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An Analysis of Mortgage Contracting: Prepayment Penalties and the Due-on-Sale Clause

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ABSTRACT

The due-on-sale clause contained in most conventional home mortgage contracts is equivalent to a prepayment penalty equal to the difference between the face value and market value of the loan. We analyze a bilateral game with asymmetric information and show that the bank demands the full penalty unless the market value of the loan is sufficiently low. In that case, the bank demands a prepayment penalty which is independent of the market value of the loan in order to induce additional prepayments. We also demonstrate, by a risk-sharing argument, that the due-on-sale clause is optimal in some settings, even though it eliminates some beneficial home sales.

ALMOST ALL CONVENTIONAL, fixed-rate home mortgage contracts written in the last decade contain a due-on-sale clause which gives the lender the contractual right to declare the total indebtedness due immediately if the mortgaged property is transferred. As mortgage rates rise, lending institutions suffer capital losses on fixed-rate mortgage loans while borrowers earn capital gains equal to the difference between the face value and market value of their loans. The due-on-sale clause gives the lending institution the right to demand the borrower's capital gain as a prepayment penalty. In order to increase the probability of prepayment, it can be optimal for the bank to accept a partial settlement. In this paper, we analyze the bank's strategy for setting the prepayment penalty it demands under the due-on-sale clause and examine the optimality of the clause. The model we develop also applies to other types of prepayment penalties (i.e., loan covenants which require payment of a premium over market value in order to extinguish the debt prior to maturity).

Our analysis is motivated in part by issues actively debated in Congress and in recent court cases concerned with the enforceability of the due-on-sale clause. Consumer groups and realtors suggested that the due-on-sale clause is an unreasonable restraint on home sales. On the other hand, the savings and loan industry and the Federal Home Loan Bank Board (FHLBB) argued that enforcement of the due-on-sale clause is necessary to maintain the viability of the savings and loan industry and to ensure a well-functioning secondary market for mortgage loans. The regulatory authority of the FHLBB was upheld in a June 1982 Supreme Court ruling that state courts and legislatures cannot interfere with the

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FHLBB regulation that permits federal savings and loan associations to enforce due-on-sale clauses (see Fidelity Federal Savings and Loan Association et al. v. De La Cuesta et al., 458 U.S. Supreme Court 141). Subsequently, the Garn-St Germain Depository Institutions Act of 1982 established the enforceability by all types of lenders of due-on-sale clauses contained in future mortgage contracts.

A basic issue that underlies the due-on-sale clause controversy is whether the clause is part of an efficient contract. By providing the bank with the contractual right to demand a prepayment penalty, the due-on-sale clause confers upon the bank considerable ex post monopoly power (large numbers of consumers are tied to a given bank in the ex post problem). Although this monopoly power results in a cost to society because of the elimination of some beneficial home sales, the loss can be offset by the risk-sharing features of the clause.[1] For example, suppose that the bank is risk neutral while each consumer is risk averse in the utility gain from a home sale. Although the consumer's incremental utility gain from selling is not directly observable by the bank, a prepayment penalty can achieve some, but typically not perfect, risk-sharing. This risk-sharing benefit is traded off against the cost of the reduced housing turnover. We identify the conditions under which the due-on-sale clause is an efficient risk-sharing agreement. Our risk-sharing explanation for the clause is consistent with the claim of some bankers that consumers prefer ex ante the due-on-sale contract to one in which the clause is removed and the bank is fairly compensated by an appropriate increase in the contract interest rate.

Alternative rationales for the clause include controlling either interest rate risk or the change in default risk upon transferring the mortgage loan. However, the due-on-sale clause is not an efficient mechanism for accomplishing those objectives. An alternative to the due-on-sale clause for controlling default risk on fixed-rate loans is a provision permitting the bank to demand that default insurance be purchased by a buyer who assumes an existing loan. Such a provision would nearly eliminate the cost of the reduction in housing turnover caused by the due-on-sale clause. Similarly, an adjustable-rate loan or hedging in the financial market provides the lender with better protection from interest rate risk than a due-on-sale clause. Furthermore, given the superior access of financial intermediaries to the capital market, it is not clear that an efficient contract would transfer interest rate risk to consumers. Therefore, these alternative rationales for the use of a due-on-sale clause are incomplete.

While the primary goal of this paper is to explain market observations on mortgage contracting, our analysis also sheds light on the legal controversy concerning the enforceability of the due-on-sale clause. Despite its effect of reducing beneficial housing turnover, we show that the clause is efficient for some types of consumers because of the offsetting risk-sharing benefits. Absent the risk-sharing argument, there would be a conflict between the legal theories based on the sanctity of contracts and those based on the elimination of inefficient contracts. In the context of our model, the contract selected by a consumer is efficient and, therefore, the use of the due-on-sale clause implies its efficiency.

[1] Hendershott and Hu [6] provide empirical estimates of the magnitude of the cost to society from the reduction in beneficial home sales, ignoring the potential risk-sharing benefits of the clause.

Our framework also shows the potential optimality of diversity in loan contracts for a heterogeneous population.

