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Empirical Tests of Boundary Conditions for Toronto Stock Exchange Options

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ABSTRACT

Using option and stock transaction data for the period 1978–1979, three issues were investigated: first, the conformance of observed prices to various boundary conditions; second, the evolution of the market over time, as the volume of trading and the number of listed options increased; and third, to test the efficiency of the market. It was found that violations did occur. Using a trading rule based on the signal of observed violations, the results suggest that even after transaction costs the market was inefficient over the sample period.

USING SIMPLE DOMINANCE ARGUMENTS, Merton [4], Smith [6], and Galai [2] demonstrated that under very general conditions, the price of a call option must lie within certain upper and lower bounds.[1] Observations that the market price of an option did not fall within these bounds would indicate the possible existence of arbitrage opportunities.

The purpose of this paper is to examine if the prices of call options traded on the TSE obey boundary conditions which would eliminate arbitrage profits. Galai [2] performed a similar study for options traded on the CBOE, using primarily closing price data and some transaction data for a three-month period. Results of the study indicated that it was possible to earn abnormal rates of return. In a study using U.S. transactions data for options traded on the CBOE for the period August 1976 to June 1977, Bhattacharya [1] found that when transaction costs are taken into account, profits due to market mispricing disappear.

This study differs from the Galai and Bhattacharya studies in three ways. First, it uses Canadian transaction data over a two-year interval covering all of the recorded option trades on the TSE. The existence of transaction data over an extended period facilitates examination of any changes in the nature of the market over time as investors possibly become more knowledgeable about the market and as the volume of trading increases the depth of the market. Second,

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[1] See Merton [4], Smith [6], and Galai [2] for a statement and proofs of these upper and lower bounds.

the study investigates the impact, if any, of expected dividends on the frequency and size of price violations. Third, it examines whether violations tend to occur more frequency and/or with greater magnitude for in-the-money options than for out-of-the-money options. A distinction is also drawn between in-the-money and "deep"-in-the-money options.

Section I of the paper describes the structure of the market and the data used. Section II defines the appropriate lower bounds for call option prices and derives the three types of violations which will be tested. The empirical results of these tests are contained in Section II. The effects of various option characteristics, such as maturity, dividends and whether the options are in or out of the money, or "deep" in the money, on the occurrence of violations are tested. Several of these factors have a significant impact on the frequency of violations.

Section IV includes the results of two statistical tests regarding the changing nature of the TSE options market over the sample period. It is found that both the probability and the magnitude of violations have tended to increase over time. Section V examines the issue of market efficiency using two different trading rules to determine whether observed violations could be used as signals for the construction of portfolios which would earn abnormal returns. The results indicate the presence of market inefficiencies which are not eliminated by transaction costs.

I. Structure of the Market

Options may be purchased or sold on either the Toronto Stock Exchange (TSE) or the Montreal Stock Exchange (MSE), depending upon the exchange on which the underlying security is listed. There are a number of securities which are interlisted and option trading on these securities can occur on either exchange.[2] For interlisted securities and their options, the bulk of trading occurs on the TSE. The clearing and settling of options transactions occur through a clearing member of TransCanada Options, Inc. (TCO), the clearing organization.

Underlying the liquidity of the TSE options market is the market maker,[3] who is associated with a member firm of the stock exchange and who has been approved by the TSE staff to make a market in the options of one or more underlying securities. By trading for the account of the member firm with which he or she is associated, the market maker is responsible for providing liquidity and promoting fair and orderly markets. To this end, the market maker has the right to initiate transactions in the option market without payment of brokerage

[2] As of the end of the first quarter of 1980, 14 underlying securities were listed on the TSE, 11 on the MSE, and 21 were interlisted. The bulk of trading on underlying securities which are interlisted and their options is on the TSE.

[3] Typically, the market maker can have a principal appointment in the options of a minimum of two and a maximum of three underlying securities. These restrictions have been relaxed under certain situations. For example, market makers who have had no previous experience may limit their participation to the options of one underlying security for the first three months of their appointment. Second, there is a "grandfather" clause that permits market makers who had in excess of three principal appointments prior to the adoption of new rules for market makers in 1981 to maintain these appointments.

fees. However, there are two sources of direct transactions costs incurred by the market maker—clearing charges and a floor charge. In the options market, the clearing charge is 25¢ per contract (of 100 shares) for exercised options and 10¢ per contract for all other transactions. The floor charge is 10¢ per contract for all option transactions.

In meeting their market-making responsibilities, the market makers enter the equity market for transactions through their associated firm. The transactions costs faced are equivalent to those paid by other professional traders in the equity market: a 20¢ fee per fill (in the market maker's case, 100 shares) and a fee of 1/80 of 1% of the value of the equity shares traded. The costs of transacting in either the equity or the option market for the market maker are thus minimal.

Phillips and Smith [5] argue that any study of market efficiency must include all costs associated with trading, including the indirect trading cost associated with the bid-ask spread in the options and equity markets. We return to the trading cost argument in the empirical section of the paper. It is important to recognize that the operations of the TSE equity market are very different from those in the U.S. exchanges. The former is a pure agency market where there are no specialists standing ready to transact at the bid and ask prices and provide liquidity services on an ongoing basis. The observed bid-ask spreads in equities on the TSE is, therefore, a biased estimate of indirect trading costs (and hence the cost of maintaining liquidity) since actual transactions will be within the stated bid-ask spread.

The allocation of market maker appointments is performed by the TSE staff and entry does not require the purchase of a seat. The organizational structure of the market is very different from that found in the CBOE, where market makers must purchase a seat. In the TSE options market, seat prices exist only for the member firm and their values will reflect rents obtainable from all of the activities in which the member is engaged. These include underwriting, equity, and options trading (the latter perhaps as a market maker).[4]

Consistent with the CBOE, the options traded are standardized with respect to maturity dates and exercise prices. There are three expiry cycle dates—January, February, and March. Trading in options of a particular expiration month normally commences approximately nine months earlier. Exercise prices are generally set at $2.50 intervals for underlying securities trading at or below $35 and at $5 intervals for securities trading above $35.

