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Author(s): Amihud Dotan and S. Abraham Ravid
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On the Interaction of Real and Financial Decisions of the Firm Under Uncertainty

AMIHUD DOTAN and S. ABRAHAM RAVID*

ABSTRACT

This study analyzes the interaction between the optimal level of investment and debt financing. For this purpose, a model is structured in which a firm, facing an uncertain price, has to decide on its optimal level of investment and debt. The amount of investment sets a limit on output whose optimal level is determined after price is realized. The debt involved is risky (there exists a possibility of bankruptcy).

The analysis proves that investment and its optimal financing have to be simultaneously determined and that a negative relationship exists between operating and financial leverage. We also demonstrate that as the tax rate increases, optimal capacity decreases and optimal leverage increases. An analysis of the impact of changes in the expected price shows that under some conditions, an increase in expected price would lead to an increase in optimal investment (firm size) and a decrease in optimal debt.

THE FOCUS OF THIS WORK is on the linkage between the firm's financing decision and its production policies. Although intuitively such interdependence seems quite appealing, the sheer complexity of the task has deterred most authors who have therefore treated financing and production as separate issues. Finance literature, naturally, focused on the capital structure question. The publication of the pathbreaking Modigliani and Miller papers [21, 22] was followed by a host of works which endeavored to reconcile the Modigliani and Miller maximum leverage position [22] with observed capital structure. Notable examples are: Kraus and Litzenberger [15]; Scott [27]; Brennan and Schwartz [1] (the bankruptcy approach); Jensen and Meckling [12] (agency costs); Miller [20]; De Angelo and Masulis [4]; and empirical examinations by Kim [13] and Castanias [2] (personal taxes approach). The assumption of stochastic operating cash flows used in most of these papers implicitly reflects independence of production and financing decisions. This important simplification allowed tractable analysis and useful insights. Our paper, however, will show how the production decisions which affect these cash flows interact with capital structure determination.

This interdependence has been recognized by several authors during the last decade (for irrelevance conditions, see Stiglitz [30]). These latter papers have, however, analyzed only specific subsets of issues. Stiglitz [31] showed how

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investment with a known distribution of returns affects financial structure, but no explicit relationship was derived. Myers [23, 24] showed how investment decisions, i.e., acceptance or rejection of projects, affected the optimal financial structure of a firm and why investment in turn, should be affected by leverage. However, taxes and bankruptcy were not considered. Long and Racette [17] showed that the cost of capital of a firm facing stochastic demand was affected by the level of production, but they analyzed a competitive firm with no leverage. Subrahmanyam and Thomadakis [32] studied the impact of "real" variables, mainly the capital-to-labor ratios and monopoly power on the firm's beta. Hite [10] examined the impact of leverage on the optimal stock of capital employed by the firm and its capital-to-labor ratio.

Hite's paper is perhaps the closest to what we have in mind. However, his model incorporated some strong assumptions about the tax structure and explicitly assumed away default. The risk he treated in his model is then the systematic CAPM market risk. Because of these assumptions, a major conclusion of Hite's analysis is that increased leverage will reduce the cost of capital. Hite's debt is riskless and hence, increased debt does not increase the interest rate paid to bondholders. Also, since debt is explicitly modelled as a *proportion* of capital investment, increased debt increases the tax shelter of debt, thus lowering the implicit cost of capital.

In our analysis, the proportional automatic link between investment and the level of debt is dropped.[1] Leverage is an independent decision variable, and we obtain conclusions which are diametrically opposed to Hite's. Specifically, the increased probability of zero accounting income implied by a larger amount of debt (in which case no tax shields can be used) *increases* the "user cost of capital" (à la Hite), and lowers optimal capital input. This analysis provides theoretical support to the common wisdom of negative relationships between financial and operating leverage.

Our paper is more general than De Angelo and Masulis' work [4] in that it endogenizes the nondebt-related tax shields—where the tax-related benefits and "costs" are added to production benefits to be compared with capital cost. Most of the paper is based on a risk-neutral valuation of cash flows. However, a final section provides preliminary extension to a nonrisk-neutral world. Results are essentially unaffected. Additional elements, such as personal taxes as well as agency costs, can, in principle, be appended to the model. However, we do not feel they would illuminate further issues of major concern to us, while they would greatly contribute to the complexity of the presentation.

In deriving our results, we could choose between two types of firm behavior models—one is a model which highlights factor choices (see, e.g., Hartman [9], Holthausen [11]). The other (often used in the public utilities literature) focuses on pricing and the capital investment decision (see, e.g., Panzar [25], Crew and Kleindorfer [3]). We selected the latter type of model for reasons to be discussed later. However, the former type can also be applied with similar qualitative results.

[1] We owe the initial observation to Avner Kalay.

The next section presents the model. Subsections C and D contain the main results.