We also examine the bank's optimal strategy for setting the ex post penalty it demands on loans with a prepayment penalty such as the due-on-sale clause. In practice, banks frequently relax the terms of the due-on-sale clause and accept a partial settlement. Specifically, we often observe banks allowing the loans to be assumed at a "blended rate" which is higher than the rate on the existing loan, but less than the current market interest rate for a similar loan. This observation leads to some additional questions, such as under what circumstances does a waiver of a portion of the penalty occur and what is the form of the resulting partial settlement? Therefore, we examine a bilateral game with asymmetric information between the owner of a residential home and the bank that holds the loan on the house. We show that the bank's optimal strategy is to insist upon the full penalty when the contractual prepayment penalty is below a critical level (which is satisfied if the market value of the mortgage is near face value). Otherwise, the optimal strategy is to demand a prepayment penalty equal to this critical value. Because it is optimal for the lender to waive a portion of a fixed prepayment penalty in some circumstances, the design of the contractual prepayment penalty is especially important in mortgage contracts with ex post inflexibility. The value of mortgage-backed securities could be increased and the cost of mortgage funds reduced by specifying ex ante a prepayment penalty of the form identified by our ex post analysis. This could be accomplished by making the contractual prepayment penalty a function of an observed interest rate and the time remaining to maturity, analogous to the payments on adjustable rate loans being contingent on observed rates.

In Section I we develop a bilateral game with asymmetric information and use the model to study a bank's ex post optimal monopoly strategy under the due-on-sale clause and prepayment penalties. In Section II we discuss the potential ex ante efficiency of the due-on-sale clause and prepayment penalties. We conclude in Section III.

I. The Model and Bank Strategies

We consider a two-period (three-date) economy with a continuum of consumers and a competitive banking industry. Consumers (borrowers) are assumed to be nonsatiated while banks are assumed to be risk neutral in the consumer's gain from moving. At time zero, consumers enter into a two-period mortgage contract which calls for payments at the end of each period. The contract gives borrowers the option to prepay their loans at the end of the first period and gives the bank the right to demand that they prepay the face value of their loans if the mortgaged property is sold. At time zero, the banks and consumers have identical information, which includes a common prior distribution of consumers' gains from selling their homes at the end of period one.[2]

[2] In the analysis of the bank's ex post strategy given the form of the contract, the bank will optimize given its beliefs about the distribution of gains. Each consumer can have more information than the bank about the actual distribution from which their gain is drawn. Our ex ante analysis, however, requires that the bank and consumers have identical beliefs about the distribution of gains.

At the end of the first period, a realization from the common prior distribution of gains provides each consumer with private information describing the gain from a sale.[3] This gain is the incremental utility received from selling (rather than remaining in or renting) the house. For example, the gain could be a consequence of a relative preference for a different house or location. It does not refer to any increase or decrease in the market value of the house (because this wealth effect occurs without selling). We denote the dollar equivalent of the incremental gain by g. We assume all consumers have a nonnegative gain (since those consumers with negative gains would not sell) and denote the density function of this gain by $f(g)$ and the cumulative distribution function by $F(g)$. The density, which is assumed to be continuous, satisfies $f(g) > 0$ if and only if $g \in [0, \not{g}]$. We let g_i denote the independent drawing of consumer i from the common distribution of gains.

The bank is allowed to demand a prepayment penalty of p, which under the due-on-sale clause equals the difference between the remaining principal balance and the market value of the existing mortgage loan.[4] If the bank knew each consumer's gain, g_i, it would perfectly price discriminate by demanding a penalty of p when $g_i > p$ and g_i otherwise. However, we focus upon situations in which the bank does not know each consumer's subjectively-determined, ex post gain, g_i, and, therefore, consumers have ex post private information which prevents the bank from perfectly discriminating among them. Further, in our two-period environment, a consumer has only one opportunity to sell. Therefore, we analyze a one-shot bilateral game with asymmetric information in which the bank sets its terms (any relaxation from the contractual prepayment penalty) and then, based on independent realizations of the gain drawn from an identical distribution, each consumer decides whether to accept the bank's prepayment terms and sell. At the time the parties enter into the mortgage contract, both the bank and consumers have alternative market opportunities, but ex post the bank possesses some monopoly power. It is not optimal for the consumer to reveal private information to the bank by making an offer. Instead, in the context of the hypothesized information structure, it is optimal for the uninformed bank to make an offer (i.e., prepayment terms) that the consumers accept or reject based upon their independent realizations.

The monopolist's problem is to maximize shareholder wealth, which is equivalent to maximizing the revenue from prepayment penalties in this environment. We initially ignore the bank's prepayment penalty limit of p (or equivalently, suppose p is infinite). In this case, the bank is a monopolist facing a demand function given by $q = 1 - F(g)$ where q is the probability of prepayment (i.e., the quantity of prepayments is normalized to express q as the probability that a consumer will have a gain of at least g) and g is the price charged by the bank to release the consumer from the debt. The (unconstrained) demand function is g

[3] For our analysis of the ex post strategies of the bank, we can interpret the gain from prepaying as the certainty equivalent of the consumer's updated distribution of gains, i.e., the consumer can continue to face uncertainty.

[4] The amount of the prepayment penalty is known in the ex post problem, but can be stochastic ex ante (as it is under a due-on-sale clause).

$= F^{-1}(1 - q)$ when written as the usual Marshallian inverse expressing price as a function of quantity. We restrict the distributions analyzed so that marginal revenue goes from positive to negative values. The Marshallian inverse demand function, $g = F^{-1}(1 - q)$, is continuously differentiable in the interior since $F'(\cdot) = f(\cdot) > 0$ in its interior. Therefore, the marginal revenue function is also continuous. In our model, the marginal opportunity cost of an additional prepayment is zero because consumers have only one opportunity to prepay their loans. Hence, the bank's maximum profit is obtained by determining the prepayment probability that maximizes the revenue from prepayments, and we can set marginal revenue equal to zero to determine the price, p^*, which is the unconstrained profit-maximizing prepayment penalty.