A. Data Description

The option data used in this study are taken from the Toronto Option Tape (TOT), compiled by Turnbull and Halpern,[5] which records every option transaction on the TSE from January 3, 1978 to December 31, 1979. For each option transaction, the transaction price, the trading volume, and the stamp time are

[4] Phillips and Smith [5] argue that when the opportunity cost of the seat is included as a cost in the measurement of economic income, market makers will earn only a normal rate of return. If markets are efficient, this conclusion appears to be impossible to contradict since it is the result of the definition of economic income.

[5] For a more detailed description see Turnbull and Halpern [9].

recorded. Also the nearest stock price transaction at or before the option trade is available, with the stock price, the trading volume, and the stock stamp time.

The TOT has been compiled from the TSE computer tapes on which all daily option and stock trades are recorded. Any errors on the TOT reflect errors made in the original recording of the data at the TSE.

The sample was divided into two groups in an attempt to isolate the possible impact of the variability of the exchange rate on the existence of violations. The first group included options on nondividend paying stocks[6] and all options on stocks which paid dividends in Canadian dollars during the two-year sample periods;[7] the second group included options on stocks which paid dividends during the sample period in U.S. dollars. The first group is referred to as the Canadian dollar sample. The size of this group underwent rapid growth during the two-year sample period. In January 1978, options could be traded on 7 stocks and by December 1979, options could be traded on 36 stocks. The second group is composed of four companies: Alcan, Brascan, Inco, and Moore Corporation. The options of three of these companies were traded over the whole of the sample period. Brascan options started to trade in June 1978. Since the results for the two groups are similar, the results for the U.S. dollar dividend-paying stocks are not reported. In rare instances where differences do occur, they are noted in the text.[8]

Many of the options and underlying stocks traded on the TSE are also traded on the MSE. Trades that occur on the MSE are not recorded on the TOT. Up to September 1, 1979, an option on the TSE was not allowed to trade until there had been a stock trade on either the TSE or, if interlisted, on the MSE. From September 1, 1979 if the stock was listed on an exchange in the U.S. and a trade occurred on one of these exchanges before any stock traded on the TSE or MSE, option trading for that stock could start. In the recording of the first option trade, if the stock trade occurred on an exchange other than the TSE, a special identification code was used to replace the unobservable stock price.

In general, if a stock trades on the TSE and other exchanges, the problem of nonsimultaneity of the recording of stock and option prices on the TOT is exacerbated. The option trader on the floor of the TSE will be aware of the prices on the MSE, as these prices are displayed in the exchange. Although prices on the American exchanges are not displayed, traders can contact their member firms for this information. This implies that option trading on the TSE may be in response to stock trading on the other exchanges. For stock transactions in 1978, 86% of the total dollar value and 89% of the total volume for both the TSE and MSE occurred on the TSE. For 1979, the percentages were 87 and 89%, respectively. The problems caused by trading on the MSE are unlikely to be serious, given the dominance of the TSE. However, for the American exchanges, this may present a problem in the last four months of the sample period, during

[6] In our sample, only two securities did not pay dividends—Range Oil and Dome Petroleum. Options on the former began trading in June 1979 whereas options on the latter traded over the entire sample period.

[7] Seagram's, one of the options in the sample, is classified as a Canadian dividend paying company. In 1980, it started to pay U.S. dollar dividends.

[8] The results for the U.S. dollar dividend stocks are available from the authors on request.

which time option trading could begin after a stock trade on an American exchange had taken place.

Dividend data for the relevant stocks were collected from the *TSE Review* and the *Financial Post Dividend Record.* Interest rate data were gathered from the *Bank of Canada Review.* For 91- and 182-day maturities, weekly interest rates were collected; for options with maturities greater than 182, a one-year treasury bill rate was used. Monthly quotes were available for this maturity. Exchange rate data were gathered from the *Globe and Mail.*

II. Lower Bounds for the Pricing of Call Options

Definitions

The first lower bound for an American call option is that its value cannot be less than its immediate value if exercised; i.e.,

$$C(S;\ T,\ X) \geq \text{Max}[0,\ S - X], \tag{1}$$

where $C(S;\ T,\ X)$ is the market value of the American call option with exercise price X and time to maturity T; S denotes the current value of the underlying stock. It is assumed that there are no transaction costs.

If, over the life of the option, it is known with certainty that the stock will pay dividends d_j at calendar time t_j, $j = 1, \cdots, n$ then

$$C(S;\ T,\ X) \geq \text{Max}[0,\ S - XB(T) - \textstyle\sum_{j=1}^{n} d_j B(T_j)], \tag{2}$$

where $B(T_j)$ is the present value of a riskless pure discount bond paying one dollar with maturity of T_j periods; and $T_j \equiv t_j - t$, t denoting the current time. Galai [2] refers to the above condition as the strong European call dominance condition.

For an option written on a stock which is expected to pay dividends over the life of the option, a lower bound which is at least as strict as (2) can be obtained:

$$\begin{aligned} C(S;\ T,\ X) \geq \text{Max}\ \{0,\ &\underset{i}{\text{Max}}\ [S - XB(T_i) - \textstyle\sum_{j<i} d_j B(T_j)], \\ &(S - XB(T) - \textstyle\sum_{j=i}^{n} d_j B(T_j))\}. \end{aligned} \tag{3}$$

This bound examines whether it is optimal to exercise the option before the stock goes ex-dividend. Galai [2] refers to the above condition as the strong dominance condition.[9]

The above conditions will be subject to empirical tests. The first type of deviation is

$$\varepsilon_1 \equiv S - X - C(S;\ T,\ X). \tag{4}$$

From (1) if the option is priced rationally, then ε_1 should be nonpositive. The second type of deviation is

$$\varepsilon_2 \equiv S - XB(T) - \textstyle\sum_{j=1}^{n} d_j B(T_j) - C(S;\ T,\ X), \tag{5}$$

[9] If dividends are stochastic, then similar forms of inequalities can be derived (see Chapter 5 of Jarrow and Rudd [3]).

and the third type of deviation is

$$\varepsilon_3 \equiv \underset{i}{\text{Max}} \{S - XB(T_i) - \textstyle\sum_{j<i} d_j B(T_j)\} - C(S;\, T,\, X). \tag{6}$$

From (2) and (3), respectively, if the option is priced rationally, then ε_2 and ε_3 should be nonpositive. If any of the ε_i, $i = 1, 2, 3$ are positive, then there is said to be a type i violation.

