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On Jumps in Common Stock Prices and Their Impact on Call Option Pricing

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ABSTRACT

The Black-Scholes call option pricing model exhibits systematic empirical biases. The Merton call option pricing model, which explicitly admits jumps in the underlying security return process, may potentially eliminate these biases. We provide statistical evidence consistent with the existence of lognormally distributed jumps in a majority of the daily returns of a sample of NYSE listed common stocks. However, we find no operationally significant differences between the Black-Scholes and Merton model prices of the call options written on the sampled common stocks.

EMPIRICAL EVIDENCE CONFIRMS THE systematic mispricing of the Black-Scholes call option pricing model. These biases have been documented with respect to the call option's exercise price, its time to expiration, and the underlying common stock's volatility. Black [3] reports that the model overprices deep in-the-money options, while it underprices deep out-of-the-money options. By contrast, MacBeth and Merville [10] state that deep in-the-money options have model prices that are lower than market prices, whereas deep out-of-the-money options have model prices that are higher. These conflicting results may perhaps be reconciled by the fact that the studies examined market prices at different points in time and these systematic biases vary with time (Rubinstein [16]). Employing over-the-counter data, Black and Scholes [4] also document that the model underprices both calls written on low volatility stocks and calls near to expiration, while the model overprices calls written on high volatility stocks.

A number of explanations for the systematic price bias have been suggested (Geske and Roll [6]). Among these is the fact that the Black-Scholes assumption of a lognormally distributed security price fails to systematically capture important characteristics of the actual security price process. Merton [11, 12] has put forward an option pricing model that explicitly admits jumps in the underlying security return process, and which may resolve these pricing discrepancies. According to the Merton specification, the arrival of normal information leads to price changes which can be modelled as a lognormal diffusion, while the arrival of abnormal information, which can be modelled as a Poisson process, gives rise

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to lognormally distributed jumps in the security return. If the underlying security return follows the Poisson jump-diffusion process, then the resultant equilibrium call option price will systematically differ from the Black-Scholes equilibrium call option price.

The purpose of this paper is to contrast the Merton call option pricing formula with the corresponding Black-Scholes formula. Can this more general specification eliminate the systematic biases of the Black-Scholes option pricing model? Actual common stock returns and actual call option contracts are employed to address this question. Empirical implementation of the Merton option pricing model requires estimation of the Poisson jump-diffusion process for security returns. This paper provides maximum likelihood estimation of this security return process. Efficient estimation of the security return process permits efficient estimation of the Merton option pricing model, and hence an empirically efficient comparison with the Black-Scholes option pricing model.

The plan of this paper is as follows. The statistical estimation of the Poisson jump-diffusion process for security returns is detailed in Section I. We implement the maximum likelihood estimation procedure on the basis of daily returns to 30 NYSE listed common stocks. In Section II, we introduce and compare the Black-Scholes and Merton option pricing formulae. Assume an investor believes that security return dynamics follow a lognormal diffusion process and, as such, uses the Black-Scholes call option pricing model, when a Poisson jump-diffusion process and the Merton call option pricing model are actually appropriate. Conditions on the actual security return process and the call contract required for operationally significant differences between the investor's appraised value of the option and its actual value are examined. Systematic differences between Black-Scholes model prices and Merton model prices of all call option contracts written on our sample of 30 NYSE listed common stocks are empirically investigated in Section III. Section IV provides a summary and our conclusions.

I. Statistical Estimation of the Poisson Jump-Diffusion Process for Security Returns

Formally, we express the Poisson jump-diffusion process by the following stochastic differential equation

$$\frac{dS}{S} = \alpha\, dt + \sigma\, dZ + dq \tag{1}$$

where

S ≡ security price,
α ≡ instantaneous expected return on the security,
σ^2 ≡ instantaneous variance of the security return, conditional on no arrivals of "abnormal" information,
Z ≡ standardized Wiener process,
q ≡ Poisson process, assumed independent of Z, and
λ ≡ intensity of the Poisson process (mean number of "abnormal" information arrivals per unit time).

