bar{r}_k - \bar{d}) - \lambda \left[\frac{\partial CV(\tilde{\pi}, \tilde{W})}{\partial A_k} + \frac{\partial CV(\tilde{\pi}, \tilde{W})}{\partial D} \right] - \lambda \left[\frac{\partial CV(\tilde{\pi}, \tilde{\pi})}{\partial A_k} + \frac{\partial CV(\tilde{\pi}, \tilde{\pi})}{\partial D} \right] \right\}, \quad k = 1, 2, \cdots N \tag{9}$$

[4] For a discussion of the recent development and controversy surrounding the function of capital and the proper forms of capital regulation see, e.g., [10], [7], and [9]. Koehn and Santomero [5] use a minimum capital to asset ratio as the capital requirement. Since there is a one-to-one correspondence between capital-to-asset ratio and deposit-to-asset ratio, the two forms of capital requirements are equivalent.

When the capital constraint is binding (which we will assume hereafter), $\gamma_1 > 0$ and Equation (8) becomes an equality.[5] Equation (9) states that the bank should invest in risky assets or projects until the marginal expected returns of each asset or project are equated. The marginal expected return γ_1 is the direct contribution to the value of the bank and has three distinct components. The first component $(\bar{r}_k - \bar{d})$ represents the *expected spread* of the investment in asset or project k. But investment in a risky asset may alter both the internal risk and the external risk of the bank and thus must be explicitly accounted for. The impact of the changes in risk is contained in the second and third terms of (9). The second term adjusts for the impact due to the external risk, while the third term adjusts for the impact due to the internal risk. Note that in a mean-variance framework (e.g., [5]), the effects of external risk are not explicitly recognized in the portfolio selection and adjustment process.

Since interest rate deregulation affects, in particular, the cost of deposits, and thus the internal and the external risks of the bank's cash profits, the portfolio adjustment process and the risk-related behavior of the bank must also be affected. Thus, the effect of capital regulation on portfolio behavior may differ according to whether Regulation Q is in effect or is phased out with the interest rate on deposits set by the market forces. The detailed analysis of the impact of capital and interest rate regulations is examined in the next two sections.

II. Portfolio Behavior Under Regulation Q

When Regulation Q is in effect, the interest rate on deposits cannot exceed a ceiling imposed by the Fed. Assume that the bank set the deposit rate at the maximum allowable rate $\hat{d}$. This rate is known with certainty for the relevant decision period. Thus, the variances of deposit cost, σ_d^2, and covariances of deposit cost with returns of assets, σ_{jd}, and with that of other firms in the economy, σ_{dw} are all equal to zero. To solve for the portfolio decisions explicitly, we need to solve equations (7), (8) (with equality), and (9) simultaneously. First recognize that (7) and (8) with the capital constraint binding reduces to

$$\textstyle\sum_{j=1}^{N} A_j = (1 + C)K. \qquad (10)$$

Substituting the partial derivatives of the covariances and Equation (10) into (9) and simplifying, we obtain:

$$2\lambda \textstyle\sum_{j=1}^{N} A_j \sigma_{kj} + R\gamma_1 = \bar{r}_k - \hat{d} - \lambda \sigma_{kw}, \qquad k = 1, 2, \cdots N. \qquad (11)$$

The right-hand side of (11) represents the expected spread of investing in asset or project k adjusted for the external risk of asset or project k. Note that since the regulator set the deposit rate at the maximum $\hat{d}$, all variance and covariance terms related to $\hat{d}$ do not affect portfolio decisions. Denoting this expected risk-

[5] In light of the recent debate regarding the stringent level of capital requirement, such assumption seems to be reasonable. One important aspect of our analysis is the optimal portfolio adjustment and risk exposure when this constraint is relaxed.

adjusted spread as α_k,

$$\alpha_k = \bar{r}_k - \hat{d} - \lambda\sigma_{kw} \qquad k = 1, 2, \cdots N, \tag{12}$$