It should be emphasized that the tests proposed here are not valid for testing market efficiency.[10] As Galai points out, these tests can only indicate whether the option and stock markets are not synchronized or are not continuously in equilibrium. Given the nonsimultaneity problems due to the trading of the underlying security on other exchanges, the results of these tests must be interpreted with caution.

B. *The Bid-Ask Spread and Transaction Data*

The above boundary conditions use equilibrium prices, which may differ from actual transactions prices. Bhattacharya, following Phillips and Smith, argues that financial assets are generally bought at the ask price and sold at the bid price. To the extent that the option transaction price observed was initiated as a sell order by a customer and executed at the bid price, a violation of a boundary condition may be observed when, in fact, there is none if the transaction were at the equilibrium price. As the transactions data does not identify whether the recorded transaction was at the bid or ask price, the magnitude of the bias cannot be determined *a priori.*[11] Also, transactions do not necessarily occur at the extreme of the bid-ask spread.[12]

III. Empirical Results

A. *The Effects of Option Maturity*

In Table I (top portion), the total number of violations by type and days to maturity are presented, along with the average dollar violation per option.

[10] See Galai [2], p. 194.

[11] Let C_E represent the equilibrium value of the option, and C_B and C_A the bid and ask prices, respectively. Define $C_B \equiv C_E - \Delta_{CB}$, $C_A \equiv C_E + \Delta_{CA}$, where Δ_{CB}, $\Delta_{CA} \geq 0$ and is a measure of the divergence from the equilibrium price. A similar notation is used for the stock price S. If options and securities are traded at equilibrium prices, the relationship for no type 1 violation is $C_E \geq S_E - X$. However, when transactions occur at prices different from their equilibrium values, the existence of a false violation and its magnitude will depend upon which of the following four possible combinations occur:

$$C_A \geq (S_A - X) - (\Delta_{SA} - \Delta_{CA}), \quad \text{overestimation occurs if} \quad \Delta_{SA} - \Delta_{CA} > 0;$$

$$C_A \geq (S_B - X) + (\Delta_{SB} + \Delta_{CA}), \quad \text{underestimation occurs;}$$

$$C_B \geq (S_A - X) - (\Delta_{SA} + \Delta_{CB}), \quad \text{overestimation occurs;}$$

and

$$C_B \geq (S_B - X) + (\Delta_{SB} - \Delta_{CB}), \quad \text{underestimation occurs, if} \quad \Delta_{SB} - \Delta_{CB} > 0.$$

[12] This fact was confirmed in discussions with a number of professional traders.

Table I

The Effects of Option Maturity

	Violation								
	Type 1			Type 2			Type 3		
Maturity (days)	No.	%	Average $ of Violation	No.	%	Average $ of Violation	No.	%	Average $ of Violation
0–7	4,696	37	0.23	5,814	18	0.20	179	2	0.16
8–30	3,113	24	0.29	5,790	17	0.29	1,504	13	0.17
31–60	3,174	25	0.37	10,048	31	0.37	2,912	25	0.27
>60	1,803	14	0.72	11,337	34	0.49	6,849	60	0.35
Total	12,786			32,989			11,444		

	Maturity Category (days)									
	0–7		0–30		31–60		>60		Total	
Dollar Amount of Violation	No.	%	No.	%	No.	%	No.	%	No.	%
0.01–0.50	10,129	95	9,327	90	13,126	81	15,318	76	47,900	84
0.51–1.00	456	4	832	8	2,253	14	2,897	15	6,438	11
>2.00	104	1	248	2	755	5	1,774	9	2,881	5
Total	10,689		10,407		16,134		19,989		57,219	

There is a positive relationship between the number of days left to maturity and the average size of the violation for each type of violation. It is also observed that 37% of all type 1 violations occur in the 0- to 7-day maturity category, and that 86% of violations occur when the maturity of the option is 60 days or less. This negative relationship between option maturity and the frequency of violations is not found for type 2 or 3 violations.

Of the 57,219 violations observed, 58% occur in the type 2 category. The nature of the test is such that it will detect more type 2 than type 1 violations, particularly for long maturity options.[13] In addition, the requirement that type 3 violations include only options which are expected to pay dividends over their remaining lives reduces the number of type 3 violations. This is especially true in the short maturity option categories. It is observed that 85% of the type 3 violations occur with options which mature in greater than 30 days.

The average magnitude of the type 1 and 2 violations are similar to those reported by Galai for CBOE options. However, for type 3 violations the observed average of $30 per contract is significantly lower than the $41.70 reported by Galai.

In Table I (bottom portion), it is seen that as the maturity of the option increases, the frequency of the dollar size of violations shifts to the right. For options with maturity between zero and seven days, 95% of the violations are

[13] The type 2 violation can be written in terms of the type 1 violation:

$$\varepsilon_2 = \varepsilon_1 + X[1 - B(T)] - \textstyle\sum_j d_j B(T).$$

As it might be expected that $X[1 - B(T)] - \sum_j d_j B(T_j)$ is more often positive than negative, then there will tend to be more type 2 violations than type 1. This is more likely to be the case for long maturity options.

not greater than 50¢. For options with maturity greater than 60 days, 76% of the violations are not greater than 50¢.

B. In- and Out-of-the-Money Options

The propensity for violations to occur when the option is in or out of the money was examined.[14] By definition, type 1 violations cannot occur when the option is selling out of the money.[15] This is not the case for type 2 and 3 violations. For the Canadian dollar sample, there was a total of 315,202 option transactions. Of these, 137,180 occurred when the option was out of the money.[16] There were 32,991 type 2 violations with only seven of the violations occurring on options which were out of the money. All seven violations occurred on options with maturities greater than 60 days.[17] There were 11,444 type 3 violations and none occurred on out-of-the-money options. Given the low frequency of violations for out-of-the-money options, they are not reported as a separate group.

