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Differential Information and Performance Measurement Using a Security Market Line

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ABSTRACT

An uninformed observer using the tools of mean variance and security market line analysis to measure the performance of a portfolio manager who has superior information is unlikely to be able to make any reliable inferences. While some positive results of a very limited nature are possible, e.g., when there is a riskless asset or when information is restricted to be "security specific," in general anything is possible. In particular, a manager with superior information can appear to the observer to be below or above the security market line and inside or outside of the mean-variance efficient frontier, and any combination of these is possible.

MEAN-VARIANCE THEORY PREDICTS that if we plot expected returns against beta coefficients, all securities plot on a single line, known as the security market line (SML). Since β_i, the beta coefficient for security i, is interpreted as a measure of the riskiness of security i, the market line is a graphical representation of the linear relationship between risk and return. What are we to make of deviations from the SML, i.e., of securities or managed portfolios that do not plot on such a line? If we can retain our intuitive interpretation of β_i as the riskiness of asset i and the market line as the appropriate expected return needed to "reward" agents taking on various degrees of risk, then vertical deviations from the market line represent abnormal returns that differ from what is merited. This paper and the companion piece, Dybvig and Ross [10], explore in detail the validity of this intuition and the question of what SML deviations really measure. The companion piece analyzes SML deviations caused by the choice of an inefficient market or reference portfolio. In this paper, we analyze SML deviations caused by superior performance based on superior information.

The intuitive explanation of deviations from the SML has motivated many calls for using this analysis to measure the performance of portfolio managers.[1] The most important recent critique of these arguments was mounted by Roll [20, 22, and especially 21] in his series of papers criticizing existing applications and tests of the CAPM. Roll pointed out that the intuitive argument is not theoretically correct: the interpretation of β_i as a measure of riskiness is based on the validity of mean-variance theory. However, if mean-variance theory were valid,

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[1] See, e.g., Jensen [16, 17], Friend and Blume [13], or Sharpe [26].

there would be no abnormal returns, since all securities would plot on the SML. Roll called for an alternative model to explain deviations from the SML. In Roll's own alternative model, the market portfolio is misspecified, and the index used as a proxy for the market portfolio is inefficient. (This is the case studied in the companion piece to this paper, Dybvig and Ross [10].) Misspecification is the only source of deviations from the SML in Roll's model. Superior performance—based on superior information—is ruled out *a priori.* As observed by Mayers and Rice [8], this means that deviations from the SML cannot possibly measure such performance.[2]

Our paper follows Mayers and Rice in the sense that superior performance arises because of superior information.[3] By this we mean that the portfolio manager may have information that is useful for portfolio selection, but is not possessed by the observer. Contrary to Mayers and Rice, we find that differential information disrupts the validity of SML analysis, since it takes us outside the domain of mean-variance analysis. Even if underlying asset returns and the manager's signal are joint normally distributed, the portfolio return includes the product of the asset returns and the portfolio manager's strategy, which is a function of his information. This product might, for example, be skewed to the right (if better news causes the manager to pick an investment with higher expected return), in which case the portfolio would be more attractive to an investor than a normally distributed return with the same mean and variance. For this reason, mean-variance-based performance measurement can fail, since it may interpret superior performance it "does not understand" as inferior performance. A related intuition is that correct interpretation of performance should, in principle, be from the perspective of the informed manager, while SML deviations take the uninformed observer's perspective. These objections are valid for the Sharpe and Jensen measures as well. For example, we show that the portfolio manager's mean and variance can be dominated by the uninformed observer's mean-variance frontier and still be preferred by an investor.

The intuition behind some of our results is closely related to the concerns that a successful market timer will be more highly invested in the market, on average, when the market is up, and that a changing beta over time might invalidate SML analysis. Existing modifications of the SML analysis for nonconstant beta miss the point; what really matters is nonconstancy of beta *times the risk premium on the market portfolio.* Another problem with some of the proposed modifications of SML analysis is the errors in variables problem caused by using an *estimated* time series of beta coefficients. We do not explore these problems explicitly; we

[2] Roll's model does admit a type of inferior performance by managers who choose inefficient portfolios. Therefore, by default, relatively superior performance is attained by managers who can find the efficient frontier. In Dybvig and Ross [10], we show that, provided there is a riskless asset, SML analysis will correctly identify efficient portfolios as doing better than an inefficient index, but that SML analysis may also incorrectly identify arbitrarily inefficient portfolios as superior performers. In any case, it is doubtful that seeking the mean-variance frontier is what is usually intended in discussions about superior portfolio performance, especially among practitioners.

[3] We should emphasize that our analysis is related to the performance issue raised by Mayers and Rice. This paper does not, however, bear on their discourse on CAPM testability in general and Roll's critique in particular. The arguments of Mayers and Rice concerning CAPM testability have been countered by Roll [22, 23].

refer the reader to Bhattacharya and Pfleiderer [4] who have some new results in this direction.

It is interesting to consider why our conclusion differs from the Mayers and Rice [18] conclusion that SML analysis is generally valid when superior performance is caused by imperfect information. Mayers and Rice base their conclusion on two results, one interpreted as saying that SML analysis correctly measures the performance of agents whose information is "security specific," and a second result that is interpreted as being valid more generally. In fact, the second result is not very general, since it is based on assumptions that in effect assume again that information is "security specific." The failure of their arguments to be general can be seen using a simple "market timing" example (like the one given in our Section I).