I. The Model

The model will be developed stage by stage until full complexity is attained. The purpose is to provide the reader with a rigorous path from profit maximization under uncertainty to firm value maximization, including debt and taxes, examining along the way the impact of closer and closer integration of real and financial elements.[2]

The assumptions of the model are as follows: the firm is a competitive firm which operates for a single period. At the beginning of the period, it purchases z units of capital goods, at a constant cost of β dollars per unit. For simplicity, z is used as a capacity constraint on production (see similar models by Crew and Kleindorfer [3], Sherman and Visscher [28], and others). This capacity investment decision is based upon a stochastic price, the distribution of which is known ex ante, i.e., $\tilde{P} = p + \tilde{u}$ where u is a random variable with a cumulative distribution function $F(u)$ so that $E(u) = 0$.[3] At the end of the period, price is realized. The firm then produces output Q and liquidates. We first compute optimal z for the case of no taxes and no debt. We then impose taxes, and finally we allow the firm to take on debt. The objective throughout is to maximize the value of the firm.

A. The No Tax Case

If no taxes are paid, then end-of-period cash flows to owners are:[4]

$$y = \begin{cases} z(p + u) - c(z) & u > c'(z) - p \\ Q(u)(p + u) - c[Q(u)] & u < c'(z) - p \end{cases} \tag{1}$$

where c is the twice-differentiable convex cost function. Optimal quantity, Q, is determined *ex post*, as P is realized, so that $c'[Q(u)] = p + u$. If Q exceeds z, only z units are produced. The net expected future value of cash flows to equity holders is given by the expected end-of-period cash flows less the future value of the initial investment[5]

[2] If ex ante pricing or quantity setting is assumed, then this must be optimized together with capital and debt.

[3] Hite's model assumes a downward sloping demand function, but as long as the quantity decision is assumed to be ex post, qualitative conclusions should not be affected.

[4] Strictly speaking, $c'(z) \equiv \left.\dfrac{\partial c}{\partial Q}\right|_z$.

[5] For convenience, our expressions are in terms of future value rather than present value. Also, following the entire public utility literature (see, e.g., Crew and Kleindorfer [3]), it is assumed that the lower limit of u will still not render $p + u$ negative. Further, there is at least a small range of u for which $AVC < p + u < c'(z)$ (where AVC stands for average variable cost), so that production takes place. If for some u, $p + u < AVC$, the results still follow.

$$V = \int_{c'(z)-p}^{\infty} [z(p+u) - c(z)]f(u)\,du$$
$$+ \int_{-\infty}^{c'(z)-p} \{Q(u)(p+u) - c[Q(u)]\}f(u)\,du - \beta z(1+R_F) \qquad (2)$$

where R_F is the riskless rate of interest, and $f(u)$ is $F'(u)$ (the density function of u). At this juncture, z is the only decision variable, yielding the following first-order condition:

$$\frac{\partial V}{\partial z} = \int_{c'(z)-p}^{\infty} [p + u - c'(z)]f(u)\,du - \beta(1+R_F) = 0 \qquad (3)$$

Equation (3) is a familiar representation of marginal conditions: the expected end-of-period profit from an additional capacity unit is equated at the margin to its end-of-period cost.

B. The Competitive Case with Corporate Taxes

Corporate taxes (levied at a rate t) reduce income (of course) but add a depreciation tax shield, $t\beta z$. They also add a domain where accounting income is negative and no tax is paid but, of course, neither are tax shelters obtained (a similarity to DeAngelo and Masulis [1980] is becoming apparent. Also, as in their case, our one-period model does not allow for tax carry-backs and carry-forwards).

Cash flows to securityholders are then:

$$Y = \begin{cases} [z(p+u) - c(z)](1-t) + t\beta z & u > c'(z) - p \\ [Q(u)(p+u) - c(Q(u))](1-t) + t\beta z & u^b \le u \le c'(z) - p \\ Q(u)(p+u) - c(Q(u)) & u \le u^b \end{cases} \qquad (4)$$

u^b is the break-even value of u for which accounting income is zero. Thus,

$$u^b = \frac{\beta z + c(Q(u^b))}{Q(u^b)} - p$$

The future value of the firm (defined as before) is then as follows:

$$V = \int_{c'(z)-p}^{\infty} [z(p+u) - c(z)](1-t)f(u)\,du$$
$$+ \int_{-\infty}^{c'(z)-p} [Q(u)(p+u) - c(Q(u))]f(u)\,du$$
$$- t\int_{u^b}^{c'(z)-p} [Q(u)(p+u) - c(Q(u))]f(u)\,du$$
$$+ t\beta z[1 - F(u^b)] - \beta z(1+R_F) \qquad (5)$$

First-order conditions are given by

$$\frac{\partial V}{\partial z} = \int_{c'(z)-p}^{\infty} [p+u-c'(z)](1-t)f(u)\,du - \beta[1+R_F-t(1-F(u^b))] = 0 \qquad (6)$$