The issue of whether violations occur more frequently for options that are deep in the money compared to in the money was also examined. As there is no natural definition of deep in the money, it was arbitrarily decided to define an option as deep in the money if the stock price was at least 20% above the exercise price and in the money if the stock price was less than 20% above the exercise price. This distinction has the effect of dividing the sample into two groups of almost equal size. Fifty-one percent or 28,961 of the violations are defined as occurring for deep-in-the-money options while 49% are defined as being in the money.

Table II divides the sample by type of violation, number of days to maturity, and the in-the-money category.[18] For type 1 and 2 violations and options with maturities greater than 7 days, violations greater than 50¢ occur more frequently for deep-in-the-money options. For type 3 violations, it is only for options with a maturity of over 60 days that violations greater than 50¢ occur more frequently for deep-in-the-money options. However, it is observed that for all types of

[14] An option is classified as in (out) of the money if the stock price is greater (less) than the exercise or strike price.

[15] From the definition, $\varepsilon_1 \equiv S - X - C(S;\, T, X)$. If the option is out of the money, $S - X < 0$, so that $\varepsilon_1 < 0$.

[16] As the check on the accuracy of the data, all type 1 violations over \$3 were examined. An observation was deleted if there was an obvious keypunch error. A typical case would be the stock price at \$20, the exercise price at \$10, and the option price recorded as \$1. A total of 193 observations were deleted.

[17] The low frequency of violations for out-of-the-money options is not surprising. For a type 2 violation to occur,

$$S - XB(T) - \textstyle\sum_{j=1}^{n} d_J B(T_j) > C$$

which can be written in the form

$$X[1 - B(T)] - \textstyle\sum_{j=1}^{n} d_j B(T_j) > C - (S - X).$$

If the option is out of the money, then $S - X < 0$, so that the right-hand side of the above inequality can be written $C + |S - X|$ for $S < X$. In this case, it is unlikely that the above inequality will occur except for options of long maturity written on stocks with zero or small dividends.

[18] The dollar violations per contract is obtained by multiplying the per share violation by 100, the number of shares covered in an option contract.

Table II

Number of Violations (by Type of Violation), Dollar Amount of Violation, Number of Days to Maturity, and In-the-Money Category

Dollar Amount of Violation	0–7		8–30		31–60		Greater Than 60 Days	
	DI	I	DI	I	DI	I	DI	I
Violation Category: Type 1								
0.01–0.50	973	3,526	1,407	1,513	2,048	735	1,045	302
0.51–1.00	67	91	97	35	230	43	87	54
>2.00	21	18	37	24	59	59	176	139
Total	1,061	3,635	1,541	1,572	2,337	837	1,308	495
Grand Total	4,696		3,113		3,174		1,803	
Violation Category: Type 2								
0.01–0.50	1,132	4,325	2,022	2,942	4,133	3,636	4,760	3,425
0.51–1.00	128	164	421	237	1,356	343	1,549	470
>2.00	28	37	87	81	475	105	862	271
Total	1,288	4,526	2,530	3,260	5,964	4,084	7,171	4,166
Grand Total	5,814		5,790		10,048		11,337	
Violation Category: Type 3								
0.01–0.50	30	143	361	1,082	1,153	1,421	3,312	2,474
0.51–1.00	4	2	19	23	158	123	528	209
>2.00	0	0	6	13	23	34	167	159
Total	34	145	386	1,118	1,334	1,578	4,007	2,842
Grand Total	179		1,504		2,912		6,849	

Notes: DI denotes deep in the money. I denotes in the money.

violations, the average magnitude is greater for deep-in-the-money options than for in-the-money options.[19]

C. *The Effects of Dividends*

In Table III, the number of violations is related to the number of dividends to be paid over the life of the option and the maturity of the option. A similar pattern is observed for type 1 as for 2 violations. The largest proportion of violations is found for options which have no dividends remaining. The overall

[19] The following table is constructed by aggregating across maturities in Table II.

Type of Violation	In the Money: Number	In the Money: Average Dollar Size	Deep in the Money: Number	Deep in the Money: Average Dollar Size
1	6,539	0.27	6,247	0.42
2	16,036	0.27	16,953	0.46
3	5,683	0.26	5,761	0.35

Table III

Number of Violations Based on Number of Dividends Left Before Exercise Date

Maturity (days)	Violation											
	Type 1				Type 2				Type 3			
	No. of Dividends				No. of Dividends				No. of Dividends			
	0	1	2	3	0	1	2	3	0	1	2	3
0–7	4,572	124	0	0	5,795	19	0	0	0	179	0	0
8–15	2,318	795	0	0	5,350	4,400	0	0		1,504	0	0
31–60	2,284	890	0	0	8,460	1,588	0	0		2,912	0	0
>60	766	886	132	19	5,185	5,113	967	72		5,452	1,303	94
Total	9,940	2,695	132	19	24,790	7,160	967	72		10,047	1,303	94

average proportion for type 1 violation with no dividends remaining is 78%, and for type 2 violations, 75%. For all three violation types and for options with maturity of less than 60 days, no violations were observed when the underlying stock had in excess of one dividend expected before the maturity of the option contract. This is to be expected since dividends are paid quarterly, or in some cases semiannually, and it would be highly unusual to have two or more dividends expected for an option with maturity of less than 60 days. By construction, the type 3 violation only considers options which have at least one dividend remaining. The presence of more than one dividend will affect only the long maturity options; 80% of the violations occur on options with a maturity of greater than 60 days.

Table IV shows the influence of the number of dividends remaining on the dollar size of the violation. Considering first the grand average values, for type 1 violations, there is a positive relationship between the number of dividends remaining and the average size of the violation. For type 2 violations, comparing the average sizes of the violations when there are no dividends and one dividend expected, it is observed that the presence of the dividend lowers the average size of the violation.

For the overall average results, for either deep-in-the-money or in-the-money categories, the positive relationship between number of dividends remaining and dollar violation exists only if the zero dividend case is excluded.

