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An Investigation of Commodity Futures Prices Using the Consumption-Based Intertemporal Capital Asset Pricing Model

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ABSTRACT

In this paper we extend the multigood futures pricing model of Grauer and Litzenberger [9] to a dynamic discrete time setting. We then test the model using data on futures prices for corn, wheat, and soybeans. The parameter estimates we obtain are similar to those obtained by other researchers using stock return data. The model itself is rejected and we offer some suggestions as to which assumption may be violated. We also give an interpretation to the Hansen-Singleton nonlinear instrumental variables estimation technique used in our empirical work.

THE PRICING OF FUTURES contracts has continued to receive attention in the literature, ever since Keynes first advanced the hypothesis, in 1923, that hedgers purchase insurance against unfavorable future price movements by selling futures contracts to speculators. In the futures pricing models of Grauer and Litzenberger [10], Richard and Sundaresan [19], and Cox, Ingersoll, and Ross [5], among others, agents implement their consumption plans by investing in financial assets. They hedge against unanticipated movements in relative prices of commodities by trading in futures contracts. An implication of this trading policy is that the stochastic properties of futures prices are intimately related to the stochastic properties of spot prices and consumption. In this paper we extend the two-period Grauer-Litzenberger futures pricing model to a dynamic environment and test it using methods that do not require explicitly specifying the stochastic processes for futures prices and the quantities consumed.

The motivation for this analysis comes from the following considerations. If agents share risk through futures markets and if asset returns over time are independent and identically distributed and follow a joint Gaussian process, then, "returns" from futures contracts should be consistent with the Sharpe-Lintner Capital Asset Pricing Model (CAPM). Several authors have examined returns from futures contracts using the CAPM and have come up with different findings. Dusak [7] estimated the systematic risk for corn, wheat, and soybean futures during 1952–67 to be close to zero. She found the average return on the three

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contracts to be close to the risk-free rate which is consistent with her estimate of zero systematic risk. Bodie and Rosansky [2] in their study of a well-diversified portfolio of commodity futures found the average return during 1950–76 to be well in excess of the risk-free rate. However, they estimated the beta to be negative when measured relative to the S&P 500, which is inconsistent with the CAPM. Both the above studies assume that the betas which measure the systematic component of the risk in futures contracts are constant (nonstochastic) and that the S&P index of 500 stocks is a good approximation to the aggregate wealth portfolio. Carter, Rausser, and Schmitz [4] found that the systematic risks in futures contracts studied by Dusak [7] were significantly different from zero when the betas are allowed to be stochastic and when the proxy for the aggregate wealth portfolio includes futures contracts. Breeden [3] used the consumption CAPM to study futures prices. He estimated the consumption betas for several commodities and found them to be significantly different from zero, but he did not perform a full test of his pricing model.

In our study, we use the Intertemporal Capital Asset Pricing Model (ICAPM) instead of the standard CAPM. Our approach avoids measuring the return on the aggregate wealth portfolio and is consistent with fairly arbitrary systematic stochastic variations in the betas. However, it requires measuring the per capita consumption of agents in the economy. We estimate and test our model using econometric techniques that do not require explicitly specifying the stochastic processes for futures prices and quantities consumed. Our theoretical model is similar to the one Hansen and Singleton [14] use in studying stock returns. The framework for our analysis is a representative agent economy. The agent chooses consumption-investment plans to maximize the expected value of a time-additive Von Neumann-Morgenstern utility function. To derive empirically testable implications, we assume that the agent's preferences across goods belong to the homothetic class and exhibit constant relative risk aversion (CRR) for the composite consumption good. This parameterization was chosen because (a) it was used in related empirical studies of stock returns (e.g., Grossman and Shiller [11] and Hansen and Singleton [14]) and (b) it has played an important role in previous theoretical studies on asset pricing since it leads to closed form solutions.

The rest of the paper is organized as follows. Section I describes the theoretical model. Section II discusses the Generalized Method of Moments (GMM) estimates of the parameters and the testing of the model. Section III contains the summary of the results and the concluding discussions.

I. The Model

Consider a frictionless economy in which there are N goods. Assume that there is a representative agent who chooses a stochastic consumption plan so as to maximize the expected value of his or her time-additive utility function given by

$$E_0[\textstyle\sum_{t=0}^{\infty} \beta^t U(c_t)], \qquad 0 < \beta < 1 \tag{1}$$

where β is the time discount factor, c_t is an N vector of per capita consumption of the N goods by the agent, and $U(\cdot)$ is a strictly concave, thrice differentiable utility function with $U'(\cdot) > 0$, $U''(\cdot) < 0$, and $U'(0) = \infty$. Let $E_t(\cdot)$ denote the expectation operator conditioned on information available at time t.