As intuitively expected, taxes will reduce optimal z^*. To see this, multiply both terms of Equation (3) by $(1-t)$. Now, the first term in the revised Equation (3) is the same as the first term in Equation (6) but $\beta(1+R_F)(1-t) < \beta[1+R_F - t(1-F(u^b))]$. Therefore, for z^* that solves Equation (3), $\left.\frac{\partial Vt}{\partial z}\right|_{z^*} < 0$ (where V_t is the value of the firm in the case with taxes). This implies that a smaller z is required to restore equality.[6]

The interpretation of Equation (6), however, is similar to the interpretation of Equation (3). The marginal expected after tax profit from an additional capacity unit including the expected depreciation tax shield benefits $(\beta t(1-F)u^b))$ are equated to the expected marginal capacity cost, $\beta(1-R_F)$.

C. Partial Financing with Risky Debt

Consider now the case in which the firm is allowed to borrow the amount D at the rate of interest R. The cash flows to bondholders and stockholders detailed below are similar to DeAngelo and Masulis' [4] framework except that exogenous upper limits on tax shields are not imposed and the nondebt tax shields themselves, as we noted earlier, are endogenized.

We assume now that upon default, the firm incurs a fixed bankruptcy cost, B. This assumption is necessary within our framework only to guarantee an optimal capital structure in the absence of personal taxes (see later discussion). Most of the interactions we point out will take effect even if $B=0$.

Y_e, the cash flows to stockholders, are as follows:

$$Y_e = \begin{cases} [z(p+u)-c(z)](1-t)+t\beta z - D[1+R(1-t)] & u > c'(z)-p \\ [Q(u)(p+u)-c(Q(u))](1-t) & u^b < u \le c(z)-p \\ \quad + t\beta z - D[1+R(1-t)] & \\ Q(u)(p+u)-c(Q(u))-D(1+R) & u^* < u \le u^b \\ 0 & u \le u^* \end{cases} \qquad (7)$$

Bondholders receive:

$$Y_d = \begin{cases} D(1+R) & u > u^* \\ Q(u)(p+u)-c(Q(u))-B & u^B \le u \le u^* \\ 0 & u < u^B \end{cases} \qquad (8)$$

u^b is, again, the value of u for which accounting income is equal to zero. Both debt and depreciation tax shields are then rendered worthless. Here,

$$u^b = \frac{\beta z + RD - \pi(u^b)}{Q(u^b)}$$

[6] Note that this results from the fact that the tax effect is not symmetrical on marginal contribution and marginal cost of capital. While the marginal contribution is always taxed, marginal cost may not be tax deductible (when $u < u^b$).

where $\pi(u) = Q(u)\cdot p - c(Q(u))$. u^*, in turn, is the value of u for which the end-of-period cash flows are just sufficient to cover promised payments to bondholders (if $u < u^*$, the firm defaults)[7],

$$u^* = \frac{D(1+R) - \pi(u^*)}{Q(u^*)}.$$

Finally, u^B is the value of u for which the end of period cash flows are equal to the bankruptcy cost, B,

$$u^B = \frac{B - \pi(u^B)}{Q(u^B)}.$$

We can now examine Equations (7) and (8) in detail. As before, if u is such that the optimal production quantity exceeds z, only z is produced, and stockholders receive total firm profit plus the depreciation tax shield less payments to bondholders. If the optimal quantity, $Q(u)$, is feasible (the second domain), then $Q(u)$ is produced and stockholders receive similar cash flows. If accounting income is negative, no tax is paid and no tax shelters are realized. Stockholders retain profits less payments to bondholders. If profits are insufficient to cover principal and interest payments, stockholders receive nothing. As for bondholders, they receive their promised payment, $D(1 + R)$, if the firm can afford that ($u > u^*$). Otherwise, they receive profits less the bankruptcy cost. If income cannot even cover the bankruptcy cost, then both bondholders and stockholders receive nothing.

The interest rate paid to bondholders, R, must guarantee that expected payments equal the return obtained at the risk-free rate. It follows, then, that

$$E(Y_d) = D(1 + R_F) \tag{9}$$

where $E(Y_d)$ is explicitly defined as follows:

$$E(Y_d) = \int_{u^B}^{u^*} [Q(u)(p + u) - c(Q(u))]f(u)\,du$$

$$+ D(1 + R)(1 - F(u^*)) - B[F(u^*) - F(u^B)] \tag{10}$$

In the spirit of Scott [27] or Fama [5], we can expect capital markets under risk neutrality to require equation (9) to hold. Condition (9) also constitutes a sufficient guarantee for bondholders that their rights will not be violated. This is because in the context of a one-period model, no additonal debt can be issued which would increase the risk of default. Given this constraint, stockholders cannot, in effect, transfer any wealth away from bondholders. If risk aversion on the part of securityholders were assumed, a more complicated guarantee would be required. But, as we shall see in the last section, our main results will not be affected by such a formulation.