Considering the overall averages for all violation types, apart from two exceptions, the size of the violation is greater for deep-in-the-money than for in-the-money options. The two exceptions occur for type 2 and 3 violations with two dividends remaining.

D. Conclusions

The violation definitions—see equations (3), (4), and (5)—assume all relevant data are recorded at the same point in time. Given the absence of continuous trading in stock and option markets, any empirical test is faced with the problem of nonsimultaneity. Therefore, the results of this section must be interpreted with caution.

Table IV

Dollar Value of Average Violation per Share Classified by Maturity, Number of Dividends Left Until Maturity, and In-the-Money Category

Maturity Category (days)	No. of Dividends Left 0		1		2		3	
	DI	I	DI	I	DI	I	DI	I
				Type 1				
0–7	$0.32	$0.20	$0.34	$0.18	$0	$0	$0	$0
8–31	0.37	0.23	0.28	0.20	0	0	0	0
31–60	0.39	0.41	0.32	0.30	0	0	0	0
>60	0.72	1.08	0.52	0.56	1.72	1.06	1.88	1.58
Overall Average	$0.41	$0.25	$0.40	$0.30	$1.72	$1.06	$1.88	$1.58
Grand Average	$0.33		$0.36		$1.37		$1.69	
				Type 2				
0–7	$0.28	$0.18	$0.16	$0.06	$0	$0	$0	$0
8–30	0.37	0.24	0.21	0.17	0	0	0	0
31–60	0.48	0.28	0.23	0.22	0	0	0	0
>60	0.71	0.44	0.39	0.28	0.45	0.49	0.90	0.88
Overall Average	$0.50	$0.26	$0.36	$0.26	$0.45	$0.49	$0.90	$0.88
Grand Average	$0.38		$0.31		$0.47		$0.89	
				Type 3				
0–7	NA		$0.25	$0.14	$0	$0	$0	$0
8–30	NA		0.22	0.16	0	0	0	0
31–60	NA		0.29	0.24	0	0	0	0
>60	NA		0.37	0.28	0.39	0.42	0.97	0.68
Overall Average			$0.34	$0.23	$0.39	$0.42	$0.97	$0.68
Grand Average			$0.29		$0.40		$0.76	

Notes: NA = not applicable due to design of the test. DI denotes deep in the money. I denotes in the money.

Some general observations for all violation types can be made. Violations rarely occur when the option is out of the money. There does not appear to be a relationship between the number of days to maturity on the option and the number of violations. Galai noted that this relationship was negative. For our sample, this relationship weakly holds for type 1 violations but not for type 2 or type 3 violations.

Considering the average size of the violation, there are three geneal observations that can be made. First, there tends to be a positive relationship between the maturity of an option and the average dollar size of the violations. Second, the size of the violation tends to be greater for deep-in-the-money than for in-the-money options. This can be explained by the fact that deep-in-the-money option prices tend to follow the price of the underlying stock very closely. Galai suggests that under these circumstances, it is difficult to maintain the option and stock markets in continuous equilibrium or even within boundary conditions. Third, the average size of violation tends to increase with the number of dividends expected to be paid before the exercise date.

IV. The Nature of the Market over Time

Given the rapid growth in the volume of trading over the two-year period, it might be expected that the nature of the market changed as investors and traders became more familiar with the pricing of options and as the market gained more liquidity. However, the increased trading volume was due not only to increased transactions in the options of securities which were available for trading during 1978 but also to a dramatic increase during 1979 in the number of securities on which options were written.

The first hypothesis to be tested examines whether the probability of observing a violation changed over time. If investors have become more knowledgeable about the pricing of options, then, *ceteris paribus*, it might be expected that the probability of a violation occurring will have decreased over time. However, as noted, other things did not remain constant since the number of underlying securities on which options could be traded increased. Thus, both liquidity through the market making function and the ability to analyze and price options may actually have decreased with the rapid growth in optionable securities. The null hypothesis is that there is no change, and the alternative hypothesis is that the probability has changed.

The sample is split into two one-year intervals and the probability of a violation occurring is estimated by the frequency of a given type of violation to the total number of transactions during the interval. Assuming that violations of a given type are independent events, a statistic for testing the equality of the probabilities for two Bernoulli random variables is constructed; the statistic is asymptotically normally distributed. The results are shown in Table V where it is seen that the null hypothesis, that the probability of a violation has not changed over the two-year period, can be rejected. The results indicate that the probability of a violation has increased over time.[20]

The second hypothesis concerns the magnitude of the violations. Irrespective of whether the probability of a violation occurring has changed, the mean magnitude of the violations of a given type might have changed over the two-year period. The null hypothesis is that the mean value has not changed, and this is tested against the alternative hypothesis that the mean has changed. The test statistic is described by a t-distribution which, given the large number of observations, is asymptotically normally distributed.[21] The results are shown in Table VI. The null hypothesis can be rejected for type 1 and 3 violations, and

[20] The results for the U.S. dollar dividend paying stocks are quite different; only for the type 2 violation can the null hypothesis be rejected at the 95% confidence level. Again, it is observed that the probability of a violation has increased over the two-year period.

[21] If $X_1 \sim N(\mu_1, \sigma_1^2)$, $X_2 \sim N(\mu_2, \sigma_2^2)$, variances are unknown, and samples are independent, then $[(\bar{X}_2 - \bar{X}_1) - (\mu_2 - \mu_1)]/[S_1^2/n_1 + S_2^2/n_2]^{1/2} \sim t$ with degrees of freedom equal to minimum $[n_1 - 1, n_2 - 2]$, where $S^2 = \sum_{j=1}^{n} (X_j - \bar{X})^2/(n - 1)$, $\bar{X}$ is the estimated sample mean, and n is the number of observations in the sample.

Table V

Frequency of Violations Test

Year	Violation			No. of Transactions
	Type 1	Type 2	Type 3	
1978	0.035079	0.087309	0.023072	66,877
1979	0.042042	0.109341	0.039871	248,325
Z-statistic*	8.52	17.5	23.9	

* Z is a zero-mean unit variance asymptotically normally distributed random variable.