The agent substitutes consumption today for consumption in the future by trading in financial assets. These assets include default-free one-period nominal bonds and futures contracts in the goods.[1] Prices are denoted in some arbitrary unit of account. Let

- n_t denote the number of unit discount bonds the agent holds at the end of period t—each unit discount bond will pay one unit at the beginning of period $t + 1$,
- $F(t, T, j)$ denote the futures price at time t for delivery of 1 unit of commodity j at time T,
- $M(t, T, j)$ denote the number of futures contracts on commodity j for delivery at time T held by the agent at the end of time t,
- r_{t+1} denote one plus the rate of return on a nominally risk-free unit discount bond that pays one unit at time $t + 1$,
- w_t denote the vector of endowments received by the agent during time t, and
- P_t denote the vector of nominal prices of the n commodities in this economy.

Any feasible consumption-investment plan should satisfy the following sequence of budget constraints:

$$P_t' c_t + \frac{n_t}{r_{t+1}} \le n_{t-1} + P_t' w_t + \textstyle\sum_T \sum_j [\{F(t, T, j) - F(t-1, T, j)\} M(t-1, T, j)] \quad (2)$$

It can be shown (see French [9] and Jagannathan [16]) that a first-order necessary condition for the maximization of (1) subject to (2) is:

$$\beta^t V_1(Y_t, P_t) F(t, T, j) = E_t[\beta^T V_1(Y_T, P_T)\{\Pi_{s=t+1}^T r_s\} F(T, T, j)],$$
$$j = 1, 2, \cdots, N \quad \text{and all values of} \quad T > t \quad (3)$$

In Equation (3), Y_t denotes the nominal consumption expenditure of the representative agent at time t, $V(Y_t, P_t)$ denotes indirect utility for consuming goods worth Y_t units of account when nominal prices are given by P_t, and $V_1(Y_t, P_t)$ denotes the first partial derivative of $V(Y_t, P_t)$ with respect to Y_t. French [9] showed that $\{\Pi_{s=t+1}^T r_s\} F(t, T, j)$ is the random time T payoff obtained by following a specific investment strategy that required investing $F(t, T, j)$ units of account at time t in a specific portfolio.[2] For this reason, we will call

$$\left[\{\Pi_{s=t+1}^T r_s\} \frac{F(T, T, j)}{F(t, T, j)}\right]$$

[1] The restriction of trading to futures contracts and risk-free bonds is without loss of generality, since the pricing relationship we derived will be valid even if agents have access to other financial assets as well.

[2] Consider investing $F(t, T, j)$ units in one-period bonds and buying r_{t+1} futures contracts on commodity j for delivery at date T. The value of such a decision at time $t + 1$ will be $[r_{t+1} F(t + 1, T, j)]$. Consider investing this amount in one-period bonds and buying $(r_{t+1} r_{t+2})$ futures contracts on commodity j for delivery at time T. This will result in $[r_{t+1} r_{t+1} F(t + 2, T, j)]$ at time $t + 2$. If our agent follows such a strategy until time T, he or she will end up with $\{\Pi_{s=t+1}^T r_s\} F(t, T, j)$ units at time T.

the nominal return on the "French" portfolio on commodity j with $T-t$ periods to maturity.

Now assume that $U(\cdot)$ is homothetic[3] and equals $G(u(c_t))$, where $u(\cdot)$ is linearly homogeneous and $G(\cdot)$ is any monotone function. This assumption enables us to collapse the quantities of the N goods consumed into one single quantity index. Using methods similar to Grauer and Litzenberger [10], it is shown in the Appendix that Equation (3) becomes:

$$E_t\left[\beta^{(T-t)}\frac{G^1(y_T)}{G^1(y_t)}\{\Pi_{s=t+1}^{T} r_s\}\frac{F(T,T,j)}{F(t,T,j)}\frac{i_t}{i_T}\right]=1 \tag{4}$$

In Equation (4), i_t denotes the ideal price index at time t, $y_t = Y_t/i_t$ denotes the real consumption expenditure at time t and $G^1(\cdot)$ denotes the first partial derivative of $G(\cdot)$ with respect to its argument. In our empirical work, we will assume that the representative agent's preferences can be represented by a utility function that exhibits constant relative risk aversion for consumption expenditure—i.e., we will assume that $G^1(y) = y^\alpha$ where α is a nonpositive constant and is a measure of the coefficient of relative risk aversion. This functional form has been used in earlier empirical and theoretical studies of stock and bond returns by other authors, as mentioned earlier. Equation (4) can then be written as:

$$E_t\left\{\beta^{T-t}\left(\frac{y_T}{y_t}\right)^\alpha\left[\{\Pi_{s=t+1}^{T} r_s\}\frac{F(T,T,j)}{F(t,T,j)}\frac{i_t}{i_T}\right]-1\right\}=0 \tag{5}$$