We will now compute the total expected value of the firm combining expected

[7] Assuming that $u^* \leq u^b$ is equivalent to assuming $D < \beta z$. But this is required because for $D > \beta z$ no debt tax shelters are allowed by law.

payments to stockholders and bondholders. From Equation (7), expected payments to stockholders will be,

$$E(Ye) = \int_{u^*}^{c'(z)-p} [Q(u)(p + u) - c(Q(u))]f(u)\, du$$

$$- t\int_{u^b}^{c'(z)-p} [Q(u)(p + u) - c(Q(u))]f(u)\, du$$

$$+ (1 - t) \int_{c'(z)-p}^{\infty} [z(p + u) - c(z)]f(u)\, du$$

$$+ t\beta z[1 - F(u^b)] - D(1 + R)(1 - F(u^*) + tRD[(1 - F(u^b)] \quad (11)$$

Adding Equation (10) to Equation (11), one obtains that the end of the period expected value of the firm is[8] $V = E(Y_e) + E(Y_d) - \beta z(1 + R_F)$, or more explicitly,

$$V = \int_{u^B}^{c'(z)-p} [Q(u)(p + u) - c(Q(u))]f(u)\, du$$

$$- t \int_{u^b}^{c'(z)-p} [Q(u)(p + u) - c(Q(u))]f(u)\, du$$

$$+ (1 - t) \int_{c'(z)-p}^{\infty} [z(p + u) - c(z)]f(u)\, du + t[1 - F(u^b)](\beta z + RD)$$

$$- B[F(u^*) - F(u^B)] - \beta z(1 + R_F) \quad (12)$$

A casual glance at the equations above provides an initial reasoning as to why our results differ from results found in much of the finance literature. The distribution of cash flows is crucially dependent on the decision variables of the firm, specifically z and D. The following computations will demonstrate this contention more clearly, and will provide explicit relationships between optimal z and optimal D. Taking derivatives with respect to z and D, one obtains,[9]

$$\frac{\partial V}{\partial z} = \int_{c'(z)-p}^{\infty} [p + u - c'(z)](1 - t)f(u)\, du$$

$$- \beta[1 + R_F - t(1 - F(u^b))] = 0 \quad (13)$$

[8] Note that since the expected value of debt is kept constant, maximization of the value of the firm or the value of stockholders equity will yield the same results.

[9] In the derivations, we used the following substitutions:

(a) $\dfrac{\partial R}{\partial D} = \dfrac{1 + R_F}{D\left[1 - F(u^*) - \dfrac{Bf(u^*)}{Q(u^*)}\right]} - \dfrac{1 + R}{D} > 0$ (b) $\dfrac{\partial R}{\partial z} = 0$ (c) $\dfrac{\partial u^*}{\partial z} = 0.$

The implied condition that B is not large enough to render the denominator negative, which is carried through the paper, directly follows from Equation (14a).

$$\frac{\partial V}{\partial D} = t[1 - F(u^b)]\left[\frac{1 + R_F}{1 - F(u^*) - \dfrac{Bf(u^*)}{Q(u^*)}} - 1\right]$$

$$- \frac{Bf(u^*)(1 + R_F)}{Q(u^*)[1 - F(u^*)] - Bf(u^*)} = 0 \tag{14}$$

Note that Equation (13) is almost identical to Equation (6) except that u^b is of course different. We can also show that

$$\frac{\partial u^b}{\partial D} = \frac{R + D\dfrac{\partial R}{\partial D}}{Q(u^b)} > 0 \quad \left(\text{see footnote 9 for derivation of } \frac{\partial R}{\partial D}.\right) \tag{15}$$

Inspecting Equation (13), we can see that the level of debt affects capacity choice because of its impact on the zero income level and hence on the probability that depreciation tax benefits will in fact be used. As more debt is taken, u^b increases (see Equation (15)) and consequently the expected capacity cost goes up.

To discuss the impact of capacity on debt, one may write Equation (14) as

$$t[1 - F(u^b)] = (1 + R_F)\left\langle\frac{t[1 - F(u^b)] - \dfrac{Bf(u^*)}{Q(u^*)}}{1 - F(u^*) - \dfrac{Bf(u^*)}{Q(u^*)}}\right\rangle \tag{14a}$$

or less explicitly as,

$$\frac{\partial V}{\partial D} = t[1 - F(u^b)]\left(R + D\frac{\partial R}{\partial D}\right) - B\frac{\partial F(u^*)}{\partial D} = 0 \tag{16}$$

Here, the expected tax advantage of an additional unit of debt is balanced by the change in the expected bankruptcy cost. But the ability to exploit the tax benefits depends crucially on u^b, the point where accounting income is equal to zero. u^b, in turn, is determined also by the choice of z. (Note that if $B = 0$, we still have an "optimal" capital structure—the firm chooses D so that $F(u^b) = 1$, zero income is guaranteed and no taxes are ever paid. This, however, is a less interesting case from our point of view.) We can now formalize this interdependence via two short propositions:

PROPOSITION 1. *Optimal z is a decreasing function of D.*

Discussion: This proposition shows that the optimal investment decision is different for an all equity firm than for a levered firm. Clearly, by Equations (13) and (16), investment and capital structure must be jointly determined. However, here we point out the direction of change in the optimal value of one variable when another is changed for exogenous reasons. Otherwise stated, a corollary could be formulated.