The "security-specific" result says that superior performers with security-specific information will plot above the SML. By security-specific information, Mayers and Rice refer to information that tells us something about the returns to specific securities but nothing about the market as a whole (or more precisely the returns to the index portfolio). A pure example of security-specific information might be knowledge of which of two firms will win a lawsuit or whether an acquisition will take place (in the absence of synergy or differential managerial talent). Operationally, almost any "stock tip" about a single firm should be thought of as security specific, since knowing something good or bad about a single firm does not, effectively, give us any useful information about the market as a whole.

The Mayers and Rice derivation of the "security-specific" result goes well outside the mean-variance framework by assuming complete markets, quadratic utility, and a number of other strong assumptions, some of which are difficult to interpret. We extend and validate the Mayers and Rice result by removing all but a few simple assumptions, and we explore its scope and limitations. Since this result gives an interesting class of situations in which SML analysis is justified, our final conclusion is more temperate than Roll's assertion that SML analysis is never justified, while still far from the Mayers and Rice assertion that it is almost always justified.

In Section I, we present a plausible example that illustrates what can go wrong with the use of SML analysis in the presence of market timing. Section II sets up the general theory of superior performance under differential information in mean-variance models. Section III presents the positive result that significantly generalizes the Mayers and Rice "asset-specific" result. Section IV presents an example that shows what can go wrong with SML analysis when there is no riskless asset, even in the absence of market timing. Section V discusses directions for future research.

I. What Can Go Wrong

This section is devoted to a simple "market timing" example that calls into serious doubt the use of abnormal returns analysis to measure performance. The example is particularly puzzling, since the observer not only plots the manager below the market line, but also he views the manager as mean-variance inefficient as well. The resolution of this apparent contradiction lies in the fact that although both agents view their anticipated returns as normal, given their information, the manager's return is not normal since it is the product of normally distributed asset returns and normally distributed asset weights. The correlated part of the two normal variables multiply to form a chi-squared term, which is positively skewed and preferred by the clients to a normal distribution with the same mean and variance.

To construct the example, consider a model with two assets: a riskless asset with return r and a risky asset with return $\tilde{x} = r + \pi + \tilde{s} + \tilde{\varepsilon}$, where π is a risk premium, $\tilde{s}$ is the signal observed by the manager, and $\tilde{\varepsilon}$ is an unobserved noise term. Since the value chosen for r does not affect the conclusion of the analysis, we will take $r = 0$. The noise terms $\tilde{s}$ and $\tilde{\varepsilon}$ are independent normal random variables both having mean zero, and π is a positive constant. We assume that $\tilde{s}$ has variance σ_s^2, that $\tilde{\varepsilon}$ has variance σ_ε^2, that the manager has useful information ($\sigma_s^2 > 0$), and that the manager's information is not complete ($\sigma_\varepsilon^2 > 0$).

The manager controls an initial investment of 1 under a constant absolute risk-aversion utility function $u(\tilde{w}) = -e^{-A\tilde{w}}$ for some $A > 0$. The manager's problem, given the signal s, is to choose $\gamma(s)$ to maximize

$$E[-e^{-A\gamma(s)(\pi+s+\tilde{\varepsilon})}].$$

Using the formula for the normal moment generating function and taking the log of minus the objective function, the manager's problem, conditional on state s, is equivalent to:

Choose $\gamma(s)$ to maximize

$$\gamma(s)\{\pi + s - A \operatorname{var}[\gamma(s)(\pi + s + \tilde{\varepsilon})]/2\} = \gamma(s)(\pi + s) - \gamma(s)^2\sigma_\varepsilon^2 A/2.$$

Consistent with the spirit of mean-variance theory, the maximand depends only on preferences (parametrized by A) and the first two conditional moments of the payoff distribution: the mean $\gamma(s)(\pi + s)$ and variance $\gamma(s)^2\sigma_\varepsilon^2$. The first order conditions for this problem imply that

$$\gamma(s) = \frac{\pi + s}{\sigma_\varepsilon^2 A} \tag{1}$$

is the manager's portfolio choice.

The usual or unconditional abnormal returns δ_γ to a portfolio $\gamma(s)$ are defined generally by

$$\delta_\gamma = E[\gamma(\tilde{s})\tilde{x}] - r - \beta_\gamma\{E[\alpha(\tilde{s})\tilde{x}] - r\}, \tag{2}$$

where

$$\beta_\gamma = \operatorname{cov}[\alpha(\tilde{s})\tilde{x}, \gamma(\tilde{s})\tilde{x}]/\operatorname{var}[\alpha(\tilde{s})\tilde{x}]$$

is the beta coefficient of the portfolio $\gamma(s)$, and $\alpha(s)$ is the market index or benchmark portfolio. (If there is no riskless asset, it is generally ambiguous what interest rate to use in defining abnormal returns. When there is a riskless asset, as in this example, its return is a natural choice.) The expectations used in defining δ_γ and β_γ are the observer's (i.e., unconditional) expectations. The portfolio $\alpha(s)$ is assumed to be known to the observer and does not depend on s;

we will therefore write α instead of $\alpha(s)$. Now we are ready to compute the various expectations used in defining δ_γ.