Equation (5) is the starting point for our empirical study. In the next section, we discuss the estimation of the coefficient of relative risk aversion α and the time discount factor β based on observations on real per capita consumption (nondurables + services), nominal 1-month returns on risk-free bonds, and the futures prices for corn, wheat, and soybeans. Other authors have studied this asset pricing model. Grossman and Shiller [11] have shown how to identify α and β based on a joint lognormality assumption on consumption and returns. Hansen and Singleton [14] make use of the intertemporal correlation of these variables to obtain maximum likelihood estimates of the parameters. Since we are parameterizing preferences, we want to leave the nature of the distribution of the forcing variables in the economy unspecified. Hence, maximum likelihood methods are not convenient to use. We therefore use the Generalized Method of Moments suggested by Hansen and Singleton [13] for estimation of the parameters in Equation (5). In Section II, we discuss the GMM method and the results of our empirical study.

II. Generalized Method of Moments Estimates of Parameters

In the last section, we considered futures contracts that required delivery at a specific time T in the future, to facilitate deriving the pricing relationship. For empirical work, however, it is more convenient to consider contracts entered into

[3] Homotheticity is equivalent to assuming that agents will consume one percent more of every good for every percent increase in their wealth if prices remain the same.

at time t for delivery at time period $t + k$ for some positive integer k. Using such a notation, Equation (5) becomes:

$$E_t\left[\beta^k\left(\frac{y_{t+k}}{y_t}\right)^\alpha \left\{(\Pi_{s=t+1}^{t+k}\ r_s)\ \frac{F(t+k,\ t+k,\ j)}{F(t,\ t+k,\ j)}\ \frac{i_t}{i_{t+k}}\right\}\right] = 1 \tag{6}$$

Let

$$u_{j,t+k} = \beta^k\left(\frac{y_{t+k}}{y_t}\right)^\alpha \{\Pi_{s=t+1}^{t+k}\ r_s\}\ \frac{F(t+k,\ t+k,\ j)}{F(t,\ t+k,\ j)}\ \frac{i_t}{i_{t+k}} - 1 \tag{7}$$

The unobserved random variable $u_{j,t+k}$ depends on the observed data and the unknown parameters describing preferences. Equation (6) implies that $E_t(u_{j,t+k}) = 0$, for $j = 1, 2, \cdots, J$, where J is the number of commodities being studied. Let Z_t denote the q-dimensional vector of elements included in the information set of the agent at time t. If our agent's expectations are rational and he or she makes use of all this information and possibly more in making portfolio decisions, then $u_{j,t+k}$ should be orthogonal to Z_t, i.e., it will not be possible to predict $u_{j,t+k}$ as being different from zero based on Z_t. We can state this algebraically as follows:

$$E(W_{jtk}) = 0, \qquad \text{for} \quad j = 1, 2, \cdots, J \quad \text{and all} \quad k \tag{8}$$

where $W_{jtk} = u_{j,t+k} \cdot Z_t$. Under the assumption that the $\{W_{jtk}\}_{t=0}^{\infty}$ sequence is stationary[4] and ergodic with finite second moments, we can use the Hansen-Singleton [13] nonlinear instrumental variables procedure to estimate and test the system of equations given by (8).

The Hansen-Singleton nonlinear instrumental variables procedure we use can be given the following interpretation. Let us first assume that α and β are known. Define W_{tk} as $W'_{tk} = [W'_{1tk},\ W'_{2tk},\ \cdots,\ W'_{Jtk}]$, where $'$ denotes transposition. If Equation (8) is true, it can be shown that $\bar{W}_{tk}$ is asymptotically normally distributed with zero mean and covariance matrix $\frac{S}{T}$ where

$$\begin{aligned} \bar{W}_k &= \frac{1}{T} \textstyle\sum_{t=1}^{T}\ W_{tk}, \text{ and} \\ S &= \textstyle\sum_{j=-k+1}^{k-1} E(W_{tk} W'_{tk+j}) \end{aligned} \tag{9}$$

and T is the number of observations. We can check whether the data are consistent with Equation (8) by the Wald test statistic L given by

$$L = T\bar{W}'_k S^{-1} \bar{W}_k \tag{10}$$

which has an asymptotic chi-square distribution with Jq degrees of freedom

[4] There is evidence indicating that futures prices for different delivery months behave differently. However, this does not necessarily imply that the $\{W_{jtk}\}_t$ sequence is not a stationary stochastic process. Anderson [1] has pointed out that futures prices for grains for every delivery month are more volatile in summer than in winter. While our estimation strategy can potentially allow for such seasonal patterns in volatility for futures prices, it is likely to affect the validity of our asymptotic inference due to the relatively small sample size we have used. Unfortunately the small sample properties of our nonlinear estimation methods are not known.