The introduction of the binding limit, p, on the prepayment penalty charged by the bank does not affect the solution provided $p \geq p^*$, because the unconstrained solution satisfies the feasibility condition and is therefore optimal. The assumption that marginal revenue goes from positive to negative values implies that, if the unconstrained penalty exceeds the allowed contractual penalty, the contractual ceiling is the constrained optimal. The Marshallian inverse demand function can be modified to reflect the contractual limit on the prepayment penalty by letting $d = \min[g, p]$ denote the truncated demand function. In the region where the market value of the mortgage is low enough so that $p \geq p^*$, the probability of prepayment, q^*, and the gain to the bank, p^*, are independent of p and the market value of the mortgage (see $p = p_h$ in Figure 1). In contrast, if $p \leq p^*$, the marginal revenue is negative to the right of the kink in the truncated demand curve and the bank insists upon the entire contractual prepayment penalty p. In this case, the probability of prepayment declines along the demand curve with increases in p (see $p = p_l$ in Figure 1). From this analysis we obtain the following proposition.

PROPOSITION 1. *The bank demands a prepayment penalty equal to the minimum of the unconstrained profit-maximizing penalty and the penalty allowed under the contract. Hence, the penalty demanded is equal to a constant whenever the contractual constraint on the penalty is not binding.*[5]

This proposition establishes that the bank is willing, under some circumstances, to relax the contractual penalty. This occurs despite the resulting ex post adverse selection (i.e., the consumers who accept the bank's offer are those with the highest gains, including those who would have been willing to pay the full contractual penalty).[6] In order to induce additional prepayments, the bank can allow consumers to retain a portion of the capital gain on their mortgage loans in the event the house is sold. When relaxation occurs it is of a special form and conditionally independent of the allowed penalty. In particular, there is no

[5] Proposition 1 is robust to a degree even if we drop the restrictions on marginal revenue. If p^* is the unconstrained profit-maximizing penalty, then p^* is also profit-maximal, subject to the constraint that the penalty does not exceed p, provided this constraint is not binding. However, in the absence of marginal revenue going from positive to negative values, it is possible that the constrained profit-maximizing penalty is less than p in the region where the constraint is binding (i.e., $p < p^*$).

[6] In a dynamic world, intertemporal adverse selection considerations also affect the time profile of the bank's concession in order to induce consumers to reveal their type.

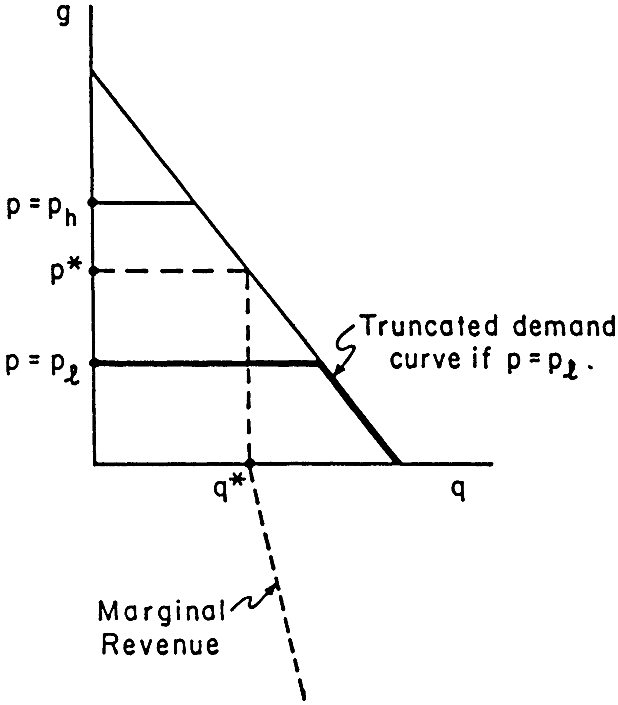

Figure 1. Truncated Demand Curve, $d = \min(g, p)$. Adjusting p above p^* does not move p^* or q^*. Changes in interest rates affect the market value of the mortgage and therefore, p. However, p^* is fixed as interest rates change, provided the distribution of consumer gains (and the demand curve) are independent of interest rates. When $p = p_\ell$, the contractual limit is low enough that the monopolist requires the entire contractual penalty, as the marginal revenue is negative below the kink in the truncated demand curve.

concession in the region where the market value of the existing loan is greater than $B - p^*$, where B denotes the remaining principal balance of the loan. In the region where the market value is less than $B - p^*$ the bank charges p^*, a constant penalty which is independent of the market value of the existing mortgage loan (see Figure 2). The simple nature of our result does not, however, apply to discrete distributions.[7] Because the bank always demands a positive penalty ex post, our model illustrates the inefficient exclusion of beneficial trades that is a consequence of single-price monopoly. If there were no prepayment

[7] That the proposition does not hold in the case of a discrete distribution is illustrated by an example in which 60 percent of the consumers have a gain of 20 and the remainder have a gain of 10. In this example, the unconstrained optimal penalty is $p^* = 20$. If there is a ceiling of p, then the constrained optimal penalty is

$$\begin{cases} 20 & \text{if } p \geq 20 \\ p & \text{if } 16\tfrac{2}{3} \leq p \leq 20 \\ 10 & \text{if } 10 \leq p \leq 16\tfrac{2}{3} \\ p & \text{if } p \leq 10. \end{cases}$$

In particular, the penalty is below $\min[p, p^*]$ for $10 < p < 16\tfrac{2}{3}$.