Table VI

Mean Magnitude of Violations Test (per one share of equity)

Year	Violation		
	Type 1	Type 2	Type 3
1978	0.293374	0.374970	0.265746
	(0.173415)	(0.225153)	(0.171961)
	(2,346)	(5,839)	(1,543)
1979	0.359255	0.363438	0.308319
	(0.520576)	(0.380214)	(0.260656)
	(10,440)	(27,152)	(9,901)
t-statistic	5.921	1.591	2.627

Note: The numbers in parentheses are the estimated variance and the number of observations, respectively.

the results show that the mean magnitude of the violations has increased over time.[22] This is not the case for the type 2 violation.[23]

The results suggest that over the two-year period, both the frequency of violations and their magnitude have tended to increase. The number of securities on which options could be traded and the volume of option trading per security underwent rapid growth during the two-year period. In January, 1978, options could be traded on 7 stocks; in December, 1979, options could be traded on 36 stocks. The results are consistent with the inability of the market to adjust to such changes.[24]

[22] A crude test of the robustness of these results to the distributional assumption was performed by taking the natural logarithm of each violation and recalculating the test statistic. The conclusions remain unchanged.

[23] For the U.S. dollar dividend-paying stocks, the null hypothesis cannot be rejected for any type of violation.

[24] In comparing the results of the U.S. and Canadian dollar sample, it must be remembered that, for the former, there were three underlying securities until August 1978; in June 1978, Brascan was added to the list of underlying securities paying U.S. dollar dividends. Although this increase in the number of securities should not be discounted, it is not as dramatic as that for the Canadian dollar sample. Therefore, the U.S. dollar sample provides the closest approximation to the ceteris paribus assumption. Since the U.S. dollar sample displays no significant changes over the two years even with increased trading activity, the results for the Canadian dividend sample are consistent with the inability of the market to adjust to the very large increase in volume in 1979 associated with the trading of options on underlying securities existing in 1978 and also the new option contracts based on new underlying securities.

V. Market Efficiency: An Ex Ante Test

The observation at time t of a violation implies that if an investor could enter the market and transact at the option and stock prices that generated the violation, then, ignoring transaction costs, it would be possible to form a portfolio with zero initial wealth which would earn nonnegative and, in some states, positive rates of return. This implies that the options market is inefficient since investors are able to consistently earn above normal profits for the level of risk taken. Normally, investors are unable to trade at the prices observed at the time of the violation. Therefore, the violation will be used as a signal. The profitability of a trading strategy implemented upon the observation of the signal is investigated. Since there is no guarantee that the prices used in the strategy are equal to the prices which led to the signal, there is no guarantee of receiving nonnegative returns.

A. Methodology and Results

The occurrence of a type 2 violation is used as a signal to implement a trading rule. Upon observing a violation at time t, the investor enters the option and stock markets at $t + 1$ and constructs a portfolio with a short position in the stock and an investment of the proceeds of the short sale in: (i) treasury bills with a face value at maturity of X; (ii) treasury bills with a face value d_j at each T_j; and (iii) the call option. The investor's net proceeds from the portfolio at $t + 1$ is $\Delta(t + 1)$ which is defined by

$$\Delta(t + 1) \equiv -[S(t + 1) - XB(T - 1) - \textstyle\sum_{j=1}^{n} d_j B(T_j - 1) - C(S;\, T, X)] \quad (7)$$

which may be positive, negative, or zero. If $\Delta(t + 1)$ is positive (negative), the portfolio is not (is) self-financing and the deficiency (excess) must be financed by borrowing (lending). At the maturity of the option, the individual pays (receives) an amount equal to $-\Delta(t + 1)/B(T - 1)$, and the net dollar value of the portfolio is

$$-\Delta(t + 1)/B(T - 1) + \begin{cases} 0; & S^* \geq X \\ X - S^*; & S^* < X \end{cases} \quad (8)$$

where S^* is the value of the stock at maturity. If the portfolio is self-financing ($\Delta(t + 1) \leq 0$), the net dollar value of the portfolio at maturity is nonnegative.[25]

To implement the trading rule a number of points should be noted.

First, the portfolio may produce a loss when the option matures, since the investor may have had to finance the portfolio by borrowing (i.e., $\Delta(t + 1) > 0$), and debt repayment is required.

Second, although the transactions required to build the portfolio should be made simultaneously, the structure of the options tape forces a sequential strategy. The TOT records an option transaction price and the price of the

[25] In this strategy, the transactions costs for the market maker are negligible, and it is assumed that the full proceeds of the short transaction in the underlying security can be used to purchase the treasury bills and the call option. For the market makers this is, in fact, the case.

closest stock trade that occurred *at* or *before* the option trade. If a violation signal is observed, a position will be taken using the data of the next option trade, provided the stamp time of the associated stock trade was subsequent to the time at which the violation was observed. Taking this position necessitates first a short position in the stock, and then with the proceeds, a long position in the option and an investment in debt.[26]

Third, due to this sequential order and the resulting time delays in achieving the required portfolio, the risk of the strategy is increased. The time was recorded between the initial violation and the stamp time of the next acceptable option trade. The length of time the individual had to wait to implement the trading rule can be used as a crude measure of the risk of the strategy: the longer the individual had to wait before successfully building the portfolio, the greater the chance of prices changing from those recorded at the time of the violation.[27] In fact, the length of time required to complete the transaction would be one indication of the liquidity of the market.[28]

Fourth, to determine the actual extent of the financing required to build the portfolio, bid and ask prices should be used. Unfortunately, these data were not available since TOT contains actual transactions prices.

Fifth, a short sale is permitted only if it constitutes either an up-tick or zero up-tick. This constraint was met by using only subsequent option trades where the associated stock trade occurred at a price no lower than that in the previous transaction.