The expected return to the manager's portfolio, as seen by the observer, is

$$\begin{aligned} E[\gamma(\tilde{s})\tilde{x}] &= E_s\left\{E\left[\frac{\pi + \tilde{s}}{\sigma_\varepsilon^2 A}(\pi + \tilde{s} + \tilde{\varepsilon}) \mid s\right]\right\} \\ &= E\left[\frac{\pi + \tilde{s}}{\sigma_\varepsilon^2 A}(\pi + \tilde{s})\right] \\ &= \frac{\pi^2 + \sigma_s^2}{\sigma_\varepsilon^2 A}. \end{aligned} \tag{3}$$

Every fixed portfolio with positive weight on $\tilde{x}$ is efficient from the point of view of the observer. Therefore, the return to the observer's index must be of the form $\alpha\tilde{x}$ for some fixed $\alpha > 0$. The variance of this index, as seen by the observer, is

$$\begin{aligned} \operatorname{var}(\alpha\tilde{x}) &= E\{[\alpha\tilde{x} - E(\alpha\tilde{x})]^2\} \\ &= E\{[\alpha(\pi + \tilde{s} + \tilde{\varepsilon}) - E(\alpha(\pi + \tilde{s} + \tilde{\varepsilon}))]^2\} \\ &= \alpha^2 E[(\tilde{s} + \tilde{\varepsilon})^2] \\ &= \alpha^2(\sigma_s^2 + \sigma_\varepsilon^2). \end{aligned} \tag{4}$$

The covariance of the manager's return with this index, as seen by the observer, is

$$\begin{aligned} \operatorname{cov}(\gamma(\tilde{s})\tilde{x}, \alpha\tilde{x}) &= E\{[\alpha\tilde{x} - E(\alpha\tilde{x})][\gamma(\tilde{s})\tilde{x}]\} \\ &= E_s E[\alpha(\tilde{s} + \tilde{\varepsilon})(\pi + \tilde{s})(\pi + \tilde{s} + \tilde{\varepsilon})/(\sigma_\varepsilon^2 A) \mid s] \\ &= E_s\left\{\alpha \frac{\pi + \tilde{s}}{\sigma_\varepsilon^2 A}[\tilde{s}(\pi + \tilde{s}) + \sigma^2]\right\} \\ &= \frac{\alpha\pi}{\sigma_\varepsilon^2 A}[2\sigma_s^2 + \sigma_\varepsilon^2]. \end{aligned} \tag{5}$$

Noting that the excess return $E(\alpha\tilde{x}) - r$ on the index is simply $\alpha\pi$, we have assembled the information needed to compute the abnormal return of the managed portfolio as viewed by the observer:

$$\begin{aligned} \delta_\gamma &\equiv E(\gamma(\tilde{s})\tilde{x}) - r - \frac{\operatorname{cov}(\gamma(\tilde{s})\tilde{x}, \alpha\tilde{x})}{\operatorname{var}(\alpha\tilde{x})}[E(\alpha\tilde{x}) - r] \\ &= \frac{\pi^2 + \sigma_s^2}{\sigma_\varepsilon^2 A} - \frac{(\alpha\pi)^2(2\sigma_s^2 + \sigma_\varepsilon^2)/(\sigma_\varepsilon^2 A)}{\alpha^2(\sigma_s^2 + \sigma_\varepsilon^2)} \\ &= [(\pi^2 + \sigma_s^2)(\sigma_\varepsilon^2 + \sigma_s^2) - \pi^2(2\sigma_s^2 + \sigma_\varepsilon^2)]/[\sigma_\varepsilon{}^2 A(\sigma_s^2 + \sigma_\varepsilon^2)] \\ &= \sigma_s^2[(\sigma_\varepsilon^2 + \sigma_s^2) - \pi^2]/[\sigma_\varepsilon^2 A(\sigma_s^2 + \sigma_\varepsilon^2)] \\ &< 0, \end{aligned} \tag{6}$$

provided that $\pi^2 > \sigma_\varepsilon^2 + \sigma_s^2$ and that σ_ε^2 and σ_s^2 are both positive. Therefore, the informed manager appears inferior on the basis of the SML analysis.

Surprisingly, the manager's portfolio appears (is!) inferior in mean and variance as well. (This can be viewed as a counterexample to an assertion to the contrary by Roll [22, 23].) The observed variance of the manager's portfolio is

$$\begin{aligned}
\operatorname{var}[\gamma(\tilde{s})\tilde{x}] &= E\{[\gamma(\tilde{s})\tilde{x} - E(\gamma(\tilde{s})\tilde{x})][\gamma(\tilde{s})\tilde{x}]\} \\
&= E_s E\left\{\left[\frac{\pi + \tilde{s}}{\sigma_\varepsilon^2 A}(\pi + \tilde{s} + \tilde{\varepsilon}) - \frac{\pi^2 + \sigma_s^2}{\sigma_\varepsilon^2 A}\right]\left[\frac{\pi + \tilde{s}}{\sigma_\varepsilon^2 A}(\pi + \tilde{s} + \tilde{\varepsilon})\right]\right\} \\
&= E_s\left\{\left[\frac{(\pi + \tilde{s})^2}{\sigma_\varepsilon^2 A} - \frac{\pi^2 + \sigma_s^2}{\sigma_\varepsilon^2 A}\right]\frac{(\pi + \tilde{s})^2}{\sigma_\varepsilon^2 A} + \frac{(\pi + \tilde{s})^2}{\sigma_\varepsilon^4 A^2}\sigma_\varepsilon^2\right\} \\
&= \frac{\pi^4 + 6\pi^2\sigma_s^2 + 3\sigma_s^4}{\sigma_\varepsilon^4 A^2} - \frac{\pi^4 + 2\pi^2\sigma_s^2 + \sigma_s^4}{\sigma_\varepsilon^4 A^2} + \frac{(\pi^2 + \sigma_s^2)\sigma_\varepsilon^2}{\sigma_\varepsilon^4 A^2}
\end{aligned}$$

(where we have used the fact that $\tilde{s}$ normal with mean zero implies that $E(\tilde{s}^4) = 3\sigma_s^4$)

$$= \frac{4\pi^2\sigma_s^2 + 2\sigma_s^4 + (\pi^2 + \sigma_s^2)\sigma_\varepsilon^2}{\sigma_\varepsilon^4 A^2}. \tag{7}$$