COROLLARY 1. *Optimal operating leverage is a decreasing function of financial leverage.* This corollary is supported by recent empirical evidence provided by Mandelker and Rhee [18].

Proof: To prove the proposition, consider Equation (13). We can write it as $\frac{\partial V}{\partial z}(z^*, D) = 0$, where z^* is the optimal value of z for a given D. Total differentiation with respect to D yields:

$$\frac{dz^*}{dD} = -\left(\frac{\partial^2 V}{\partial z \partial D}\right) \Big/ \left(\frac{\partial^2 V}{\partial z^2}\right) \tag{17}$$

However,

$$\frac{\partial^2 V}{\partial z^2} = -(1 - t) \int_{c'(z)-p}^{\infty} c''(z) f(u)\, du - \beta t f(u^b) \frac{\partial u^b}{\partial z} < 0$$

since

$$\frac{\partial u^b}{\partial z} = \frac{\beta}{Q(u^b)} > 0$$

(which is also required for the second-order conditions). Also,

$$\frac{\partial^2 V}{\partial z \partial D} = -t\beta f(u^b) \frac{\partial u^b}{\partial D} < 0 \quad \text{(by Equation (15))} \tag{18}$$

From Equations (17) and (18), it follows that indeed, $\frac{dz^*}{dD} < 0$. Q.E.D.

This result contrasts with Hite's [10] analogous theorem (Proposition 2) where an increase in financial leverage leads to an increase in the optimal stock of capital. Hite's result, however, is a direct consequence of the way debt is formulated in his model, i.e., as a proportion of the capital investment, and of the fact that in his model the firm can utilize debt-related tax shields at all times. (There is no possibility of loss or default.)

In our model, debt is independent of the capital investment variable, thus a change in debt does not have an automatic effect on the user cost of capital.[10] However, an increase in debt reduces the expected tax shelter of capital by increasing the probability of accounting loss. Consequently, expected user cost of capital goes up and optimal investment will decrease. An idea similar to ours is found in DeAngelo and Masulis [4] where the value of the debt-related tax

[10] The debt-related tax shelter in Hite's model, using our notation, is $R\gamma\beta z$ which directly introduces a positive relationship between z and γ (or D); in our model, the expression for the debt tax shelter is simply RD. Alternatively, if Hite used our formulation, the link he proposes, in Proposition 2, would vanish.

shield decreases as debt increases because of the presence of other tax shelters. However, their nondebt tax shields are exogenous.[11]

Corollary 2, to be presented shortly, will attempt to shed some more light on the "real" financial link as we see it, i.e., how optimal leverage, D, varies when z changes. As can be expected, similar results emerge.

COROLLARY 2. *Simultaneous optimization of z and D will result in a higher optimal D than if D were optimized given the optimal level of z for an all equity firm.*

Discussion: This corollary invalidates again the widespread use of exogenous, independently distributed EBIT (earning before interest and taxes). If the distribution of EBIT is based upon a given level of z, optimal for an all equity firm, then it will not be correct to use this data to compute optimal financial leverage. This is because as soon as any debt is taken z must be different as well.

Proof: Assume $\bar{z}$ (optimal capacity level) is derived for $D = 0$. Next, the optimal level of D is derived, using equation (14), for $z = \bar{z}$. According to Proposition 1, a nonzero level of D will entail an optimal z^* lower than $\bar{z}$. Since $\frac{\partial^2 V}{\partial z \partial D} < 0$ (Equation (17)), one must now increase D to restore equality in the first-order conditions. Q.E.D.

D. Comparative Statics

It is clear from the foregoing that exogenous variables, related either to production or to the capital structure, will affect the optimal values of both endogenous variables. Comparative statics tend to be very difficult to perform in our framework. We will, however, investigate the impact of a change in two important exogenous variables, t and p, on optimal z and optimal D.