Finally, the problem of nonsimultaneity of the data biases the results against finding that the market is inefficient. In this case, the problem is exacerbated by the possibility of the stock trading on other exchanges. The recording of a violation may arise because of the nonsimultaneity of the data even though the option is priced rationally. Thus, if the violation is used as a signal for a trading rule and the signal is false, this decreases the chance of earning a positive net dollar payoff at maturity on the portfolio. Alternatively, suppose the signal is true, but a transaction occurs in another market which results in an adjustment in the stock or option price. By the time the subsequent transactions are completed on the TSE, there is a reduced chance of earning a positive net dollar payoff at maturity.

Table VII shows the mean and standard deviation of the net dollar payoff per underlying share on the arbitrage portfolio. For the cells in the first row, the elapsed time between the violation and an acceptable call option observation to form the arbitrage portfolio was not greater than 30 seconds. It is anticipated

[26] To illustrate how the trading rule was implemented, suppose a violation is observed at 11:00 A.M. A position was taken using the data of the next option trade at 11:06 A.M. as long as the associated stock price was after 11:00 A.M. and no later than 11:06 A.M.; the latter is guaranteed by the construction of the tape.

[27] It is assumed that any riskless borrowing/lending can be undertaken without delay.

[28] It is possible that if the options market is particularly active, there may be a delay between the time the trade is executed and the stamp time placed on the order. This would tend to overestimate the waiting time necessary before a transaction could be undertaken. In conversation with market makers, it was determined that this delay is not frequent and when it does occur is not substantial (an estimate was approximately two minutes).

Table VII

Net Dollar Payoff Per Underlying Share of the Arbitrage Portfolio by Time Taken to Form the Initial Portfolio, Dividends Remaining, and In-the-Money Category

	Dividends Remaining					
	0		1		2	
Time Length (minutes)	DI	I	DI	I	DI	I
0–0.50	0.89*	0.44*	0.24*	0.84*	0.22*	0.42*
	2.50	1.19	4.78	1.70	1.13	1.21
	3045	2035	999	534	129	65
0.51–5.0	0.59*	0.26*	0.63*	0.23*	0.38*	0.03
	1.02	0.75	1.24	1.01	0.76	0.73
	986	937	358	256	25	9
>5.0	0.60*	0.21*	0.29*	0.34*	0.24*	−0.01
	1.83	1.02	1.00	1.04	0.94	0.86
	3976	4500	1418	845	185	99
Total No. of Observations: 20,423						

Notes: DI denotes deep in the money. I denotes in the money. In each cell, the first entry is the mean dollar value per underlying share, the second is the standard deviation, and the third is the number of observations.

* Statistically significant from zero at 1% level, two-tailed test.

that these observations are least affected by the risk induced by the sequential construction of the portfolio. The mean returns are positive, and all but one are statistically different from zero.[29] Whether the option was initially deep in the money or in the money does not seem to have any systematic effect on the mean return of the portfolio.[30] Similarly, the number of dividends remaining does not seem to have any systematic effect.[31] Although the results are not presented in Table VII, no evidence was found of any systematic effect between the maturity of the option generating the violation and the mean dollar return on the portfolio. Finally, the mean dollar return on the different arbitrage portfolios were statistically larger than zero. This does not imply rejection of the null hypothesis, that the option market was efficient.[32]

[29] Results, which will be presented in Table VIII, indicate that the distribution is highly skewed, implying that the normality assumption is violated. Thus the tests of statistical significance are unreliable.

[30] For the Canadian dollar sample, the total number of type 2 violations was 32,991. Given the short sale restriction, it was only possible to form 20,432 portfolios. A similar type of comment applied for the U.S. dollar sample.

[31] Portfolio payoffs have not been presented for the cases where there are three dividends remaining. There are a total of 31 instances where a portfolio was constructed, 17 in the shortest time interval and 12 in the longest.

[32] Certainly, if the portfolio is self-financing [i.e., $\Delta(t + 1) < 0$], the market cannot be efficient. Suppose that $\Delta(t + 1) > 0$. This implies that the payoff on the portfolio must be sufficient to compensate for the risk of the portfolio. An observation of a positive dollar payoff for the portfolio only implies that the returns were in excess of the risk-free rate. Until the necessary equilibrium return is specified, no test of efficiency is possible.

A trading rule was specified in which an additional restriction was used in constructing the portfolio: the portfolio must be self-financing ($\Delta(t + 1) < 0$). From (8), this implies that the dollar payoff per underlying share would be positive.[33] The investor would form the portfolio only if prices had not changed "too much" from those of the initial violation.[34] It was observed that there was only a 16% reduction in the number of portfolios formed when the requirement that the portfolios be self-financing is imposed. This implies that initial violations tend to be followed by a subsequent violation. Galai [2, p. 205] reports a similar finding for CBOE options.

The results of using this trading rule are presented in Table VIII. Since the rule results in the removal of all portfolios with negative net dollar payoffs, the mean dollar returns are larger than those reported in Table VII; for some categories, the increase in mean dollar payoffs is dramatic. Again, no systematic relationship was found between the mean dollar returns and the maturity of the option, the number of dividends remaining, and whether the option was deep in the money or in the money. The magnitude of the mean dollar returns are anything but economically insignificant, recalling that the dollar violation per contract is obtained by multiplying the per share violation by 100, the number of shares covered in an option contract.

While these results tend to indicate that the option market is inefficient, this conclusion must be treated with caution. To implement the rule, the investor must short the security upon observing the violation signal and then purchase the option and treasury bills. However, once the stock is sold short, an adverse change in price may occur before the option is purchased, and the investor may decide to abort the strategy but be saddled with the results of the short position. Given that it could take a long time to achieve the portfolio, there could be substantial risk. The results in Table VIII ignore this risk. By construction, the results are reported only for those cases where it was possible to construct a self-financing portfolio.

As a further check, the trading rule was applied to firms whose stocks and options were actively traded. This has a number of benefits. First, the bid-ask spread will be lower, the more actively traded the securities,[35] and second, there will be less risk due to either long lags before a position can be taken or to the trading of the underlying equity security on other exchanges.

The two firms chosen were Dome Petroleum and Norcen Energy.[36] The options

[33] This implies that all the results presented in Table VIII (and in Table IX) are statistically different from zero.