(11) then becomes

$$2\lambda \textstyle\sum_{j=1}^{N} A_j\sigma_{kj} + R\gamma_1 = \alpha_k \qquad k = 1, 2, \cdots N. \tag{13}$$

Equations (10) and (13) are a system of $N + 1$ equations with $(N + 1)$ unknowns (N assets and the multiplier γ_1) and therefore can be solved for the optimal asset allocation as follows (the derivation of (14) is available from the authors):[6]

$$A_k^* = (2\lambda)^{-1} \textstyle\sum_{j=1}^{N} \nu_{kj}\alpha_j - \left[\dfrac{(2\lambda)^{-1} \sum_{i=1}^{N} \sum_{j=1}^{N} \nu_{ij}\alpha_j - (1 + C)K}{\sum_{i=1}^{N} \sum_{j=1}^{N} \nu_{ij}}\right] \cdot \textstyle\sum_{j=1}^{N} \nu_{kj} \qquad k = 1, 2, \cdots N. \tag{14}$$

The terms ν_{ij} above represent the (i, j) elements of the inverse of the variance-covariance matrix of asset returns.[7] The term inside the square bracket is equal to $(2\lambda)^{-1}R\gamma_1$, since the multiplier is

$$\gamma_1 = \frac{(2\lambda)^{-1} \sum_{i=1}^{N} \sum_{j=1}^{N} \nu_{ij}\alpha_j - (1 + C)K}{(2\lambda)^{-1}R \sum_{i=1}^{N} \sum_{j=1}^{N} \nu_{ij}}. \tag{15}$$

A close examination of the optimal asset allocation process in Equation (14) reveals that the process consists of two effects (two terms). The first term in (14) involves the weighted average of all expected risk-adjusted spreads, α_j. The weights are the respective elements of the inverse of the variance-covariance matrix, ν_{kj}. This weighting reflects the effect of the internal risks of the asset portfolio. This weighted average value is further discounted by the market price of risk, $(2\lambda)^{-1}$. The second term reflects the effect of the capital requirement on optimal asset allocation, and it consists of two factors. The first factor (terms inside the square bracket) is equal to $(2\lambda)^{-1}R\gamma_1$, which is the value of the multiplier related to the capital constraint of Equation (4) adjusted for market risk. This factor is independent of the individual asset k being considered and applies to *all* assets. The effect of the capital requirement on individual asset allocation is incorporated in the second factor, $\sum_{j=1}^{N} \nu_{kj}$ which incorporates the internal risk associated with that particular asset. Since each asset differs in its contribution to internal risks, the impact of capital on portfolio allocation also differs.

Thus, when Regulation Q is binding, the bank's asset investment is a function of expected spread adjusted for external risk, α_j, the level of capital, K, the allowable deposit/capital ratio, C, and the internal risk contained in ν_{ij}. Ignoring the asset's internal as well as external risks would not lead to optimal asset allocation decisions consistent with shareholder-wealth maximization.

Now consider the effect of changes in capital regulation on the bank portfolio

[6] Note that by summing up all optimal asset allocations in Equation (14), $\sum_{k=1}^{N} A_k^* = (1 + C)K$, as expected.

[7] See Merton [6] and Koehn and Santomero [5] for applications of the inverse matrix elements in similar expressions of optimal asset decisions using different approaches.

decision problem. As expected, a more stringent capital requirement (decreases in C) would increase the value of the multiplier γ_1, since $\partial\gamma_1/\partial C < 0$. As the capital requirement is changed, the bank would readjust its portfolio so as to maximize its value. The effect on asset adjustment can be evaluated by taking the partial derivative of (14) with respect to C,

$$\frac{\partial A_k^*}{\partial C} = K \frac{\sum_{j=1}^N \nu_{kj}}{\sum_{i=1}^N \sum_{j=1}^N \nu_{ij}}. \tag{16}$$

While the denominator is positive since it is assumed that the variance-covariance matrix is symmetric and positive definite, the numerator can have positive or negative sign. Thus, it is possible that imposing a more stringent capital constraint on the bank can, in fact, induce the bank to increase its investment in a more risky asset or project.[8] This conclusion is not surprising since the response to a capital requirement change in (16) is a function of only the internal risk (ν_{kj}) and the level of capital K. On the other hand, as explained earlier, the optimal allocation process also involves the effect of external risks (the first term in (14)). Depending on the relative size of the first term in (14), which includes external risk, the effect of a more stringent capital requirement can be either positive or negative.