From total differentiation of D^* and z^* with respect to t we obtain:

$$dz^* = \begin{vmatrix} \frac{\partial^2 V}{\partial z \partial t} & \frac{\partial^2 V}{\partial z \partial D} \\ \frac{\partial^2 V}{\partial D \partial t} & \frac{\partial^2 V}{\partial D^2} \end{vmatrix} \Big/ |H| \, dt \tag{19}$$

$$dD^* = \begin{vmatrix} \frac{\partial^2 V}{\partial z^2} & \frac{\partial^2 V}{\partial z \partial t} \\ \frac{\partial^2 V}{\partial D \partial z} & \frac{\partial^2 V}{\partial D \partial t} \end{vmatrix} \Big/ |H| \, dt \tag{20}$$

[11] Although our model does not include personal taxes, one could conjecture that even in the presence of such taxation, increased investment by each and every firm will result in a lower level of equilibrium aggregate debt. The reason is that increased investment will yield higher non debt-related tax shields. By examining the De Angelo and Masulis [4] argument (Figure 2, p. 13), this is clearly seen to shift the debt-supply curve so that at equilibrium, a lower level of aggregate debt will result. The reasoning behind this result is similar to ours. Increased nondebt-related tax shields reduce the probability of using debt-related tax shields and thus increase the cost of debt.

where

$$|H| = \begin{vmatrix} \dfrac{\partial^2 V}{\partial z^2} & \dfrac{\partial^2 V}{\partial z \partial D} \\ \\ \dfrac{\partial^2 V}{\partial D \partial z} & \dfrac{\partial^2 V}{\partial D^2} \end{vmatrix}$$

If a maximum exists, it must be that $|H| > 0$. We can now prove the following proposition.

PROPOSITION 2. *An increase in the tax rate will lead to a higher level of optimal debt and a lower level of optimal capacity.*

Proof: The cross-derivative of V with respect to z and t is:

$$\frac{\partial^2 V}{\partial z \partial t} = -\int_{c'(z)-p}^{\infty} [p + u - c'(z)]f(u)\, du + \beta(1 - F(u^b)) \tag{21}$$

From comparison to Equation (13), it can also be shown that at $z = z^*$, Equation (21) is negative.[12]

The cross-derivative of V with respect to D and t is given below:

$$\frac{\partial^2 V}{\partial D \partial t} = (1 - F(u^b))\left(R + D\frac{\partial R}{\partial D}\right) > 0 \tag{22}$$

We know from second-order conditions and Equations (17) and (18) that

$$\frac{\partial^2 V}{\partial z^2} < 0; \quad \frac{\partial^2 V}{\partial D^2} < 0 \quad \text{and also that} \quad \frac{\partial^2 V}{\partial D \partial z} < 0.$$

Finally, substituting Equations (21) and (22) into the total derivatives of z^* and D^* with respect to t (Equations (19) and (20)), we obtain that $\dfrac{dz^*}{dt} < 0$ and $\dfrac{dD^*}{dt} > 0$. Q.E.D.

The intuition behind this result is quite appealing. An increase in the tax rate causes an increase in the optimal level of debt, since the value of the debt tax shelter increases. This increase should be coupled with a lower optimal investment, but here the intuition must be supported by a proof because there are several additional implications of a higher tax rate. An increase in taxes is accompanied by a decrease in the user cost of capital as the depreciation tax shelter increases. However, the marginal after-tax profit decreases as well. (Note our earlier result for the case with no debt.) The empirical implication of this propostion is quite clear—an increase in the corporate tax rate should be accompanied not only by more debt financing, but by lower investment as well.

As for the derivatives of z^* and D^* with respect to p, we obtain equations identical to Equations (19) and (20) except that t is replaced by p.

[12] Note that $\dfrac{\partial u^b}{\partial t} = \dfrac{\partial u^*}{\partial t} = \dfrac{\partial R}{\partial t} = 0.$

Again, we note from second-order conditions and Equations (17) and (18) that $\frac{\partial^2 V}{\partial z^2} < 0$; $\frac{\partial^2 V}{\partial D^2} < 0$, and $\frac{\partial^2 V}{\partial z \partial D} < 0$. We can now derive $\frac{\partial^2 V}{\partial z \partial p}$ and $\frac{\partial^2 V}{\partial D \partial p}$.

First,

$$\frac{\partial^2 V}{\partial z \partial p} = (1 - t)[1 - F(c'(z) - p)] - \beta t f(u^b)\frac{\partial u^b}{\partial p} \tag{23}$$

Also,

$$\frac{\partial u^b}{\partial p} = \frac{D}{Q(u^b)} \cdot \frac{\partial R}{\partial p} - 1.$$

It can be shown[13] that $\frac{\partial R}{\partial p} < 0$ and hence, $\frac{\partial u^b}{\partial p} < 0$. It follows that $\frac{\partial^2 V}{\partial z \partial p} > 0$. In other words, Equation (21) implies that an increase in price will cause the previous level of z^* $\left(\text{where } \left.\frac{\partial V}{\partial z}\right|_{z^*} = 0\right)$ to be too low and will lead to capacity expansion. This is of course very logical—if the firm can expect a higher price, it will expand production capacity.