under the null hypothesis. However, we do not know the true parameters α and β. One natural way to estimate these parameters is to choose values that make L as small as possible. Under fairly weak assumptions about the stochastic process for $\{W_{tk}\}_t$, Hansen [12] has shown that the estimate $\hat{\alpha}_T$ and $\hat{\beta}_T$ of the parameters that minimizes L is best in the sense that any other estimator will have a greater asymptotic standard error. Since the number of unknown parameters is only 2 and there are Jq equations, the estimation procedure sets exactly two linear combinations of the Jq equations to zero. If the data indeed support the model, we should expect the other $(Jq - 2)$ linear combinations of the Jq equations to be close to zero also. We can check for this by looking at the minimized value of the criterion function L given in (10). By analogy to the case when we know α and β, we may expect L to have a chi-square distribution but with two fewer degrees of freedom. Hansen [12] has shown that it is indeed distributed as chi-square $(Jq - 2)$, under the null hypothesis that the model is true. Sufficient conditions for the consistency and asymptotic normality of the estimators and the validity of the test statistics will be satisfied if $\{W_{tk}\}_t$ is a stationary and ergodic process at the true parameter values α and β and if the functions $\{f_j(\cdot)\}_{j=1}^{J}$ satisfy certain continuity properties given in Hansen [12], where

$$f_j(X_{t+k}, Z_t, \alpha, \beta, k) = W_{j+k} \tag{11}$$

and

$$X'_{t+k} = [y_{t+k}, y_t, r_{t+1}, \cdots, r_{t+k}, F(t+k, t+k, j), F(t, t+k, j), i_t, i_{t+k}] \tag{12}$$

The function $f_j(\cdot)$ maps the vector X_{t+k} of observable random variables to the unobserved vector of random disturbances given by W_{jtk}. Using Theorem 3.1 of Hansen [12] it can be shown that the estimator $\hat{\gamma}'_T = (\hat{\alpha}_T, \hat{\beta}_T)$ is asymptotically normally distributed with mean γ_0 and covariance matrix $\frac{1}{T}(d'_0\, S^{-1}\, d_0)^{-1}$ where $d_0 = E[\partial f(\cdot) \,|\, \partial\gamma]$ is assumed to exist and be of full rank. In this expression, f denotes a J-element vector whose jth element is the vector valued function $f_j(\cdot)$ and $\partial f/\partial\gamma$ denotes the first partial of the function $f(\cdot)$ with respect to γ evaluated at γ_0. The asymptotic properties do not change if S and d_0 are replaced by any consistent set of estimates S_T and d_T.

We use monthly consumption data in our study and therefore assume that agents make decisions once a month. On the first day of every month they observe at the very least the futures prices and the nominal risk-free return available for the month before deciding how much to consume.[5] We examine futures contracts on corn, wheat, and soybeans traded on the Chicago Board of Trade during 1960:1 to 1978:12. There are 5 delivery months for corn and wheat and 7 delivery months for soybeans, in any given year. Hence, corn and wheat futures can be traded for delivery in March, May, July, September, and December. Soybeans futures can be traded for delivery in January, March, May, July,

[5] If agents make decisions at finer intervals than 1 month or if the consumption data are incorrectly aligned with the data on futures prices, then our statistical model is misspecified even if Equation (7) is true. Shiller [22] discusses how to derive testable implications that do not require aligning the consumption and return data correctly.

August, September, and November. Therefore, French portfolios with any given maturity length can be only traded in 5 selected months in a year for corn and wheat and 7 selected months of any year for soybeans.

In our study, we consider French portfolios with 2 months to maturity for corn, wheat, and soybeans. We limit our attention to contracts for delivery in March, May, July, and September, which are the only delivery months common to all three commodities. Hence, there are four observations per year on the return on the French portfolio for each commodity. We use the constant 1, the past 2-month growth rates in per capita real consumption, the 2-month holding period real return obtained by rolling over 1-month bonds for 2 months, and the spot premium given by the ratio of current futures price to the spot price[6] as elements of the instrument vector Z_t. The spot premiums were included as elements of the instrument vector, since they are a measure of the difference between the cost of carrying the commodity into the future and the service flow from carrying the inventory and may have value as instruments. When all of them are used, Z_t will be a 6 element vector and W_{tk} will be an 18 element vector. Otherwise Z_t will be a 4 element vector and W_{tk} a 12 element vector.

Estimates were obtained using monthly, seasonally adjusted, nondurables + services consumption series from the CITIBASE data tape. The observations on consumption were divided by the monthly estimates of population published by the Bureau of Census to get per capita consumption values. The monthly nominally risk-free return data were obtained from Ibbotson and Singuefield [15]. Futures prices were obtained from the Center for the Study of Futures Prices, Columbia University. Spot prices were obtained from the *Wall Street Journal.* Nominal returns were converted to real returns using the implicit price deflator corresponding to the consumption measure we use.[7] Table I presents the summary statistics for the data used in this study. It can be seen from Table I that the average of the 76 2-month holding period real returns on the French portfolios for corn and wheat considered in the study were 0.205% and 0.172%,

[6] In general, the ratio of spot to futures prices will exhibit seasonal tendencies. For example, the spot price will be higher (than the futures) before the harvest and lower during the harvest. Hence, the spot-to-futures ratio was divided by the corresponding figure for the previous year, with the hope that it will help in small samples.