The effect of changes in the capital requirement on the bank's internal risk as measured by $CV(\tilde{\pi}, \tilde{\pi})$, however, is more clear (the derivation of (17) is available from the authors):

$$\frac{\partial CV(\tilde{\pi}, \tilde{\pi})}{\partial C} = \frac{\partial \sigma_\pi^2}{\partial C} = \frac{2(1 + C)K^2}{\sum_{i=1}^N \sum_{j=1}^N \nu_{ij}} > 0. \tag{17}$$

Thus, a more stringent capital requirement (lower C) would induce a banking firm to adjust its portfolio holdings to lower internal risk, a result intended by the regulator.

The effect on the expected cash profit when the capital requirement is changed can be easily obtained by taking a partial derivative of the expected profit with respect to C. Using (16) to simplify, we obtain:

$$\frac{\partial E(\tilde{\pi})}{\partial C} = K \frac{\sum_{i=1}^N \sum_{j=1}^N (\bar{r}_i - \hat{d})\nu_{ij}}{\sum_{i=1}^N \sum_{j=1}^N \nu_{ij}} = \sum_{i=1}^N (\bar{r}_i - \hat{d}) \frac{\partial A_i^*}{\partial C}. \tag{18}$$

It can be easily seen from (18) that the directional effect of changes in capital requirement on expected bank profit is indeterminant *a priori.* This ambiguity holds even if we can assume a positive expected spread for all asset returns, i.e., $(\bar{r}_i - \hat{d}) > 0$ for all i. In this case, the effect of changes in capital requirement is dependent on the relative size and direction of adjustment of *each* individual asset, i.e., $\partial A_i^*/\partial C$.

Equations (16) through (18) indicate that when an interest rate ceiling on the deposit is prevalent, a shareholder-wealth-maximizing bank reacts to a more stringent capital requirement (i.e., lower value of C) by readjusting its asset

[8] Koehn and Santomero [5] show that, in fact, a more stringent capital requirement would have increased the relative investment in more risky assets. Since their approach does not explicitly include external risks, comparison of results must be done with great care.

portfolio, possibly shifting its allocation more towards risky assets. The magnitude of the readjustment, however, depends on the variances and covariances of the assets. This portfolio readjustment will reduce the bank's internal risk (Equation (17)). However, the bank may increase or decrease its expected portfolio profit.

Koehn and Santomero [5] have recently analyzed banks' portfolio reaction to changes in capital requirements within a mean-variance efficient portfolio framework. They conclude that a more stringent capital requirement would reduce portfolio risk and reduce expected portfolio profit unambiguously over the entire permissible efficient frontier. Furthermore, if the utility function exhibits constant or decreasing relative risk aversion, the bank would invest relatively more in risky assets. Our conclusions agree with Koehn and Santomero's result on the direction of the portfolio variance, but do not coincide with the effect they found on individual asset adjustment and expected bank profit. The difference is due to the fact that their analysis restricts portfolio adjustments in the permissible risk-return frontier while ours is not similarly restricted.

Furthermore, since the capital amount K in Equations (16) through (18) is separable, the level of capital is a scale decision and is independent of the portfolio decision when the deposit rate is risk-free as correctly assumed by Koehn and Santomero. However, the separation of decisions may not hold when the interest rate ceiling is eliminated and the deposit rate is uncertain, a topic we will analyze next.