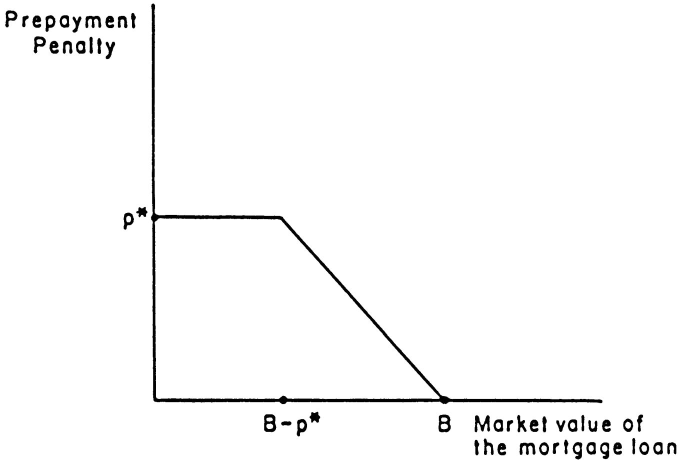

Figure 2. The Relationship Between the Market Value of the Loan and the Prepayment Penalty.

penalties (e.g., assumable loans) or if the bank could perfectly price discriminate, profitable trades would not be excluded. The ex ante pricing of assumable contracts would compensate the bank for the absence of prepayment penalties.

Proposition 1 can be applied to the due-on-sale clause. Variation in the market value of the loan payments due on existing loans would result from differences in initial terms (e.g., the contract interest rate) or in current market conditions. The optimizing behavior of the bank is such that it will not demand repayment of face value on loans in which the current market value of the loan payments is exceptionally low (e.g., those loans written at an especially low interest rate). The acceptance of partial settlements by the bank does not imply that the bank is "neighborly" enough to forego some of its own profit in order to aid consumers. In time periods when values of old mortgages are low, the bank will appear flexible. In practice we sometimes observe a direct reduction in the prepayment penalty as illustrated by mail offers by banks to particular consumers allowing immediate repayment of the mortgage at a specified discount. In other instances, the bank will offer to reduce the interest rate on a loan in return for an acceleration of payments (shortening the term of the loan) or offer to lend additional funds in order to increase both the principal balance and contract interest rate of the refinanced loan. More commonly, we observe an indirect concession in the form of below-market financing to the new purchaser of the property (which the current owner captures through a higher sales price, assuming a competitive market for housing). Our model predicts that the interest rate on the refinanced loan should be such that the loan's market value equals the market value of the existing loan plus the optimal prepayment penalty and any additional cash advanced. Therefore, the blended rate on a refinanced loan with the same remaining principal balance and term to maturity is less than the market rate if the market value of the existing loan is low enough and equal to the market rate otherwise.

In order to examine the empirical implications of our model when only prices and quantities are observed by an econometrician, we examine the theoretical

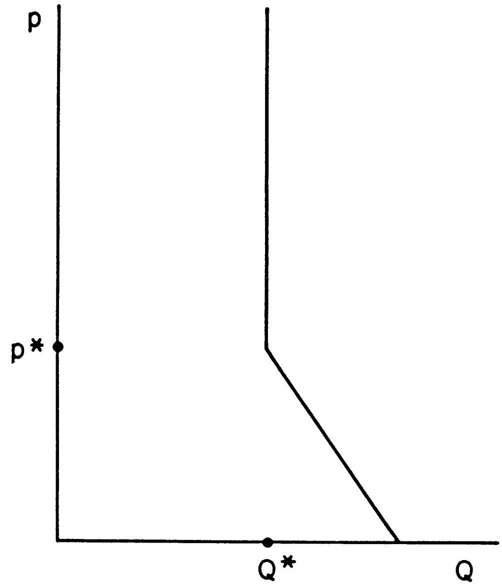

Figure 3. The Relationship Between Prepayment Volume and the Contractual Prepayment Penalty.

tradeoff between the contractual prepayment penalty, p, and volume of prepayments, Q. If all consumers draw from the same distribution, Proposition 1 implies that the prepayment volume is fixed at Q^* for $p \geq p^*$ (since the bank fixes the penalty quoted at p^* when the allowed penalty is higher) and moves down the demand curve for $p < p^*$ (see Figure 3). The segment of the demand schedule that Q moves along must induce negative marginal revenue and at the top of the segment (p^*) the marginal revenue is zero (because marginal cost is zero). In fact, these restrictions exhaust the implications on the volume-prepayment penalty tradeoff. Given any such tradeoff there is some underlying distribution of consumer gains that supports the solution.