[34] The feasibility of this trading rule was confirmed in discussion with traders involved in the option market.

[35] This result has been documented in a number of studies; these studies are summarized in Tinic and West [7, chap. 4].

[36] Since 1979 option trading volume was much greater than in 1978, it was decided that trading activity in both options and the underlying securities in 1979 would determine the composition of the subsample. In 1979, Norcen and Dome Petroleum dominated option trading activity and comprised 49% of the total number of contracts traded in the Canadian sample. For the universe of stocks on which options could be written, Norcen and Dome Petroleum were the second and third most actively traded stocks. For 1978, Norcen was the most actively traded option and stock, while Dome Petroleum was ranked fourth.

Table VIII

Net Dollar Payoff per Underlying Share of the Arbitrage Portfolio Using the Trading Rule that Portfolio Be Self-Financing

Time Length (minutes)	Dividends Remaining 0 DI	0 I	1 DI	1 I	2 DI	2 I
0–0.50	1.04	0.53	0.71	1.01	0.55	0.73
	2.59	1.10	2.14	1.69	1.35	1.24
	2702	1764	840	423	86	48
0.51–5.0	0.65	0.35	0.69	0.42	0.45	0.38
	1.00	0.42	1.26	1.04	0.76	0.21
	933	841	332	184	23	7
>5.0	0.74	0.33	0.46	0.60	0.48	0.30
	1.91	0.81	1.04	1.09	0.92	0.35
	3448	3740	1083	556	130	62
Total No. of Observations: 17,224						

Notes: DI denotes deep in the money. I denotes in the money. In each cell, the first entry is the mean dollar value, the second is the standard deviation, and the third is the number of observations.

on these underlying securities accounted for 36% of all type 2 violations over the two-year sample period. The results are shown in Table IX. Considering portfolios formed within 30 seconds of a violation, the mean dollar returns on deep-in-the-money options are smaller than the results reported in Table IX. The reverse situation occurs for in-the-money options. For the other time categories, no similar patterns emerged. Again, the number of dividends remaining and the in-the-money classification appear to have no systematic effect on the magnitude of the mean dollar return for the Dome Petroleum and Norcen Energy sample. Given the similarity in results for the actively-traded sample and the full sample, the results for the latter cannot be due exclusively to infrequent trading of the underlying stock or its option(s). In addition, restricting attention to the shortest portfolio formation period, where nonsimultaneity problems are least likely to be crucial, the mean dollar returns are significant in dollar magnitude.

The last issue to consider is the transactions costs of implementing the arbitrage portfolios. The direct costs to the market maker of trading are minimal, as noted in Section I. The transaction would cost 20¢ per contract to build the option part of the portfolio and 35¢ per contract if the option is exercised at the expiry date. As for the stock part of the portfolio, transactions costs per 100 shares are small. For example, for a stock transaction of 100 shares at a price per share of $75, the transactions costs to initiate the position are $1.14. The indirect costs must also be evaluated. There are a number of reasons why the indirect trading costs are not sufficiently large to offset the arbitrage profits of the market maker. First, the ex ante tests are based on actual transactions prices and should remove the selection bias noted by Phillips and Smith in using ex post trading rules. Second, since the portfolio position is taken at the next

Table IX

Actively Traded Sample, Trading Rule with Self-Financing Arbitrage Portfolio

Time Length (minutes)	Dividends Remaining: 0		1		2	
	DI	I	DI	I	DI	I
0–0.50	1.02	0.98	0.49	1.58	0.44	3.88
	2.42	1.57	1.55	2.11	1.03	3.36
	1124	458	320	171	58	3
0.51–5.0	0.62	0.35	0.82	0.18	0.24	0
	0.62	0.35	1.46	0.15	0.19	0
	546	283	197	117	14	0
>5.0	0.89	0.26	0.36	0.55	0.48	0.89
	2.44	0.37	0.85	0.88	1.08	1.43
	1584	780	485	157	87	3

Total No. of Observations: 6,397

Notes: DI denotes deep in the money. I denotes in the money. In each cell, the first entry is the mean dollar value, the second is the standard deviation, and the third is the number of observations.

acceptable trade in the equity market, there is no reason to believe that this price deviates from the mid-point of the bid-ask spread, on average. A similar argument can be made for the option transaction as well. Thus, at most, the indirect trading cost of the equity and option positions in the arbitrage portfolio will equal one-half of the bid-ask spread.[37]

To evaluate the size of the indirect costs required to eliminate trading profits, consider the results in Table VIII for the shortest time length category. With one dividend remaining, the average payoff on the portfolio was $71 per contract (71¢ per share) for deep-in-the money options and $101 per contract for in-the-money options. The average spreads in the Phillips and Smith paper for both stocks and options per contract are $20.46 and $16.05, respectively. The total of $36.51 is much less than the profits observed. Of course, these values for bid-ask spreads need not be applicable on the TSE. However, it would require very large bid-ask spreads on both markets to eliminate the portfolio profits. The only estimates of the average spreads on the TSE are found in Tinic and West [7, 8] where the estimated average spread over a 9-day period in December 1971 was $38.40 per 100 shares. However, this sample included a number of speculative mining and oil stocks which would not qualify for options trading. Removing these stocks from the sample would likely result in a reduction in the average bid-ask period.

[37] For a market maker, the indirect costs of the option contract may be significantly overstated since in the purchase of the option the market maker can buy at the bid price, i.e., transact with a nonmarket maker. In order to obtain a rapid execution of the option transaction, the price paid may be in excess of the bid price but need not attain the level of the ask price.

B. Conclusions

The results strongly suggest that the option market over the two-year period (1978–1979) was inefficient. Since the trading rule which constrains the portfolio to be self-financing involves risk, the results must be treated with caution. The existence of indirect trading costs, while reducing the arbitrage profits documented in Tables VIII and IX, are unlikely to be sufficiently large to eliminate them. The sample period exhibited substantial growth in interest in the option market, the number of underlying securities eligible for options, and the beginning of the trading of puts. Observed inefficiencies should not be generalized to current periods where the option market has matured and its growth has levelled off.

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