III. Interest Rate Deregulation and Portfolio Behavior

When Regulation Q is phased out, the interest rate on deposit is uncertain and thus its variance, σ_d^2, and covariances, σ_{jd} and σ_{dw}, must be accounted for in the portfolio decision. Following a solution procedure similar to the previous case, we can substitute the partial derivatives of the covariances and the condition of Equation (10) into (9) to obtain

$$2\lambda[\textstyle\sum_{j=1}^{N} A_j(\sigma_{kj}-\sigma_{jd}) - CK(\sigma_{kd}-\sigma_d^2)] + R\gamma_1 = \bar{r}_k - \bar{d} - \lambda(\sigma_{kw}-\sigma_{dw}) \qquad k = 1, 2, \cdots N. \quad (19)$$

Note that the right-hand side of (19) is the expected spread adjusted by the external risk factor, $\lambda(\sigma_{kw}-\sigma_{dw})$, for investing in risky asset k. Since the deposit rate is risky, $\sigma_{dw} \neq 0$, and the variability may indeed increase the risk-adjusted expected spread, albeit to a small degree. Comparing this risk-adjusted expected spread to that in Equation (11) when a deposit rate ceiling prevails, we observe an interesting effect. While in general the rate ceiling set is below that of the expected market rate $\bar{d}$, rate deregulation does not increase the risk-adjusted expected cost of deposits by the full difference between the rate ceiling and the expected rate $\bar{d}$. Examination of the left-hand side of Equation (19) reveals the impact of the loan-deposit interaction and the deposit variance σ_d^2. The necessary condition in (19) requires that the bank allocate its resources to each risky asset such that the marginal risk-adjusted spreads from each asset are all equal.

Let the right-hand side of (19) be α_k',

$$\alpha_k' = \bar{r}_k - \bar{d} - \lambda(\sigma_{kw} - \sigma_{dw}) \qquad k = 1, 2, \cdots N \tag{20}$$

and let

$$\beta_k = \sigma_{kd} - \sigma_d^2 \qquad k = 1, 2, \cdots N. \tag{21}$$

Then (19) can be written as

$$2\lambda\left[\textstyle\sum_{j=1}^N A_j(\sigma_{kj} - \sigma_{jd}) - CK\beta_k\right] + R\gamma_1 = \alpha_k' \qquad k = 1, 2, \cdots N. \tag{22}$$

Thus, the optimal portfolio composition can be obtained from solving the first-order necessary conditions in (10) and (22) (the derivation of (23) is available from the authors):[9]

$$A_k^{**} = (2\lambda)^{-1}\textstyle\sum_{j=1}^N u_{kj}\alpha_j' + CK\sum_{j=1}^N u_{kj}\beta_j - \left(\dfrac{(2\lambda)^{-1}\sum_{i=1}^N\sum_{j=1}^N u_{ij}\alpha_j' + CK\sum_{i=1}^N\sum_{j=1}^N u_{ij}\beta_j - (1+C)K}{\sum_{i=1}^N\sum_{j=1}^N u_{ij}}\right)\cdot\left(\textstyle\sum_{j=1}^N u_{kj}\right),$$

$$k = 1, 2, \cdots N \tag{23}$$

where u_{ij} is the (i, j) element of the inverse matrix U^{-1} which is derived from the variance-covariance matrix of asset return and deposit rate. The (i, j) element of U is equal to $(\sigma_{ij} - \sigma_{jd})$.

It is apparent from the above equation that the optimal investment in risky assets or projects when the deposit rate is risky is a function of not only the variances and covariances of the asset return (the internal risk) and the capital ratio C, but also of the variance of the deposit rate and the covariances of the deposit rate with the cash flows of all other firms in the economy, σ_{dw} (the external risk). These additional factors are contained in α_j', β_j, and u_{ij}. Comparing the optimal asset allocation process in (23) with that of (14) when Regulation Q prevails, one notices that there is an additional term, $CK\sum_{j=1}^N u_{kj}\beta_j$, which is absent in (14). This term includes not only the risks associated with the stochastic deposit rate, but also reflects the additional interactive effect between the capital ratio C and the uncertain deposit rate. Unlike the previous case when deposit rate is deterministic, the interaction between the capital requirement and the uncertain deposit rate must also be recognized and, furthermore, the effects must be considered jointly.