[7] According to our theoretical model, the ideal price index will depend on the linearly homogeneous part of the utility function of the representative agent. Samuelson and Swamy [21] have shown that the ideal price index can be approximated by a weighted sum of the Laspeyres and Paasche indices where:

λ_p = Laspeyres index = $P_1' \cdot C_0 / P_0' \cdot C_0$, i.e., the ratio of expenditure required to purchase the bundle of goods given by the vector C_0 when prices are P_1 to the expenditure that was actually incurred in consuming the bundle C_0 of goods when prices were P_0.

π_p = Paasche index = $P_1' \cdot C_1 / P_0' \cdot C_1$, i.e., the ratio of expenditure incurred in consuming the bundle of goods given by C_1 when prices are P_1 to the expenditure that would have been incurred if the bundle C_1 of goods were consumed when prices were P_0.

Neither of these indices are published. The published Consumer Price Index (CPI) is closer to the Laspeyres index. The implicit price deflator that we use is closer to the Paasche index. We cannot, however, average the two for our purpose, since the goods that are used to construct the CPI are not the same as those that are used to construct the nondurable + services consumption series.

Table I

Summary Statistics for Data Used in This Study[a]

Item		Mean	Std Dev	Autocorrelation of Order 1	2	3	4	5	Adjusted Box-Pierce Statistic for Lag 5
Real returns on French portfolios constructed using future contracts that mature in 2 months on (commodity)	Corn	0.00205	0.08842	0.132	0.090	0.091	−0.027	0.068	2.992
	Wheat	0.00172	0.13146	0.005	0.133	−0.029	−0.057	0.006	1.753
	Soybeans	0.02716	0.13225	0.010	0.030	−0.072	−0.090	0.004	1.164
Sample of 2-month growth rates in real per capita (nondurables + services) consumption in the U.S.		0.00410	0.00541	−0.247	−0.014	0.166	−0.204	0.050	10.70

[a] The period used is 1960–78. There were 76 observations on each of the series.

Table II

Estimated Values for the Parameters of the Model (1960–73)

Instruments Used	$\hat{\alpha}$*	$\hat{\beta}$*	χ^{2}**	d.f.
1. Deseasonalized spot-futures price ratio for corn, wheat, and soybeans	−7.728 (13.831)	1.0266 (0.0424)	18.735 (0.044)	10
2. Instruments used in (1) plus lagged 2-month growth rate in real consumption (c_{t-1}/c_{t-3}) and 2 months' real return obtained by rolling over nominal risk-free bonds for two periods, known at time t	−7.481 (12.750)	1.024 (0.040)	31.213 (0.013)	16

* Standard errors in parentheses.

** Probability values in parentheses.

respectively. The corresponding average return for soybeans is 2.716%, which is substantially higher than that for corn and wheat. The sample standard deviations are also large. There is very little autocorrelation in the returns which is consistent with market efficiency and stationarity. However, the numbers themselves should be interpreted with caution since the calendar time difference between consecutive observations on the French portfolios is not a constant. This is because contracts do not expire every 2 months.

The results of the tests are summarized in Table II. The values given in parentheses in Column 3 of Table II refer to the area in the right-hand tail of the corresponding chi-square probability density function. For example, a value of 0.044 means that the probability of drawing a value greater than 18.735 from a chi-square distribution with 10 degrees of freedom is less than 0.044. As can be seen from the table, the data reject the model at the conventional 5% significance level. The data show that the residuals of the model given by $u_{j,t+2}$, $j = 1, 2, 3$, in Equation (7) cannot be made orthogonal to the elements of the vector Z_t in the information set of the representative agent for any value of α and β.

The validity of the tests depends on the assumption that the W_t series is generated by a stationary stochastic process. We examined the estimated W_t series for possible nonstationarities. Table III presents the first five autocorrelations of the elements of the W_t vector. Since lagged variables are used as elements of the Z_t vector, the W_t series starts in 1961 although the basic data series starts in 1960. The low autocorrelation figures in Table III are consistent with the stationarity assumption underlying the estimation procedure. However, as pointed out earlier, these numbers should be interpreted with caution. Table IV presents the means and standard deviations for the elements of the W_t vector for the 1961–72 and 1973–78 subperiods. This particular partition was chosen since the oil shock occurred in 1973. As can be seen, the sample standard deviations of the elements of the W_t vector for the 1973–78 subperiod are about twice the corresponding figures for the 1960–72 subperiod. To see whether this potential nonstationarity in the distribution of the elements of the W_t vector could be the cause for the rejection of the model, the parameters of the model