The theoretical tradeoff between the allowed prepayment penalty and volume is weakened by examining the *aggregate* relationship in an environment in which consumers draw from different distributions. The aggregate version of the $p(Q)$ correspondence is generated by aggregating similar relations at the individual level (e.g., horizontal summation of curves of the sort illustrated in Figure 3). Therefore, $p(Q)$ is nonincreasing. To examine whether additional restrictions can be obtained on $p(Q)$ from aggregate data, we first consider whether an outside observer could identify the underlying information structure. For example, suppose the prepayment penalty quoted by the bank is $\min[p, p^*]$, where p^* represents the unconstrained profit-maximizing level from the unobservable prior distribution. In fact, one such distribution which will support the observed penalty is the degenerate one in which the bank knows the consumer's gain is exactly p^*. In this sense, full extraction is indistinguishable from other solutions and at the aggregate level perfect discrimination is indistinguishable from other possible outcomes. Using this perfect discrimination construction, we can generate an economy corresponding to any aggregate $p(Q)$ that is nonincreasing below p^* (including ones with flats and jumps) and vertical above p^*. These are actually the complete set of restrictions on the aggregate tradeoff between the prepayment penalty and volume. We obtained much stronger restrictions on the volume-prepayment relationship when all consumers drew their gain from an identical distribution, but the failure of those representative agent restrictions to hold in the data could merely suggest the importance of diversity in the unobservable distributions.

In general, the bank should condition the distribution of gains for a given consumer upon any observable characteristics. In fact, to the extent that certain attributes of the consumer can be costlessly disclosed and verified, the bank will mandate disclosure by drawing the least favorable inference otherwise. All consumers would try to separate themselves from those with "worse" characteristics. Full disclosure results if fraud were not possible (see Akerlof [1] and Grossman and Hart [5]). If the bank knew that consumers could communicate their gains in a costlessly verifiable manner, the bank would mandate disclosure and extract from each consumer the minimum of the full surplus and p. Perfect discrimination eliminates the deadweight loss of monopolistic pricing, but no consumer gains surplus relative to the uniform price solution and those consumers with $g_i > p^*$ are worse off, provided $p > p^*$.

II. Efficiency of Prepayment Penalties and the Due-on-Sale Clause

In this section, we develop an explanation for prepayment penalties and the due-on-sale clause based upon risk-sharing. The optimal ex ante (competitive) contract will depend upon the individual's utility function, $U(g)$, which summarizes the ex ante weighting of selling gains. For individuals whose marginal utility of consumption is low when their gain from moving is high, a positive prepayment penalty can be optimal. This occurs if the cost of foregoing opportunities to move is less than the risk-sharing benefit of a prepayment penalty. Our marginal utility hypothesis is natural under the assumption that the consumer's gain from a sale increases with a more favorable job opportunity at another location.[8] An alternative interpretation of our explanation for the due-on-sale clause is that it is a device for consumers to sell a partial claim against their human capital by making the mortgage payment contingent on whether the consumer moves.

For expositional convenience, we model the bank as risk neutral. This is equivalent to assuming that all the risk of consumers' gains from selling can be diversified away by the bank. More generally, to the extent that part of the risk is systematic, that component will be symmetrically priced by the bank and consumers (and potentially hedged by the bank in the capital market), and our analysis applies to the residual risk.

If the ex post gain from a sale were verifiable (and if we assume diminishing global marginal utility in the gain), then it follows that the optimal risk-sharing

[8] A story inconsistent with this job market hypothesis is that the consumer has a big gain from selling a home in order to reduce consumption or liquidate some wealth because of economic distress. Ex ante such a consumer would not select a loan with a prepayment penalty, if an alternative exists.

contract is one in which the bank bears the entire risk. In this case, the consumer equates marginal utility across all gains. Then the consumer's payment to the bank at the end of period 1 is set so that the consumer's gain less the payment to the bank equals a constant. This fee structure is not feasible unless the consumer's gain is verifiable.

Because the gain from sale is not generally observable by the bank, we analyze the (informationally) constrained optimal risk-sharing contract. We assume that consumers draw their gains from an identical distribution and that the bank does not observe the realized gains. Hence, the only observable characteristic upon which the payment to the bank at the end of the first period can be conditioned is whether or not a consumer prepays. The structure of risk preferences (risk-averse consumers and risk-neutral banks) implies that the efficient contract specifies a deterministic payment to the bank, conditional on whether the loan is prepaid. The payment will equal one constant if the consumer prepays the loan and another constant if the consumer does not prepay. (An analogous feature is obtained by Kahn [7] in a model of optimal severance pay under private information.) We normalize the loan payments by considering the difference between the payments on a loan with a prepayment penalty and one without a penalty.[9] The normalized payment to the bank can be written as $h(g) = k$ if the loan is prepaid and $h(g) = -c$ otherwise. With this normalization, the effective prepayment penalty is $c + k$, where c is the reduction in the scheduled payment relative to a loan without a prepayment penalty. The consumer's share (the gain less the payment) is $s(g) = g - k$ if the loan were prepaid and $s(g) = c$ otherwise. Because the consumer's net reward is not constant, the risk-sharing is not perfect. Only consumers with large enough gains prepay, i.e., a consumer prepays if $g_i \geq \hat{g} \equiv c + k$. Therefore, some consumers with positive gains find it optimal to forego the gain from prepayment and moving (thereby, eliminating beneficial home sales).