We know from (3) that, as seen by the observer, the manager's portfolio has mean return $(\pi^2 + \sigma_s^2)/(\sigma_\varepsilon^2 A)$. Since the return to any index $\alpha\tilde{x}$ is $\alpha\pi$, we know that $\alpha\tilde{x}$ has the same return as the manager's portfolio when $\alpha = \bar{\alpha} \equiv (\pi^2 + \sigma_s^2)/(\sigma_\varepsilon^2 A\pi)$. From (4), the variance of $\bar{\alpha}\tilde{x}$ is

$$\begin{aligned}
\operatorname{var}(\bar{\alpha}\tilde{x}) &= \left[\frac{\pi^2 + \sigma_s^2}{\sigma_\varepsilon^2 A\pi}\right]^2(\sigma_s^2 + \sigma_\varepsilon^2) \\
&= [\sigma_s^6/\pi^2 + \sigma_s^4\sigma_\varepsilon^2/\pi^2 + 2\sigma_s^4 + 2\sigma_s^2\sigma_\varepsilon^2 + \pi^2\sigma_s^2 + \pi^2\sigma_\varepsilon^2]\frac{1}{\sigma_\varepsilon^4 A^2}.
\end{aligned} \tag{8}$$

Combining (7) and (8),

$$\operatorname{var}(\gamma(\tilde{s})\tilde{x}) - \operatorname{var}(\bar{\alpha}\tilde{x}) = \frac{3\pi^2\sigma_s^2 - \sigma_s^2\sigma_\varepsilon^2 - \sigma_s^4\sigma_\varepsilon^2/\pi^2 - \sigma_s^6/\pi^2}{\sigma_\varepsilon^4 A^2}.$$

This expression is positive provided that $\pi^2 > \sigma_s^2 + \sigma_\varepsilon^2$, in which case $\sigma_s^2 > 0$ and $\sigma_\varepsilon^2 > 0$ imply that the first term in the numerator dominates the other terms. Therefore, the manager's portfolio appears to the observer to be mean-variance inefficient since we have shown that it has higher variance than a marketed portfolio having the same mean return. To understand why this paradoxical result obtains even though both agents view returns as normal and therefore have utility functions over mean and variance, let us reexamine the returns to the manager's portfolio:

$$\gamma(\tilde{s})\tilde{x} = \frac{\pi + \tilde{s}}{\sigma_\varepsilon^2 A}(\pi + \tilde{s} + \tilde{\varepsilon})$$

$$= \frac{\pi^2 + 2\pi\tilde{s} + \pi\tilde{\varepsilon} + \tilde{s}^2 + \tilde{s}\tilde{\varepsilon}}{\sigma_\varepsilon^2 A}.$$

The first term in the numerator is constant and the next two terms are normally distributed. The term $\tilde{s}^2$, however, has a chi-squared distribution. Since this term is skewed to the right and bounded below, the manager views this part of the return more favorably than the manager would view a normal random variable with the same mean and variance. How is it that the manager's portfolio has a nonnormal distribution even though the manager and observer both face normal returns? The answer is that the manager faces returns that are normal conditional on s, but that the product of his portfolio choice as a function of s and the return as a function of s is no longer normally distributed, and is, therefore, outside the domain (normal random variables) on which the manager's utility depends only on the mean and variance of returns. (This phenomenon appears to be general: we conjecture that, excluding degenerate cases, it is impossible to specify preferences and the joint distribution of security returns and the manager's signal that make the manager's portfolio and all assets joint normally distributed, from the perspective of both the manager and the observer.)

Of course, if we were to use the alternative motivation of mean-variance theory that the manager has quadratic utility, the manager would never choose an unconditionally inefficient portfolio, since unconditional preferences as well as conditional preferences would depend only on the mean and variance of returns. Therefore a quadratic manager's portfolio choice will never be mean-variance dominated but, rather, will generally appear to lie outside the observer's frontier.

Note, lastly, that by reversing the assumption of our example, and setting $\pi^2 < \sigma_\varepsilon^2 + \sigma_s^2$, we will have from (6) that $\delta_\gamma > 0$, and the manager will plot above the observer's SML. Depending on the exact parameter values, this case may make the manager appear to the observer as mean-variance super-efficient, efficient, or inefficient.

II. Mean-Variance Analysis and Informed Portfolio Choice

The above example shows that the presence of market timing can invalidate SML analysis. Next, we want to explore under what circumstances SML analysis can be trusted. But, first, we need to define and analyze a general mean-variance framework with differential information. As in the example, we assume that an uninformed observer is monitoring the performance of another agent, the manager, who uses superior information in the form of an observation of an information state, s, to select a portfolio.

Formally, if there are n assets, the managed portfolio $\gamma(s)$ is an n-vector of weights summing to one in every state s, i.e., with $\gamma(s)\cdot e \equiv 1$ where $e \equiv (1, \cdots,$

1). The manager's portfolio choice is most naturally interpreted to be optimal from the client's perspective, as in the example. However, we will not dwell on the particular preferences on which the choice is based, and in general we can simply assume that the portfolio choice, $\gamma(s)$, is efficient, conditional on s. Note that we are assuming away the agency problem in this context, and the performance measure is implicitly assumed to be unrelated to the manager's compensation. (See Bhattacharya and Pfleiderer [3] and Dybvig and Spatt [11] for discussions of the agency problem in the context of portfolio management.)