Table III

Sample Autocorrelations in the Elements of the W_t Vector (1961-78)

Element	Commodity Used in Constructing the "French" Portfolio	Element Z_{jt} of the Information Set Used as Instrument	Autocorrelation of Order 1	2	3	4	5	Adjusted Box-Pierce Statistic for Lag 5
W_{1t}	Corn	Constant 1	0.019	0.162	0.161	−0.075	0.107	5.381
W_{2t}	Corn	DCSP[a]	0.022	0.169	0.164	−0.075	0.109	5.694
W_{3t}	Corn	DWSP[b]	0.009	0.165	0.171	−0.090	0.116	5.894
W_{4t}	Corn	DSSP[c]	0.008	0.165	0.166	−0.080	0.114	5.739
W_{5t}	Wheat	Constant 1	0.109	−0.100	0.101	−0.023	0.022	2.516
W_{6t}	Wheat	DCSP	0.081	−0.078	0.112	−0.030	0.038	2.102
W_{7t}	Wheat	DWSP	0.110	−0.111	0.103	0.013	0.024	2.773
W_{8t}	Wheat	DSSP	0.064	−0.101	0.112	−0.039	0.040	2.313
W_{9t}	Soybeans	Constant 1	−0.002	0.052	−0.051	−0.155	0.037	2.404
W_{10t}	Soybeans	DSCP	−0.102	0.034	−0.031	−0.160	0.030	2.229
W_{11t}	Soybeans	DWSP	−0.011	0.062	−0.045	−0.154	0.033	2.396
W_{12t}	Soybeans	DSSP	−0.025	0.035	−0.035	−0.165	0.038	2.496

Note: Each element of the W_t vector is a time series of 72 observations.
[a] DCSP stands for Deseasonalized Corn Spot Premium.
[b] DWSP Stands for Deseasonalized Wheat Spot Premium.
[c] DSSP stands for Deseasonalized Soybeans Spot Premium.

were estimated separately for the two subperiods. The summary statistics for the data for the two subperiods are given in Table V and the results are given in Table VI. The sample means and sample standard deviations for the two subperiods are substantially different for the returns on the French portfolios. However, they are quite similar for the growth rate in per capita real consumption. The point estimates of the parameters for the two subperiods differ substantially. The estimate of the risk aversion parameter α for the 1960–72 subperiod averages −2.4. The corresponding figure for the 1973–78 subperiod is about −70. The standard errors of the estimates are also large. The estimates are particularly noisy for the 1973–78 subperiod. This may be because of the small (24 per series) number of observations used. However, the model itself is rejected at the 5% level in both subperiods. These results suggest that while the stationarity of the W_t process may be suspect due to the increased volatility during the 1973–78 subperiod, it may not be the cause for the rejection of the model.

In tests described so far, the model parameters were estimated using data for all three commodities simultaneously. Table VII presents the results obtained when the model was tested using data for each of the three commodities individually. Two points are worth noting. First, the data for each of the three commodities, individually, are consistent with the model. Second, the point estimates of the risk aversion parameter α vary from a high of −1.7 for soybeans to a low of −15.2 for corn, when data for the 1960–72 subperiod are used. It varies from a high of −2.4 for wheat to −23.8 for corn when data for the entire 1960–78 period are used. Estimates for the 1973–78 subperiod cannot be made for individual commodities because of the number (24 observations per series) of the observations. The standard errors of the estimates are high too. Hence, although possible,

Table IV

Mean and Standard Deviation of the Elements of the W_t Vector

Element of the W_t Vector	Commodity Used in the French Portfolio	Element Z_{jt} of the Information Set Used as Instrument	1961–78		1961–72		1973–78	
			Mean	Std Dev	Mean	Std Dev	Mean	Std Dev
W_{1t}	Corn	Constant 1	0.02431	0.1.216	0.01736	0.07132	0.03523	0.13836
W_{2t}	Corn	DCSP[a]	0.02331	0.10084	0.01667	0.07138	0.03375	0.13581
W_{3t}	Corn	DWSP[b]	0.02450	0.10414	0.01585	0.06998	0.03757	0.14290
W_{4t}	Corn	DSSP[c]	0.02419	0.10029	0.01749	0.07254	0.03471	0.13371
W_{5t}	Wheat	Constant 1	0.02318	0.13981	0.01141	0.07595	0.04168	0.20405
W_{6t}	Wheat	DCSP	0.02123	0.13579	0.01085	0.07645	0.03755	0.19679
W_{7t}	Wheat	DWSP	0.02403	0.14603	0.01083	0.07591	0.04476	0.21488
W_{8t}	Wheat	DSSP	0.02175	0.13750	0.01094	0.07620	0.03875	0.19999
W_{9t}	Soybeans	Constant 1	0.05162	0.14847	0.03580	0.08480	0.07646	0.21322
W_{10t}	Soybeans	DCSP	0.05120	0.14776	0.03536	0.08505	0.07609	0.21174
W_{11t}	Soybeans	DWSP	0.05116	0.15247	0.03517	0.08587	0.07824	0.21957
W_{12t}	Soybeans	DSSP	0.05166	0.14604	0.03568	0.08516	0.07678	0.20848

Note: Each element of the W_t vector is a time series of 72 observations.