In an ex ante competitive market, the banks offer a consumer the contract that maximizes expected utility, $\int_0^{\bar{g}} U(s(g))f(g)\,dg$, subject to the condition that the banks earn a competitive expected return, $\int_0^{\bar{g}} h(g)f(g)\,dg = 0$. Using the definitions of $s(g)$ and $h(g)$ and the condition that a consumer prepays only if

[9] The loan payments are normalized as follows. Let y denote the promised payment on a loan with no prepayment penalty and let $y - b$ denote the promised payment on a loan with a prepayment penalty equal to x. Both loans have the same original principal balance, B_0. The schedule of payments on the two loans is:

	No Prepayment Penalty		Prepayment Penalty	
Time	Sale	No Sale	Sale	No Sale
0	$-B_0$	$-B_0$	$-B_0$	$-B_0$
1	$y + y/(1 + r)$	y	$y - b + \dfrac{y - b}{1 + r} + x$	$y - b$
2	0	y	0	$y - b$

In the event of no sale, the time 1 present value of the difference between the payments on the loan with a penalty less those on the loan without the penalty is $-c = -b - b/(1 + r)$ where r is the one-period interest rate. If there is a sale at time 1, the difference between the payments is $k = x - c$. Hence, the prepayment penalty, x, is equal to $c + k$.

$g_i \geq c + k$ to rewrite the optimization problem, we obtain that the contract selected by a consumer solves

$$\max_{c,k} U(c)F(c+k) + \int_{c+k}^{\emptyset} U(g-k)f(g)\,dg$$

$$\text{subject to} \quad cF(c+k) = k(1 - F(c+k)).$$

Without loss of generality, $c + k$ is nonnegative. A positive prepayment penalty means that the payment to the bank in the event of prepayment exceeds the payment otherwise, i.e., $c + k > 0$. A zero prepayment penalty is represented by $c + k = 0$. The maximization problem selects a consumer's ex ante preferred contract from the set of feasible contracts. Therefore, our optimization problem for the efficient contract determines endogenously whether the optimal contract contains a prepayment penalty.

PROPOSITION 2. *A sufficient condition for a positive prepayment to be optimal is* $U'(0) > 2\int_0^{\emptyset} U'(g)f(g)\,dg$.

Proof: We let $x \equiv c + k$ denote the effective prepayment penalty (see footnote 9). The zero profit constraint in the ex ante maximization problem implies that $k = xF(x)$ and $c = x(1 - F(x))$. We recast the problem as an unconstrained one in a single variable, i.e.,

$$\max_{x} U(x(1 - F(x)))F(x) + \int_{x}^{\emptyset} U(g - xF(x))f(g)\,dg.$$

After differentiating the objective twice, we obtain that the first derivative is zero at $x = 0$ and the second derivative at $x = 0$ is

$$f(0)\left[U'(0) - 2\int_0^{\emptyset} U'(g)f(g)dg\right].$$

The second derivative is positive at $x = 0$ if and only if

$$U'(0) > 2\int_0^{\emptyset} U'(g)f(g)\,dg.$$

Under this condition, the first derivative is increasing near $x = 0$ so that the first derivative is positive near $x = 0$ and the objective function is locally increasing. Hence, under the hypothesized condition, the objective function is maximized with $x > 0$ so that a positive prepayment penalty (recall, $x \geq 0$ without loss of generality) is optimal. Q.E.D.

If the objective function is decreasing near $x = 0$, we cannot rule out the optimality of a positive x. However, the corner $x = \emptyset$ always yields a lower value for the objective function than the corner $x = 0$, so that $x = \emptyset$ is never optimal. One sufficient condition for $x = 0$ to be optimal is for the objective function to be globally decreasing in x.

The condition of Proposition 2 requires that the marginal utility at a zero gain exceeds twice the expected marginal utility of the gain, i.e., $U'(\cdot)$ declines rapidly relative to the density. The sufficient condition for a positive prepayment penalty given in Proposition 2 can be satisfied even if $U(\cdot)$ is not globally strictly concave. If borrowers are risk neutral with respect to their utility gain from selling, the derivative of the objective reduces to $-xf(x)$, after normalizing, so that $U'(\cdot) = 1$. The objective is then globally declining and $x = 0$ (zero prepayment penalty) is optimal. This is intuitive because in the risk-neutral case there is no risk-sharing benefit to offset the exclusion of beneficial trades. This logic extends to the case in which a borrower is only slightly risk averse. Therefore, $U(\cdot)$ globally strictly concave is neither necessary nor sufficient for a positive prepayment penalty to be optimal.

While our specification of the optimization problem addresses the optimality of a positive prepayment penalty, it can also be applied to the due-on-sale clause. If the market value of the mortgage were deterministic, the due-on-sale clause would be equivalent to a deterministic prepayment penalty. More generally, when the market value of the mortgage, M, is sufficiently low, i.e., $M \leq B - p^*$, the ex post prepayment penalty is independent of M. Under this condition, the due-on-sale clause and a constant contractual prepayment penalty of at least p^* yield identical predictions.

The utility function $U(\cdot)$ need not be observable by the bank. The consumer's knowledge of $U(\cdot)$ can be private information (we assume the density $f(\cdot)$ is common knowledge). The consumer will select (ex ante) the full information optimal contract even when $U(\cdot)$ is ex ante private information, so that self-selection constraints are not binding. This occurs because, given the known distribution of gains and optimal contract, the privately observed $U(\cdot)$ does not influence any ex post prepayment decision. Despite its ignorance of $U(\cdot)$, the bank can calculate the exact distribution of ex post prepayment decisions for a given contract, because, ex post, a consumer's decision depends only upon the relation between g_i and the prepayment penalty. In effect, each consumer can design an individual contract, because the observability of $f(\cdot)$ ensures fair pricing. By introducing diversity on $U(\cdot)$, we can explain the actual coexistence of different prepayment penalties even when the diversity is unobservable. For example, unlike FHA and VA loans, most new fixed-rate, conventional mortgage loans, have a due-on-sale provision.