Conditional on information state s, the assets have an expected returns vector $\mu(s)$ and returns covariance matrix $V(s)$. This specification is implied by, but more general than, von Neumann-Morgenstern preferences and multivariate normality of asset returns and the signal. To determine the observer's perception of asset means and covariances, we assume that the observer is completely uninformed and forms expectations rationally, i.e., that the joint distribution of asset returns as viewed by the observer is the weighting across states, s, of the conditional distributions as viewed by the manager.[4]

Letting $\tilde{x}$ be the vector of random asset returns, the observer's perception of average returns is given by

$$\begin{aligned} E[\tilde{x}] &= E_s E[\tilde{x} \mid s] \\ &= E_s[\mu(\tilde{s})], \end{aligned}$$

or, letting $\mu \equiv E_s[\mu(\tilde{s})]$,

$$E(\tilde{x}) = \mu. \tag{9}$$

The formula for the observer's covariance of returns is more interesting:

$$\begin{aligned} \operatorname{cov}(\tilde{x}_i, \tilde{x}_j) &= E[(\tilde{x}_i - \mu_i)\tilde{x}_j] \\ &= E_s E[(\tilde{x}_i - \mu_i)\tilde{x}_j \mid s] \\ &= E_s E[(\tilde{x}_i - \mu_i(\tilde{s}))\tilde{x}_j + (\mu_i(\tilde{s}) - \mu_i)\tilde{x}_j \mid s] \\ &= E_s[V_{ij}(\tilde{s}) + (\mu_i(\tilde{s}) - \mu_i)\tilde{x}_j] \\ &= E_s V_{ij}(\tilde{s}) + \operatorname{cov}(\mu_i(\tilde{s}), \mu_j(\tilde{s})). \end{aligned}$$

Using the definitions

$$V \equiv E_s V(\tilde{s})$$

and

$$\Omega \equiv E_s[(\mu(\tilde{s}) - \mu)(\mu(\tilde{s}) - \mu)'],$$

[4] Note that we are assuming very noisy or Muth-type rational expectations, in that this and other managers' information-based trading does not affect prices enough to affect the observer's expectations. Standard rational expectations models show that our approximation is valid provided there is an exogenous source of noise and the informed agents in the economy are few in number and intolerant of taking on risk. See, e.g., Hellwig [14], Diamond and Verrecchia [6], or Admati [1]. Or see Admati and Ross [2] for a discussion of this issue in the context of performance measurement.

we have that the returns variance-covariance matrix seen by the observer is

$$\mathrm{cov}(\tilde{x}, \tilde{x}) = V + \Omega. \tag{10}$$

The observer sees as variability not only the average variability seen by the manager, V, but also the shift in the manager's mean across states, Ω. This is most easily seen in the degenerate case of a perfectly informed manager. In this case $V(s) \equiv 0$, so the only variability seen by the observer is the shifting of the manager's true perception across information states. Since Ω is positive definite, (10) formalizes the sense in which the observer having inferior information implies that the observer faces more variability than does the manager, on average.

Equations (9) and (10) tell us that the means and covariance of returns to *fixed* portfolios α^1 and α^2 are given by $\alpha^1\mu$, $\alpha^2\mu$, and $\alpha^1(V + \Omega)\alpha^2$. More general formulas are needed to compute the means and covariance of returns on portfolios $\gamma(s)$ and $\eta(s)$ whose compositions are information state dependent. The mean return is

$$\begin{aligned} E(\gamma(\tilde{s})\tilde{x}) &= E_s E\{\gamma(s)\tilde{x} \mid s\} \\ &= E_s\{\gamma(\tilde{s})\mu(\tilde{s})\}. \end{aligned} \tag{11}$$

This is the same as $E\{\gamma(\tilde{s})\}\mu + \mathrm{cov}\{\gamma(\tilde{s}), \mu(\tilde{s})\}$, and, therefore, it is usually not the same as $E\{\gamma(\tilde{s})\}\mu$. The covariance of returns is given by

$$\begin{aligned} \mathrm{cov}(\gamma(\tilde{s})\tilde{x}, \eta(\tilde{s})\tilde{x}) &= E\{[\gamma(\tilde{s})\tilde{x} - E_s(\gamma(\tilde{s})\mu(\tilde{s}))]\eta(\tilde{s})\tilde{x}\} \\ &= E\{[\gamma(\tilde{s})\tilde{x} - \gamma(\tilde{s})\mu(\tilde{s})]\eta(\tilde{s})\tilde{x}\} \\ &\quad + E\{[\gamma(\tilde{s})\mu(\tilde{s}) - E_s(\gamma(\tilde{s})\mu(\tilde{s}))]\eta(\tilde{s})\tilde{x}\} \\ &= E_s[\gamma(\tilde{s})V(\tilde{s})\eta(\tilde{s})] + \mathrm{cov}[\gamma(\tilde{s})\mu(\tilde{s}), \eta(\tilde{s})\mu(\tilde{s})]. \end{aligned} \tag{12}$$

Intuitively, Equation (12) has the same message as Equation (10), namely that the covariance is the average conditional covariance plus the covariance of the conditional means. Equation (12) will be useful for computing both the numerator and denominator of the observer's perception of the beta coefficient.