[a] DCSP stands for Deseasonalized Corn Spot Premium.

[b] DWSP stands for Deseasonalized Wheat Spot Premium.

[c] DSSP stands for Deseasonalized Soybeans Spot Premium.

Table V

Summary Statistics for Data Used, by Subperiods

Item		1960–72 Mean	1960–72 Std Dev	1973–78 Mean	1973–78 Std Dev
Real returns on French portfolios constructed using futures contracts that mature in 2 months on (commodity)	Corn	−0.00505	0.05469	0.01949	0.13095
	Wheat	−0.00431	0.06665	0.01477	0.21504
	Soybeans	0.01093	0.06251	0.06234	0.21560
Sample of 2-month growth rates in real per capita (nondurables + services) consumption in the U.S.		0.00539	0.00604	0.00618	0.00515

Note: There were 52 observations per series during 1960–72 and 24 observations per series during 1973–78.

Table VI

Estimated Parameters for the Model Using Data for 1960–72 and 73–78

Instruments Used	1960–72 $\hat{\alpha}$*	1960–72 $\hat{\beta}$*	1960–72 χ^2**	1960–72 d.f.	1973–78 $\hat{\alpha}$*	1973–78 $\hat{\beta}$*	1973–78 χ^2**	1973–78 d.f.
1. Deseasonalized spot-futures price ratio for corn, wheat, and soybeans	−2.801	1.015	19.143	10	−108.122	1.273	9.997	10
	(3.530)	(0.011)	(0.040)		(59.725)	(0.154)	(0.441)	
2. Instruments used in (1) plus lagged 2-month growth rate in real consumption (c_{t-1}/c_{t-3}) and 2 months' real return obtained by rolling over nominal risk-free bonds for two periods, known at time t.	−2.057	1.005	39.404	16	−41.935	1.137	50.881	16
	(2.995)	(0.009)	(0.001)		(19.916)	(0.062)	(0.001)	

Note: There are 52 observations per series during 1960–72 and 24 observations per series during 1973–78.

* Standard errors in parentheses.

** p-values in parentheses.

Table VII

Estimated Parameters for the Model for Individual Commodities

		1960–72				1960–78			
Instruments Used	Commodity	$\hat{\alpha}$	$\hat{\beta}$	χ^2	d.f.	$\hat{\alpha}$	$\hat{\beta}$	χ^2	d.f.
1. Deseasonalized spot-futures price ratio for corn, wheat, and soybeans	Corn	−15.163	1.052	0.874	2	−23.798	1.069	0.277	2
		(12.242)	(0.039)	(0.646)		(25.973)	(0.074)	(0.871)	
	Wheat	−5.016	1.021	1.329	2	−2.374	1.084	2.416	2
		(6.958)	(0.022)	(0.515)		(20.383)	(0.063)	(0.299)	
	Soybeans	−1.649	0.999	0.193	2	−7.527	1.010	0.410	
		(6.684)	(0.022)	(0.908)		(20.311)	(0.062)	(0.815)	
2. Instruments used in (1) plus lagged 2-month growth rate in real per capita consumption (c_{t-1}/c_{t-3}) and 2 months' real return obtained by rolling over nominally risk-free 1-month bonds, known at time t	Corn	−13.625	1.043	3.869	4	−19.541	1.058	0.633	4
		(8.956)	(0.028)	(0.424)		(20.264)	(0.060)	(0.959)	
	Wheat	−3.854	1.018	2.595	4	−3.610	1.019	3.377	4
		(6.160)	(0.019)	(0.628)		(18.917)	(0.050)	(0.497)	
	Soybeans	3.642	1.008	3.537	4	−15.265	1.003	0.998	4
		(5.975)	(0.020)	(0.467)		(18.521)	(0.558)	(0.912)	

it is unlikely that the rejection of the model may be due to differences in the tastes of individuals participating in the markets for the different commodities.

The relative risk aversion coefficient α has an interesting interpretation. Merton [18] has shown that if returns follow a diffusion type stochastic process, then the expected excess return on the market portfolio will be given by:

$$\mu_m - \rho = -\alpha\sigma_m^2$$

where

- μ_m denotes the instantaneous expected real rate of return on the market portfolio,
- ρ denotes the instantaneous real riskless interest rate, and
- σ_m^2 denotes the variance of the instantaneous return on the market.