With the inclusion of the additional factors, the portfolio adjustment problem becomes more complex when the deposit to capital ratio is changed by the regulator. As shown in the previous case, the adjustment can be evaluated by taking the partial of A_k^{**} with respect to C.

$$\frac{\partial A_k^{**}}{\partial C} = K\left\{\textstyle\sum_{j=1}^N u_{kj}\beta_j - \left(\dfrac{\sum_{i=1}^N\sum_{j=1}^N u_{ij}\beta_j - 1}{\sum_{i=1}^N\sum_{j=1}^N u_{ij}}\right)\cdot\left(\sum_{j=1}^N u_{kj}\right)\right\},$$

$$k = 1, 2, \cdots N. \tag{24}$$

[9] Again, note that if we sum up all optimal asset allocations in Equation (23), $\sum_{k=1}^N A_k^{**} = (1 + C)K$, as expected.

Note that when the deposit rate is regulated and set at the maximum level d, all variances and covariances associated with the deposit rate are zero, $\beta_j = 0$ and $u_{kj} = \nu_{kj}$, so that Equation (24) reduces to Equation (16).

As in the previous case when Regulation Q prevails, the directional effect of the partial derivatives cannot be determined unambiguously. Therefore, it is possible that a more stringent capital requirement (a lower C) can induce the bank to shift more of its resources to assets with higher risk as measured by its return variances and covariances. Because of the additional uncertainty induced by the uncertain deposit rate, the effect of capital requirement on asset allocation is even more difficult to determine, as is apparent from Equation (24).

However, the more pertinent and interesting question is whether or not a more stringent capital requirement in a deposit rate deregulated environment will reduce a bank's internal risk as measured by $CV(\tilde{\pi}, \tilde{\pi})$. In the case when Regulation Q prevails, our conclusion was an unambiguous yes. In this case, however, the answer is not certain because the sign of the partial derivative of $CV(\tilde{\pi}, \tilde{\pi})$ with respect to C at the optimum is indeterminant (the exact form of the partial derivative is available from the authors). Furthermore, the expression of $\partial CV(\tilde{\pi}, \tilde{\pi})/\partial C$ and (23) show that the amount of capital, K, is an inherent part of the portfolio risk adjustment process and is *not* separable as in the case when Regulation Q prevails.[10] In other words, when Regulation Q is lifted, the portfolio risk effect on the bank is not only indeterminant but may also be a function of the existing capital (size) of the bank. Thus, the portfolio adjustments in response to changes in capital requirements may differ among banks of different sizes after interest rate deregulation. The differential impact on various bank sizes is an interesting and important area for further analysis. Note that $\partial \sigma_{\pi}^2/\partial C$ can be shown to reduce to (17) when deposit rates are regulated.

Similar to the previous case, the effects of changes in the capital requirement on expected bank profit can be evaluated by taking the partial derivative of the expected profit and using the result of (24):

$$\frac{\partial E(\tilde{\pi})}{\partial C} = K\left\{\sum_{i=1}^{N}\sum_{j=1}^{N}\bar{r}_i u_{ij}\beta_j - \bar{d} - \left(\frac{\sum_{i=1}^{N}\sum_{j=1}^{N}u_{ij}\beta_j - 1}{\sum_{i=1}^{N}\sum_{j=1}^{N}u_{ij}}\right)\cdot\left(\sum_{i=1}^{N}\sum_{j=1}^{N}\bar{r}_i u_{ij}\right)\right\}. \quad (25)$$

As in the previous case, the effect of a more stringent capital requirement may increase or decrease the expected bank profit.