By definition, the SML abnormal returns δ_γ to a portfolio $\gamma(s)$ are given by

$$\delta_\gamma = E[\gamma(\tilde{s})\tilde{x}] - r - \beta_\gamma\{E[\alpha(\tilde{s})\tilde{x}] - r\}, \tag{13}$$

where

$$\beta_\gamma \equiv \mathrm{cov}[\alpha(\tilde{s})\tilde{x}, \gamma(\tilde{s})\tilde{x}]/\mathrm{var}[\gamma(\tilde{s})\tilde{x}]$$

is the beta coefficient of the portfolio $\gamma(s)$, $\alpha(s)$ is the market index or benchmark portfolio, and r is the zero-beta or riskless rate. Until Section V, we will assume that r is the return on a riskless asset in the economy and that the benchmark portfolio α does not depend on s. The expectations used in defining δ_γ and β_γ are the observer's (i.e., unconditional) expectations. Excess returns computed using the manager's expectations (i.e., expectations conditional on s) will be indicated by $\delta_\gamma(s)$, and are given as

$$\delta_\gamma(s) = \gamma(s)\mu(s) - r - \beta_\gamma(s)[\alpha(s)\mu(s) - r],$$

where

$$\beta_\gamma(s) = \alpha(s)V(s)\gamma(s)/\alpha(s)V(s)\alpha(s).$$

We conclude this section by presenting the result that if we could measure performance conditional on the manager's information, superior performance would register correctly in SML excess returns. The result means intuitively that the source of difficulty in using abnormal returns analysis is that the observer cannot look at things from the perspective of the informed manager, i.e., cannot understand what the manager is doing without knowing the manager's information.

THEOREM 1. *Suppose that $\gamma(s)$ is positively mean-variance efficient conditional on state s and that the return on a riskless asset in the economy is used for the riskless rate in defining abnormal returns. Then the conditional (on s) abnormal return $\delta_\gamma(\tilde{s}) \geq 0$, with equality if and only if the index portfolio is efficient conditional on s. (Note that by rational expectations, both agents share the same riskless rate. We are assuming away extreme degeneracy such as the existence of arbitrage and all assets having the same expected return.)*

Proof: We will assume that both $\alpha(s)$ and $\gamma(s)$ have positive variance (the other cases are simple). Efficiency of $\gamma(s)$ conditional on s implies that

$$\alpha(s)\mu(s) - r = \beta_\alpha(s)[\gamma(s)\mu(s) - r], \tag{14}$$

where $\beta_\alpha(s)$ is the regression coefficient of α on $\gamma(s)$, conditional on s. Equation (14) can be rewritten as

$$\gamma(s)\mu(s) - r = [\alpha(s)\mu(s) - r]/\beta_\alpha(s),$$

or,

$$\delta_\gamma(s) = [1/\beta_\alpha(s) - \beta_\gamma(s)][\alpha(s)\mu(s) - r]. \tag{15}$$

Since $\beta_\alpha(s)$ and $\beta_\gamma(s)$ are regression coefficients reversing dependent and independent variables, simple linear regression theory tells us that they share the same sign and that $|1/\beta_\alpha(s)| \geq |\beta_\gamma(s)|$, and therefore that the first expression in square brackets in Equation (15) has the same sign as $\beta_\alpha(s)$. By Equation (14), this sign is the same as the sign of $\alpha(s)\mu(s) - r$, and consequently Equation (15) tells us that $\delta_\gamma(s) \geq 0$. Furthermore, regression theory tells us that equality holds if and only if $\alpha\tilde{x}$ and $\gamma(s)\tilde{x}$ are perfectly correlated conditional on s, which is equivalent to mean-variance efficiency of α conditional on s. Q.E.D.

Note: Theorem 1 is closely related to Theorem 1 of Dybvig and Ross [9]. In that context, there is no information, but the market index may be inefficient, just as the market is inefficient conditional on s.

III. What Can Go Right: A Positive Theorem on the Use of the SML

When Theorem 1 applies, the manager's own expected excess return relative to the SML is positive conditional on s, i.e., $\delta_\gamma(s) \geq 0$. If, additionally, the observer's perception of excess returns is a simple average across states of the manager's perception, i.e., $\delta_\gamma = E[\delta_\gamma(s)]$, then the observer will measure positive abnormal returns. This observation tells us that some of the problems we encounter in using δ_γ stem from the fact that δ_γ is not, in general, a simple average of $\delta_\gamma(s)$ across information states s. This leads us to ask whether there are any interesting cases in which $\delta_\gamma = E[\delta_\gamma(s)]$.

One case in which we can show that $\delta_\gamma = E[\delta_\gamma(s)]$ is a case suggested by Mayers and Rice [18], namely, that in which $\mu(s)\alpha$ and $\alpha V(s)\alpha$ are both independent of s, i.e., in which the informed manager's information does not help him to predict the return or variance of the uninformed observer's index portfolio. Mayers and Rice refer to this situation as characterized by "security-specific" information, in contrast to the "market timing" the manager could do if the condition did not hold. We have the following generalization of the Mayers and Rice result. The generalization is nontrivial because we eliminate the use of the Arrow-Debreu assumption, the assumption that both agents have the same quadratic utility function, the assumption that both agents have the same marginal utility of income, and several other explicit and implicit assumptions.

THEOREM 2. (*Based on Mayers and Rice* [18]) *Suppose that*:

(a) *There is a riskless asset, which is used for r in calculating δ_γ.*
(b) *In each information state s, the informed manager chooses a mean-variance undominated portfolio on the positive part of the efficient frontier conditional on s.*
(c) *The manager does not learn anything about the return or variance of the uninformed observer's index, i.e., $\alpha\mu(s)$ and $\alpha V(s)\alpha$ are independent of s.*
(d) *Expectations are rational and the rest of the framework is as given above.*

It follows that $\delta_\gamma \geq 0$, and equality holds if and only if α is efficient conditional on every state (*with probability* 1).