Our estimates of an average of about −8 for α during 1960–78 and −2.8 during 1960–72 reported in Tables II and VI are not inconsistent with the findings of other authors, although they used different data. The estimates obtained by Merton [18] for α average to −4.5527 during the period 7/58–6/78 and −5.5742 during the period 7/58–6/70. He used the monthly returns on the New York Stock Exchange Index and the U.S. Treasury Bill Index for the period 1926–78 and assumed that α is constant over 4-year intervals. His estimates for the various 4-year subperiods range from a high of −0.8337 to a low of −12.6527, which is consistent with the high standard errors associated with our estimates.

III. Discussion of the Results

In this study, we examined empirically futures prices for corn, wheat, and soybeans, using an Intertemporal Capital Asset Pricing Model (ICAPM) in a complete market setting. We chose the ICAPM instead of the static CAPM, since the risk in these markets may be stochastically varying through time in a systematic way.

In our tests, the model was rejected. It is possible that the asymptotic inference theory was not justified in our case due to the small sample size. It is also possible that some of the underlying assumptions were not satisfied. In what follows, we outline how the assumptions could be erroneous. We do so with the hope that it may be useful in linking up our work with other related work in this area and for future research.

Our assumptions could be erroneous for four reasons. First, in our empirical model we used the representative agent paradigm. This paradigm will not in general be valid if agents do not possess the same information set. Danthine [5] has pointed out that one of the important roles of futures markets is to aggregate and disseminate private information. In this case it will be necessary to model this aspect explicitly while deriving testable implications, which is difficult.

Second, we assumed that there was a representative agent whose preferences could be represented by a utility function which is time separable and homothetic and that there are no shocks to preferences. Some researchers have pointed out that time separability may not be a reasonable assumption. This especially is the case if we consider the consumption of leisure time as a good. Kydland and Prescott [17] and Eichenbaum, Hansen, and Singleton [8] explicitly model time nonseparability by constructing a new good that is a weighted sum of past consumption of goods and services.

Third, we assumed that the representative agent has frictionless access to markets. This is equivalent to assuming that all agents in the economy have frictionless access to financial markets. This may not be true if some agents face liquidity constraints. For such agents, the equality in Equation (3) will not hold. In such a case, it is necessary to use panel data. The panel should consist of agents who do not face financial constraints.

Finally, we ignored durable goods consumption, due to the difficulties associated with measuring the service flow from durables. Active rental markets for many durable goods do not exist, possibly due to relatively high transactions costs. For example, we do not find people renting clothes or household appliances on a regular basis. Further, due to adverse selection, markets for used durable goods are imperfect. Some authors have tried to model explicitly the household's technology for transforming the stock of durables into services that the household consumes. Kydland and Prescott [17] use this strategy in their effort to provide a quantitative explanation of business cycle behavior in the economy.

The foregoing discussion summarizes possible reasons for the data not supporting our consumption-based model of futures prices. We have pointed out the difficulties and mentioned how other researchers are trying to overcome them. Empirical research that links up prices and market-published quantities (e.g., volume, unlike accounting-based quantities like consumption) appear attractive since some of the problems in measuring "consumption" can be avoided.

Appendix

Assume that the period t preferences can be represented by $G(u(c_t))$ where $u(c_t)$ is linearly homogeneous, and $G(\cdot)$ is monotone.

PROPOSITION 1. *Let*

i_t *denote the agent's ideal price index at time t to be defined as* $i_t = 1/v(P_t)$, *where* $v(P_t) = V(1, P_t)$,

$y_t = Y_t/i_t$ *denote "real" consumption expenditure at time t, and*

$G'(y_t)$ *denote the first partial derivative of* $G(\cdot)$ *with respect to* y_t.

Then,

$$F(t, T, j)\beta^t G'(y_t) = E_t\left[\{\Pi_{s=t+1}^{T} r_s\}F(T, T, j)\,\frac{i_t}{i_T}\,\beta^T G'(y_T)\right]$$

Proof: Let $v(P_t)$ be the indirect utility function of the agent for a unit of consumption expenditure when prices are given by P_t. Then $v(P_t)$ solves the following maximization problem:

$$v(P_t) = \operatorname*{Max}_{c_t} u(c_t) \quad \text{subject to} \quad P_t' c_t = 1$$

Then, by linear homogeneity of $u(\cdot)$, $Y_t v(P_t)$ will solve:

$$Y_t v(P_t) = \underset{c_t}{\text{Max}}\; u(c_t) \quad \text{subject to} \quad P_t' c_t = Y_t$$

As $G(\cdot)$ is monotone, $Y_t v(P_t)$ will also solve:

$$\underset{c_t}{\text{Max}}\; G(u(c_t)) \quad \text{subject to} \quad P_t' c_t = Y_t$$

Hence, we get the result that:

$$V(Y_t, P_t) = G(Y_t v(P_t))$$

Substituting the ideal price index $i_t = 1/v(P_t)$ in Equation (3) of the text we get the desired result.

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