IV. Capital Requirement and Bankruptcy

This section examines the effect of the capital requirement on the probability of bankruptcy of a banking firm both before and after interest rate deregulation. To simplify our discussions, we will concentrate on the bank's internal risk and ignore the external risk. Following the work of Roy [8], Blair and Heggestad [1], and Koehn and Santomero [5], we can use Chebyshev's inequality to estimate the upper bound of the probability of failure. Given the bank's portfolio charac-

[10] Recall that the optimal asset allocation process in (23) consists of an additional term, $CK\sum_{j=1}^{N} u_{kj}\beta_j$, which incorporates the interaction between capital requirement and the uncertain deposit rate. This interactive term is absent in the case when Regulation Q prevails.

teristics described by $E(\tilde{\pi})$ and σ_π^2, the upper bound can be written as[11]:

$$\Pr\{\tilde{\pi} < K\} \leq \frac{\sigma_\pi^2}{(E(\tilde{\pi}) - K)^2} = P. \tag{26}$$

It is obvious from (26) that the higher the portfolio variance, the higher the probability of failure and the lower the expected profit, the higher the probability of failure, *ceteris paribus.* However, since both σ_π^2 and $E(\tilde{\pi})$ are functions of the required capital ratio, C, the probability of failure is also dependent on the capital requirement. Taking a partial derivative of P with respect to C and simplifying, we obtain:

$$\frac{\partial P}{\partial C} = [E(\tilde{\pi}) - K]^{-2}\left\{\frac{\partial \sigma_\pi^2}{\partial C} - 2P^{1/2}\sigma_\pi \frac{\partial E(\tilde{\pi})}{\partial C}\right\}. \tag{27}$$

Thus, the magnitude of the impact of a more stringent capital requirement on the probability of failure is a function of not only the expected profit, variance and the level of capital, but also a function of the responsiveness of the expected portfolio profit and variances (i.e., $\partial\sigma_\pi^2/\partial C$ and $\partial E(\tilde{\pi})/\partial C$). Since the portfolio adjustment processes differ between the cases when Regulation Q is in effect and when interest rate ceiling is stochastic, the impact of capital requirement changes on the probability of failure may also be different. We examine below the two cases of interest.

A. Regulation Q Prevails

Substituting Equation (18) for $\partial E(\tilde{\pi})/\partial C$ into (27), we obtain

$$\frac{\partial P}{\partial C} = [E(\tilde{\pi}) - K]^{-2}\left\{\frac{\partial \sigma_\pi^2}{\partial C} - 2p^{1/2}\sigma_\pi \textstyle\sum_{i=1}^{N} (\bar{r}_i - \hat{d}) \dfrac{\partial A_i^*}{\partial C}\right\}. \tag{28}$$

Since the first factor is nonnegative, the sign of $\partial P/\partial C$ in (28) is determined by the terms within the brackets. We already know from (18) that $\partial\sigma_\pi^2/\partial C$ is positive. Thus, a sufficient condition for $\partial P/\partial C$ to be positive (a desirable posture for capital regulation) is for $\partial E(\tilde{\pi})/\partial C < 0$, i.e., $\sum_{i=1}^{N} (\bar{r}_i - \hat{d})\cdot\partial A_i^*/\partial C < 0$. This can occur since the sign of $\partial E(\tilde{\pi})/\partial C$ is ambiguous even when we assume positive spreads between asset returns and deposit cost (see Section II). In other words, with a more stringent capital requirement (lower C), a bank can shift its asset allocation towards more risky assets or projects with higher returns (i.e., $\partial E(\tilde{\pi})/\partial C < 0$) resulting in a lower probability of failure (i.e., $\partial P/\partial C > 0$). Although the reduction of probability of failure by imposing a more stringent capital requirement is probably the intention of regulators, the portfolio adjustment towards more risky assets or projects may not be desirable. But from our analysis, this scenario is possible.