The sign of $\frac{\partial^2 V}{\partial D \partial p}$ is ambigous and depends on the specific functions involved:

$$\frac{\partial^2 V}{\partial D \partial p} = t[1 - F(u^b)]\left[\frac{\partial R}{\partial p} + D\frac{\partial^2 R}{\partial D \partial p}\right] - tf(u^b)\frac{\partial u^b}{\partial p}\left(R + D\frac{\partial R}{\partial D}\right) - B\frac{\partial^2 F(u^*)}{\partial D \partial p} \tag{24}$$

The analysis so far, enables us only to make the following conditional statement.

PROPOSITION 3. *If* $\frac{\partial^2 V}{\partial D \partial p} < 0$ *(or positive but sufficiently small), then an increase in the expected competitive price will lead to a greater optimal capacity (firm size) and a lower optimal debt, i.e., lower financial leverage.*

Proof: By inspection of Equations (19) and (20) and our previous discussion. Q.E.D.

[13] Note that

$$\frac{\partial R}{\partial p} = \frac{-Bf(u^*) - \int_{u^B}^{u^*}\left[u\frac{\partial Q}{\partial p} + \pi'(p)\right]f(u)\,du}{D\left[1 - F(u^*) - B\frac{f(u^*)}{Q(u^*)}\right]} < 0$$

since $\frac{\partial Q}{\partial p} > 0$ and $\pi'(p) > 0$ in the relevant range.

Although no unambiguous results are obtained, the analysis highlights the interdependence of the model variables. There is some intuition to the proposition: the condition $\frac{\partial^2 V}{\partial D \partial p} < 0$ means that an increase in p lowers the contribution of D to the value of the firm. The value of debt-related tax shields is lowered, less debt is taken, and hence optimal capacity is increased to capture greater profits and substitute nondebt-related for debt-related tax shields. Whether or not the condition is valid, is basically an empirical question. If it is valid, one should expect larger firms to have lower financial leverage.

In the final section that follows, we will develop a general framework for our analysis with preferences not necessarily risk-neutral.

II. Nonrisk-Neutral Valuation

In this section, we shall examine whether nonrisk-neutral valuation should affect our major results. We keep the discussion general and focus on questions of interest for our particular analysis. For a discussion of other issues in a similar framework, see Galai [6] and Green and Talmor [8].

Since debt is fully secured in our model, firm value maximization and equity value maximization should yield the same results. For reasons of convenience, we shall follow the latter approach. Specifically, based on Equation (7), we partition first-period equity value into three components. The first one is the value of the firm's operating cash flows (before taxes and debt payments). These flows are a function of z and u only (see Equation (1)). We denote this first component by $V_1(z, u)$ where V is a valuation operator. The second component is the value of payments to debtholders, which is a function of D and u (through R) only, and is denoted by $V_2\,(D, u)$. As before, debt is fully secured and R adjusts accordingly. Clearly, for risk-averse investors, promised R must be greater than for risk-neutral investors, but its main properties do not change. The third component is the value of the government claim (i.e., tax payments), denoted by V_3 and defined as $V_3 \equiv t \cdot w(V_1)$. w is a call option on V_1, the value of the firm's operating cash flows. The value of w at expiration is given by Equation (25):

$$w = \max[V_1 - (\beta z + RD);\, 0] \tag{25}$$

where $(\beta z + RD)$ is the option's exercise price. Equity value, S, is obtained by subtracting the value of debt and the value of the government claim from the first component, the value of operating cash flows. We must also deduct the initial investment, βz. Thus,

$$S = V_1 - V_2 - V_3 - \beta z(1 + R_F) \tag{26}$$

We can show that our main results, obtained under risk-neutral conditions, hold for such a general formulation as well.

Since V_1 is a function of z alone, V_2 is a function of D alone, and only V_3 is a function of both, first-order optimality conditions can be written in a most general form as follows:

$$\frac{\partial S}{\partial z} = \frac{\partial V_1}{\partial z} - t\,\frac{\partial w}{\partial z} - \beta(1 + R_F) = 0 \tag{27}$$

and

$$\frac{\partial S}{\partial D} = \frac{\partial V_2}{\partial D} - t\,\frac{\partial w}{\partial D} = 0 \tag{28}$$

In this general formulation, it can be seen that interaction between z and D occurs through the joint determination of the value of the government option, w. As we shall presently see, the main thrust of our argument remains, but to show that, we must rederive some key relationships. First, we show that $\frac{\partial^2 S}{\partial z \partial D} < 0$. This, coupled with second-order conditions (analogous to Equation (17)) will enable us to prove Proposition 1 and its corollaries.

Denote the government option's exercise price by E. It is easy to see that

$$\frac{\partial^2 S}{\partial z \partial D} = -t\beta\,\frac{\partial^2 w}{\partial E^2}\cdot\frac{\partial (RD)}{\partial D}. \tag{29}$$

By Merton's [19] Theorem 4, $\frac{\partial^2 w}{\partial E^2} > 0$. $\frac{\partial (RD)}{\partial D} > 0$ since as D increases, the promised interest rate must increase as well. It follows that $\frac{\partial^2 S}{\partial z \partial D} < 0$. In such a general formulation $\frac{\partial^2 S}{\partial z^2}$ cannot be exactly specified, but if a maximum exists it must be negative. Hence, by Equation (17), Proposition 1 and its corollaries still hold, pointing out the negative interrelationship between optimal investment and leverage.