With risk-averse consumers, the ex ante risk-sharing benefit can exceed the cost of the elimination of beneficial home sales so that a positive prepayment penalty could be optimal. Which effect dominates depends upon a consumer's utility function and the distribution of gains. For illustrative purposes, we consider $U(g) = -e^{-\alpha g}$ (constant absolute risk aversion in the gain). The sufficient condition given in Proposition 2 for a positive x to be optimal then specializes to $\frac{1}{2} > \int_0^{\bar{g}} e^{-\alpha g} f(g)\, dg$. For any fixed density, $f(\cdot)$, we can find a sufficiently big α so that the condition holds. In fact, the condition holds only for $\alpha > \alpha^*$ (since the right-hand side of the condition is decreasing in α). If the consumer is sufficiently averse to risk in the gain, a positive penalty would be optimal. The required degree of risk aversion depends on the density and, in fact, for any α there exists a density for which the condition does not hold. In cases with a continuous distribution, the optimal contract can entail both imperfect risk-sharing and a positive prepayment penalty inducing exclusion of beneficial trades.

To illustrate the tradeoff between risk-sharing and the elimination of beneficial house sales, we consider an example with two states. In this example, consumers are assumed to have strictly increasing and strictly concave $U(g)$ and to have gain Z_1 with probability ρ and gain Z_2 with probability $1 - \rho$, where $Z_1 > Z_2 \geq 0$. When none of the consumers prepay, a consumer's utility is $U(0)$. In the case in which all consumers prepay their loans, the problem is

$$\max_{k} \rho U(Z_1 - k) + (1 - \rho)U(Z_2 - k)$$

$$\text{subject to} \quad k \geq 0,$$

so that $k = 0$ and the consumer's utility is $\rho U(Z_1) + (1 - \rho)U(Z_2)$. Increasing utility implies $\rho U(Z_1) + (1 - \rho)U(Z_2) > U(0)$ so that all prepaying dominates no prepayments. Finally, we suppose that only consumers with gain Z_1 prepay. The optimization problem is then

$$\max_{c,k} \rho U(Z_1 - k) + (1 - \rho)U(c)$$

$$\text{subject to} \quad (c + k)\rho \geq c.$$

The first-order conditions imply $U'(c) = U'(Z_1 - k)$ (perfect risk-sharing) so that $Z_1 = c + k \equiv \hat{g}$. At the optimum, the budget constraint holds with equality so that $Z_1\rho = c$ and the value of the objective is then $U(Z_1\rho)$. A positive prepayment penalty is optimal if and only if

$$U(Z_1\rho) \geq \rho U(Z_1) + (1 - \rho)U(Z_2).$$

If $Z_2 = 0$ this is implied by $U(\cdot)$ concave, but this condition does not hold for Z_2 sufficiently close to Z_1 (when $\rho < 1$). For a given ρ, $U(\cdot)$, and Z_1, the monotonicity and concavity of $U(\cdot)$ imply that a positive prepayment penalty is optimal if and only if $Z_2 \leq Z_2^*$ for some Z_2^* satisfying $0 < Z_2^* < Z_1$. In the case of $Z_2 = 0$, we have no exclusion as well as perfect risk-sharing (first-best solution). On the other hand, for a given ρ, $U(\cdot)$, and Z_2, concavity of $U(\cdot)$ implies $U(Z_1\rho) - \rho U(Z_1)$ is increasing in Z_1, so we can obtain a positive prepayment penalty only if $Z_1 > Z_1^*$ for some $Z_1^* > Z_2$.[10]

The two-state example is also useful in showing that for all strictly concave $U(\cdot)$ there exists some distribution of gains for which any observed prepayment penalty is optimal. For example, if Z^* is the observed prepayment penalty, then we construct an economy in which $Z_1 = Z^*$ and $Z_2 = 0$. For any probability ρ and strictly concave $U(\cdot)$, the optimal contract entails a prepayment penalty of Z^*. If we assume a two-state unobservable distribution, then the observed contract cannot be distinguished from one resulting from a richer unobservable distribution.

In the two-state example, the positive prepayment penalty can be efficient only when an ex post monopolistic bank would charge the high gain as the penalty (for a sufficiently large permissible contractual penalty). If the bank ex post charges the low gain, i.e., $Z_2 \geq Z_1\rho$, then $U(Z_1\rho) < \rho U(Z_1) + (1 - \rho)U(Z_2)$, so that the zero penalty is optimal. Under the condition $Z_2 < Z_1\rho$, the ex ante efficient contract (zero or Z_1 prepayment penalty) depends on $U(\cdot)$ given Z_1, Z_2,

[10] We have not shown that there must exist such a Z_1^*. If Z_1^* does not exist, then a zero penalty is always optimal given the other parameters. A condition which is sufficient for such a Z_1^* to exist is for $\lim_{x\to\infty} U(x)$ to exist.

and ρ. For any fixed Z_1, Z_2, and ρ such that $Z_2 < Z_1\rho$, a positive penalty is optimal for some strictly increasing and strictly concave $U(\cdot)$.