[11] For any density function of $\tilde{\pi}$ with expected value $E(\tilde{\pi})$ and variance σ_π^2, the Chebyshev inequality states that

$$\Pr\{|\tilde{\pi} - E(\tilde{\pi})| > \ell\sigma_\pi\} \leq \left(\frac{1}{\ell}\right)^2$$

If we let $K = E(\pi) - \ell\,\sigma_\pi$ or $\ell = (E(\pi) - K)/\sigma_\pi$, Equation (26) follows.

On the other hand, it is also possible for a bank to shift its asset allocation towards lower return assets in response to a more stringent capital requirement ($\partial E(\tilde{\pi})/\partial C > 0$), but the response can be so large that it can result in an increase in the probability of failure (i.e., $\partial P/\partial C < 0$).

In sum, the analyses above indicate that capital regulation using capital ratio restriction may not necessarily reduce failure probability nor induce portfolio behavior as originally intended. Koehn and Santomero [5] arrive at the same conclusions by invoking differences in risk preferences of banking institutions as the central rationale. Our conclusions, however, follow directly from the optimizing behavior of banks.

B. Regulation Q Phased Out

To evaluate the effect of capital requirement on probability of failure when Regulation Q is phased out, one can substitute the expressions for $\partial \sigma_{\pi}^2/\partial C$ and $\partial E(\tilde{\pi})/\partial C$, respectively, into (27). However, when the deposit rate is stochastic, the signs of both partial derivatives are indeterminant *a priori*. Thus the effect of capital requirements on the chance of failure, $\partial P/\partial C$, is also indeterminant. In other words, in a deregulated environment, a more stringent capital ratio requirement may induce a result exactly opposite to that intended—an increase in the probability of bankruptcy and/or undesirable portfolio behavior. Furthermore, as indicated in the previous section, the optimal portfolio adjustment process in a deregulated environment is also dependent on the level of existing capital of the bank. Therefore, a uniform capital requirement for all banks may invoke a desirable response by some banks and exactly the opposite unintended response by others.

V. Summary and Policy Implications

This paper develops a wealth-maximizing model of a banking firm to study the joint impact of bank capital regulations and interest rate ceilings (Regulation Q) on the portfolio behavior of banks. We derive the optimal portfolio composition for the bank in cases when the interest rate on deposits is regulated and when it is deregulated. In each case, we also analyze the effect of changes in capital requirements.

When Regulation Q is binding, the bank invests in each asset or project until marginal risk-adjusted returns are equated for each asset or project. Since the deposit rate is set by the regulator, risk adjustment involves only the uncertain returns of the assets. The risks include both the internal risk and the external risk. In response to a more stringent capital requirement, our comparative statics analyses show that a wealth-maximizing bank may react by readjusting its asset portfolio, possibly shifting its resource allocation towards more risky assets or projects. The magnitude of the readjustment, however, depends on the variances and covariances of the return on the assets. This portfolio readjustment will reduce total portfolio variance, but may increase or decrease its expected portfolio profit. Since the effect of capital requirement on expected profit is indeterminant, a more stringent capital requirement may increase or decrease the probability of failure.

When Regulation Q is phased out, the interest rate on deposits is stochastic. In this case, the bank would still invest in each asset until the marginal risk-adjusted returns are equated for each asset. The risk adjustments, however, must respond to the risky deposit rate in addition to the risky asset returns, and both the internal risk and the external risk must be considered. When Regulation Q is phased out, portfolio readjustment to changes in capital requirements are more complex. The effect of a more stringent capital requirement cannot be determined without further assumptions. In addition, as opposed to the previous case (when Regulation Q is binding), the portfolio adjustment process is an inherent function of the existing level of capital of the bank. In other words, banks of different sizes (capitals) may react differently to changes in capital regulation when Regulation Q no longer exists. This result is, perhaps, the most significant result of our analysis and, as far as we know, has not been analyzed in previous literature. Since Regulation Q is destined to be lifted in 1986, this topic merits further research. Hopefully, this study can serve as a stimulus.

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