We now turn to comparative statics. Here matters are complicated somewhat, mainly because z also affects the variance of $V_1(z, u)$. To prove Proposition 2, one must essentially rederive Equations (21) and (22). We first compute $\frac{\partial^2 S}{\partial t \partial D}$.

$$\frac{\partial^2 S}{\partial t \partial D} = -\frac{\partial w}{\partial D} = -\frac{\partial w}{\partial E}\cdot\frac{\partial E}{\partial D} \tag{30}$$

Since $\frac{\partial w}{\partial E} < 0$ and $\frac{\partial E}{\partial D} > 0$ (see Equation (29)), it follows that $\frac{\partial^2 S}{\partial t \partial D} > 0$. This is similar to the result obtained in Equation (22). We now turn to the second cross-derivative:

$$\frac{\partial^2 S}{\partial t \partial z} = -\frac{\partial w}{\partial z} = -\left[\frac{\partial w}{\partial V_1}\cdot\frac{\partial V_1}{\partial z} + \frac{\partial w}{\partial \sigma^2}\cdot\frac{\partial \sigma^2}{\partial z} + \frac{\partial w}{\partial E}\,\beta\right] \tag{31}$$

where σ^2 is the (instantaneous) variance of V_1. From certain properties of options, it can be demonstrated that one would expect $\frac{\partial w}{\partial z}$ to be greater than zero at the optimal z.[14] It follows that $\frac{\partial^2 S}{\partial t \partial z} = -\frac{\partial w}{\partial z} < 0$, which is similar to the result obtained in Equation (21).

The above results, coupled with our earlier derivation of $\frac{\partial^2 S}{\partial D \partial z}$ and second-order conditions, prove Proposition 2 in a nonrisk-neutral framework as well. One must note that there is no intuitive reason to expect otherwise. Proposition 3, tentative as it is in the original risk-neutral formulation, remains tentative in our present framework as well, so we shall not specify it further.

III. Conclusions

This paper has sought to demonstrate that since cash flows to the firm are affected, and to a large extent determined, by both production and financial decisions, firm value maximization must result from a simultaneous determination of production level and capital structure. Our results indicate that an inverse relationship must exist between financial leverage and optimal capacity expansion, analogous to an inverse relationship between financial and operating leverage as supported by empirical evidence. The paper can be viewed as extending the DeAngelo and Masulis [4] work inasmuch as we endogenize the nondebt tax shield. We extend Hite's [10] paper in the inclusion of nonsolvent states of nature which reverse his conclusions about the link between real and financial variables. We also derive the impacts of changes in two exogenous variables—the tax rate and the expected price—on optimal z and optimal D. The introduction of nonrisk-neutral valuation in the final section leaves our major results intact.

The model presented is a one-period model. Extensions could include explicit consideration of personal taxes (see DeAngelo and Masulis [4]). A multiperiod extension could be attempted, although complex issues, raised by such construction even under relatively straightforward assumptions about the cash flow generating process (see multiperiod CAPM), will obviously be compounded in our case.

[14] The proof is as follows:

$$\frac{\partial^2 S}{\partial t \partial z} = -\frac{\partial w}{\partial z}$$

$$\frac{\partial w}{\partial z} = \frac{\partial w}{\partial V_1} \cdot \frac{\partial V_1}{\partial z} + \frac{\partial w}{\partial \sigma^2} \frac{\partial \sigma^2}{\partial z} + \frac{\partial w}{\partial E} \cdot \beta$$

From first-order conditions, at optimal z,

$$\frac{\partial w}{\partial z}\left[1 - t\frac{\partial w}{\partial V_1}\right] = \beta\left[\frac{\partial w}{\partial V_1}(1 + R_F) + \frac{\partial w}{\partial E}\right] + \frac{\partial w}{\partial \sigma^2} \cdot \frac{\partial \sigma^2}{\partial z}$$

Since $0 < \frac{\partial w}{\partial V_1} < 1$; $-1 < \frac{\partial w}{\partial E} < 0$ and also $\frac{\partial w}{\partial \sigma^2} \cdot \frac{\partial \sigma^2}{\partial z} > 0$, one would expect $\frac{\partial w}{\partial z} > 0$ (see Smith [29], Galai and Masulis [7]) as required. For the *B-S* model, for instance, this is strictly so since it was demonstrated (*ibid*) that $\left|\frac{\partial w}{\partial V_1}\right| > \left|\frac{\partial w}{\partial E}\right|$.

Finally, one could also use a different representation of the production process (a Leland-Sandmo type model), and this could provide further insight into the interaction of factors utilization and financial decisions.

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