We have argued above that the general specification of the optimization problem is robust to consideration of private information on $U(\cdot)$ without imposing any self-selection conditions, but that private information on the distribution of gains creates a potential ex ante adverse selection problem. For example, consider two economies in which a positive prepayment penalty is optimal in each and in which Z_1, Z_2, and $U(\cdot)$ are identical in the two settings. The probability of the high gain is greater in economy A than in economy B, i.e., $\rho^A > \rho^B$. The equilibrium utility level with public information on all the parameters is greater in economy A than B, because the positive penalty is optimal in both cases and $U(Z_1\rho^A) > U(Z_1\rho^B)$. If the probability of a large gain were private information, then all consumers would claim to have the high probability of the larger gain and the bank would not earn its required return. With ex ante adverse selection, the available mortgage contracts must differ across additional dimensions in order to induce consumers to separate themselves. For example, consumers with a high probability of moving would not favor contracts with heavy up-front fees (points).

III. Conclusion

In this paper we have examined the due-on-sale clause and prepayment penalties in home mortgage contracts. The due-on-sale clause is equivalent to a prepayment penalty which permits the bank to capture a portion of a consumer's gain from selling a house. We provide a rational explanation, based on a risk-sharing argument, for the existence of the due-on-sale clause and prepayment penalties. Specifically, we demonstrate that the clause is efficient in some environments, despite the deadweight loss it creates by excluding some beneficial home sales, because the loss is offset by the risk-sharing feature of the clause.

Our analysis of the bank's ex post value-maximizing strategy explains the observed willingness of banks to relax the terms of the due-on-sale clause and provides a specific characterization of the settlement demanded by the bank. We show that the bank demands the full penalty unless the market value of the loan is sufficiently low. In that case, the bank demands a prepayment penalty which is independent of the market value of the loan in order to induce additional prepayments. For example, the partial settlement could be in the form of an offer by the bank to accept prepayment at less than face value or, equivalently, in the form of an offer to allow the existing loan to be assumed at a blended rate which is less than the current market interest rate. The variation across banks and consumers of the dollar value of the settlement demanded by the bank can be explained by bank strategies being conditioned on different distributions of consumer gains from selling their homes or differences in the original mortgage agreement. Settlements of a given value can also be packaged in a variety of ways.

Our analysis of the bank's ex post strategy also illustrates that the due-on-sale clause is not an efficient mechanism for sharing interest rate risk. The sharing of interest rate risk is limited under the clause because of the reduced volume of prepayments which occurs in high interest rate settings (see Green and Shoven [4] for an empirical analysis of the effect of interest rates on prepayments). Further, the optimal (ex post) penalty is constant for sufficiently high interest rates. Alternatively, interest rate hedging in the capital market permits banks to transfer interest rate risk in a way that does not result in the deadweight loss associated with the due-on-sale clause.

We have shown that it is optimal for the lender to relax (ex post) the permitted contractual prepayment penalty under some conditions. This suggests that the design of the contractual prepayment penalty is important in mortgage contracts with ex post inflexibility. For example, pooling mortgage loans into a portfolio and selling claims to the portfolio to a group of diverse investors would create such rigidity. The inflexibility will be reflected in the initial terms of the mortgage loans and in prices of mortgage-backed securities. Although some of our results are robust in dynamic settings,[11] further analysis of the inflexibility problem requires extending our model to a richer dynamic environment and linking it to an intertemporal model for pricing mortgage loans (e.g., Dunn and McConnell [2]). Such an approach would also yield stronger market predictions than our current analysis.

It would also be interesting to use an explicit optimizing model to understand the rationale for other provisions besides the due-on-sale clause and prepayment penalties in home mortgage contracts. For example, the usual explanation that points (an initial payment) are an alternative to a call premium is not rich enough to explain the variety of coupon rate and point combinations available in the market. We suspect that a model of price discrimination could be useful in explaining the variety of observed terms as a way to address both ex ante and intertemporal adverse selection (see footnote 6), which we did not consider in this paper. Further, there is a basic tension between the need for a diverse menu of mortgage contract terms in order to allow consumers to select their optimal contracts and the need for some standardization of contract terms in order to achieve a well-functioning secondary market. An understanding of the fundamental role of alternative mortgage contract terms is a missing ingredient necessary to evaluate properly the tradeoff between the preferences of consumers and the need for some standardization of mortgage contract terms. The linkages among prepayment rates, the mortgage choice problem faced by consumers, and mortgage pricing models are examined in Dunn and Spatt [3].

[11] We have also extended our analysis to a dynamic setting (n periods) for some special cases. At any date, those who decline to sell retain the future opportunity to sell. The consumer decides whether to prepay on the basis of the current gain and the bank's current terms, taking into account the anticipated future surplus from the option to move. In the case in which the consumer's gains are independent over time, the consumer's future expected surplus and the bank's future expected profit are independent of the current realization. These can be represented by a constant shift down in demand and marginal revenue (so that marginal revenue is still decreasing) and a shift up in marginal cost (future opportunity cost to the bank of the consumer prepaying now). The form of the model is identical to the static model and Proposition 1 generalizes. Even in the presence of correlation in gains, at least a weaker form of Proposition 1 will obtain (see the analogous discussion in footnote 5). By an extension of a comparative statics argument to determine the impact of cost and demand shifts on the probability of prepayment, we also obtain under strong assumptions interesting horizon effects on the conditional probability of prepayment.

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