Proof: Assumptions (a), (b), and (d) ensure that the conditions of Theorem 1 are satisfied. Therefore, for all s, $\delta_\gamma(s) \geq 0$ and consequently

$$E[\delta_\gamma(s)] \geq 0, \tag{16}$$

where equality occurs if and only if α is efficient conditional on all states (to be pedantic, with probability 1). Hence, it suffices for us to show that $E[\delta_\gamma(s)] = \delta_\gamma$. By definition,

$$E[\delta_\gamma(s)] = E[\gamma(s)\mu(s)] - r - E\left\{\frac{\gamma(s)V(s)\alpha}{\alpha V(s)\alpha}[\alpha\mu(s) - r]\right\}. \tag{17}$$

But, by assumption, $\alpha V(s)\alpha$ and $\alpha\mu(s)$ are state independent and can be replaced by their averages across states, which implies that

$$E[\delta_\gamma(s)] = E[\gamma(s)\mu(s)] - r - \frac{E[\gamma(s)V(s)\alpha]}{\alpha V\alpha}[\alpha\mu - r]. \tag{18}$$

From Equation (10),

$$\begin{aligned}\mathrm{var}(\alpha x) &= \alpha V\alpha + \alpha\Omega\alpha \\ &= \alpha V\alpha + E\{\alpha(\mu(s) - \mu)(u(s) - \mu)'\alpha\} \\ &= \alpha V\alpha\end{aligned}$$

since $(\mu(s) - \mu)'\alpha = 0$ for all s by assumption (c). From Equation (12),

$$\begin{aligned}\mathrm{cov}(\alpha x, \gamma(s)x) &= E\{[\alpha V(s)\gamma(s)] + \mathrm{cov}[\alpha\mu(s), \gamma(s)\mu(s)]\} \\ &= E[\alpha V(s)\gamma(s)],\end{aligned}$$

since $\alpha\mu(s)$ is constant by assumption (c). Therefore, from the definition of δ_γ,

$$E[\delta_\gamma(s)] = E[\gamma(s)\mu(s)] - r - \frac{\mathrm{cov}[\alpha\tilde{x}, \gamma(s)\tilde{x}]}{\mathrm{var}(\alpha\tilde{x})}(\alpha\mu - r),$$

and we are done. Q.E.D.

The unusual assumption made in Theorem 2 is, of course, assumption (c), the assumption that the manager's information does not help to predict the mean or variance of the observer's index portfolio. Mayers and Rice interpret this assumption to be an assumption that the manager's information is "asset-specific," in contrast to "market timing" information. The information helps to distinguish which assets have higher return or are less risky but does not help to determine the return on the market as a whole as held by the observer. This assumption is obviously a strong one, and it plays an essential role in the proof.

A striking feature of the statement and proof of Theorem 2 is that it is never assumed that the portfolio α is efficient from the point of view of the uninformed observer. Therefore, an observer with any index α satisfying condition (c) will plot an agent with superior information on or above the SML. This observation shows that the test is somewhat immune to Roll's criticism that the actual market portfolio is not used in tests. Of course, if we use an inefficient index portfolio an agent who has no superior information may plot above the SML, and by the arguments of Dybvig and Ross [10] this will occur whenever the agent chooses a mean-variance superior portfolio. Therefore, if we use an inefficient index satisfying assumption (c), we may find that an agent with superior information plots above, but others may plot above as well, which is simply to say that the power of the test in rejecting superior performance is less when the index is inefficient.

To end the section on another positive note, the SML still provides a valid guide to marginal comparisons. The reader can easily verify that the theorems of Dybvig and Ross [10] on marginal improvements still hold conditional on the manager's information. But, if we are in a normally distributed world, this does not necessarily represent marginal *utility* improvement unless the overall return to the manager's portfolio is joint normal with that of the observer's return on the index portfolio. Of course, as we have seen in the examples, we have no reason to expect this to be the case.

This is a good place to stop and comment on some of the recent articles on performance measurement. These articles all share the theme of bringing additional information to bear on performance evaluation. This general idea is already in use by practitioners. Formal and informal interviews, both before and after the fact, play a central role in the compensation and retention of managers. These interviews help to tell the supervisor whether the manager is behaving erratically, and allows the supervisor to impose a degree of consistency on the manager.

The recent articles go beyond the existing practice by recommending specific statistical methods which can form the basis for a consistent performance evaluation policy. One interesting idea, implicit in Mayers and Rice [18] and asserted by Cornell [5] is the intuition that an informed subject will receive a better expected return on average than an uninformed investor would expect, given the portfolio composition. Verrecchia [27] proves that this assertion is false in general, since an informed agent may use the information to reduce variance, in which case the net effect on mean return is ambiguous. (Verrecchia does note, however, that the assertion is correct for log and exponential utility functions.) The usefulness of this approach requires resolution of the technical problems raised by Verrecchia (perhaps by an adjustment term) and observability of the portfolio composition at each point when the composition changes.

An alternative approach to bringing additional information to bear on the performance problem is given by Merton [19] and Henriksson and Merton [15]. This approach does not even look at portfolio returns and instead tests directly whether the manager has special information. The focus in the papers is on the question of whether stocks will outperform bonds. To detect performance in this context requires specialization of the standard Blackwell informativeness conditions to the two outcome case, as is reviewed in the papers. One weakness of this approach is that information is measured but there is no test of whether information is being used correctly. Additionally, the test will not have any power if the information is much different from what the test assumes, namely, about which will do better, stocks or bonds. (There are other tests proposed in the papers, but they are even more sensitive to misspecification and are not robust to allowing a portfolio with options or highly levered firms.) Admati and Ross [2] examine the issue in a rational expectations model. They propose a regression test with a quadratic term measuring the performance of the manager. Unfortunately, they show that such tests are statistically more complicated than had been previously suspected. Furthermore, they are heavily dependent on distributional assumptions.

IV. An Example without any Riskless Asset

In this section, we display an example that shows that Theorem 2 would not be valid if we removed the assumption (a) of the existence of a riskless asset. A striking feature of the example is that $\delta_\gamma(s) < 0$ for all s. The example has three assets and two equally probable information states. The return vectors and covariance matrices in the two equally likely information states are given by

$$\mu(1) = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \qquad v(1) = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

$$\mu(2) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \qquad v(2) = \begin{bmatrix} 3 & 3 & 2 \\ 3 & 5 & 3 \\ 2 & 3 & 2 \end{bmatrix}.$$

Averaging these with the probabilities (½, ½) for the two states gives

$$\mu = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \qquad v = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & 2 \\ 2 & 2 & 2 \end{bmatrix},$$

and

$$\Omega = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

For the uninformed observer, efficient portfolios are of the form $(\bar{\alpha}, 0, 1 - \bar{\alpha})$. We will assume that our observer chooses the particular efficient portfolio $\alpha = (½, 0, ½)$. For the informed manager, efficient portfolios are given by $(\bar{\gamma}, 1 - \bar{\gamma}, 0)$ in state 1 and $(\bar{\gamma}, \bar{\gamma} - 1, 2 - 2\bar{\gamma})$ in state 2. We have designed the covariances and returns so that returns in state 2 are just the returns in state 1 reflected across the uninformed observer's efficient set $\{(\bar{\alpha}, 0, 1 - \bar{\alpha})\}$. Therefore, with state-independent preferences, the manager will choose the same value of $\bar{\gamma}$ in both states. We take $\bar{\gamma}$ to be 2, so that $\gamma(1) = (2, -1, 0)$ and $\gamma(2) = (2, 1, -2)$.

The zero-beta rate for the uninformed observer is the rate implicit in the observer's choice of efficient portfolio, which can be calculated to be -4. Without showing the details of the calculations, the abnormal return is:

$$\begin{aligned} \delta_\gamma &= E[\gamma(\tilde{s})\tilde{x}] - r - \frac{\operatorname{cov}(\gamma(\tilde{s})\tilde{x}, \alpha\tilde{x})}{\operatorname{var}(\alpha\tilde{x})}[E(\alpha\tilde{x}) - r] \\ &= 3 - (-4) - \frac{4}{9/4}\left[\frac{1}{2} - (-4)\right] \\ &= -1, \end{aligned}$$

as required to show that a superior performer can plot below even when information is "security specific," provided there is no riskless asset. It is easy to verify that the manager's portfolio dominates the uninformed efficient frontier in this example. Together with the example of Section I, this completes the taxonomy of cases: an informed agent can plot above or below the SML, and independently can be mean-variance efficient or dominated.

V. Summary and Conclusions

This paper has shown the fragile nature of SML analysis for the evaluation of performance. Even in the absence of statistical measurement problems, a manager who makes optimal use of superior information may plot above, on, or below the SML and may plot inside, on, or outside the efficient frontier—and every combination of these cases is possible.

If the reference portfolio is known to be efficient, a manager plotting above the SML necessarily has differential information (superior information in the context of a completely uninformed observer), but we cannot tell whether or not that information is being used correctly. (A literal feature of our model is that an agent plotting below the SML relative to an efficient index would also necessarily have differential information, but this feature would go away in a model in which it is possible to waste funds, i.e., through transaction costs incurred while churning the portfolio.) On the other hand, if the observer has chosen a misspecified index, i.e., one that is inefficient given the information of the observer, then, as is shown in Dybvig and Ross [10], abnormal returns relative to the SML may simply reflect this inefficiency and not superior information. With the possibility of genuinely superior information, literally anything can happen and, in particular, it is no longer the case that the manager must plot above the observer's SML nor even outside his or her mean variance frontier.

The only positive result we have occurs with a riskless asset and a situation where the manager learns nothing about the returns on the uninformed observer's portfolio. In this case, the manager will plot above the observer's SML. But, these slim pickings are certainly far from adequate to justify the extensive use of SML analysis for performance evaluation.

We have seen that SML analysis does poorly even in a context where the agency problem is absent. It is easy to see by the arguments of Dybvig and Ingersoll [8] that when there is an agency problem; reward according to SML excess returns is easily "gamed" by an option strategy. Since there always exist options mispriced by the CAPM (as proven by Dybvig and Ingersoll), going long or short in these options (or following an equivalent trading strategy) will give a false appearance of superior performance. The mounting evidence against the validity of SML analysis prompts a call for a new performance measurement technique, perhaps based on the Arbitrage Pricing Theory (Ross [24, 25]) or some other model.

What should we require of a new performance measurement technique? First, it should correctly identify superior, ordinary, and inferior performance. Second, it should be immune to gaming, i.e., it should work correctly even when the manager being evaluated understands the measure being used. Finally, the measure should be sufficiently powerful to be useful in practice. (For instance, a measure requiring 20 years of data to get an accurate measurement will not be useful in yearly decisions of whether to retain a manager.) While there are promising measures satisfying the first two requirements (e.g., measures based on the Payoff Distribution Pricing Model of Dybvig [7]), the last requirement is likely to be the ultimate stumbling block, since the large amounts of noise in

common stock returns seem to make it impossible to measure significant superior or inferior performance over time periods short enough to be useful. If this is the case (or the curse) of performance measurement, then perhaps we must look elsewhere to resolve these issues.

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