It is assumed that upon the arrival of "abnormal" information there is an instantaneous jump in the stock price of size Y, independent of Z. The jump size has posited distribution $\ln Y \sim N(\mu, \delta^2)$. Difficulties arise with the empirical implementation and verification of this financial process.

Press [14], a priori, constrains the instantaneous expected rate of return on the security to be zero, $\alpha = 0$. This results in a security return, X, whose density is given by

$$p(x) = \sum_{n=0}^{\infty} \frac{e^{-\lambda}\lambda^n}{n!} \phi(x; n\mu, \sigma^2 + n\delta^2)$$

where

$$\phi(x; \mu, \nu^2) = (2\pi\nu^2)^{-1/2}\exp(-(x-\mu)^2/2\nu^2).$$

Let $K_i^{(p)}$ denote the i^{th} cumulant associated with the density p for any integer i. The relationship between the cumulants and moments of a probability distribution is carefully detailed by Kendall and Stuart [9]. Press establishes

$$\begin{aligned} K_1^{(p)} &= \lambda\mu \\ K_2^{(p)} &= \sigma^2 + \lambda(\mu^2 + \delta^2) \\ K_3^{(p)} &= \lambda\mu(\mu^2 + 3\delta^2) \\ K_4^{(p)} &= \lambda(\mu^4 + 6\mu^2\delta^2 + 3\delta^4). \end{aligned} \tag{2}$$

The method of cumulants sets the sample cumulants $(\bar{K}_i^{(p)})$ equal to the population cumulants, and solves the resultant equations for the parameter estimators $\bar{\lambda}$, $\bar{\sigma}^2$, $\bar{\delta}^2$, and $\bar{\mu}$:

$$\begin{aligned} &\bar{\mu}^4 - 2\frac{\bar{K}_3^{(p)}}{\bar{K}_1^{(p)}}\bar{\mu}^2 + \frac{3\bar{K}_4^{(p)}}{2\bar{K}_1^{(p)}}\bar{\mu} - \frac{\bar{K}_3^{(p)}}{2\bar{K}_1^{(p)2}} = 0 \\ &\bar{\lambda} = \frac{\bar{K}_1^{(p)}}{\bar{\mu}} \\ &\bar{\delta}^2 = \frac{\bar{K}_3^{(p)} - \bar{\mu}^2\bar{K}_1^{(p)}}{3\bar{K}_1^{(p)}} \\ &\bar{\sigma}^2 = \bar{K}_2^{(p)} - \frac{\bar{K}_1^{(p)}}{\bar{\mu}}\bar{\mu}^2 + \frac{\bar{K}_3^{(p)} - \bar{\mu}^2\bar{K}_1^{(p)}}{3\bar{K}_1^{(p)}}. \end{aligned} \tag{3}$$

However, employing the method of cumulants and a sample of monthly returns to ten NYSE listed common stocks over the period 1926 through 1960, Press frequently obtains negative estimates of the variance parameters σ^2 and δ^2.

Beckers [2], in modifying the Press procedure, sets the mean logarithmic jump size equal to zero, $\mu = 0$, rather than $\alpha = 0$. This results in the following symmetric daily security return density

$$b(x) = \sum_{n=0}^{\infty} \frac{e^{-\lambda}\lambda^n}{n!} \phi(x; \alpha, \sigma^2 + n\delta^2).$$

Letting $K_i^{(b)}$ denote the i^{th} cumulant associated with the density b for any integer i, Beckers shows

$$\begin{aligned} K_1^{(b)} &= \alpha \\ K_2^{(b)} &= \sigma^2 + \lambda\delta^2 \\ K_3^{(b)} &= 0 \\ K_4^{(b)} &= 3\delta^4\lambda \\ K_5^{(b)} &= 0 \\ K_6^{(b)} &= 15\delta^6\lambda. \end{aligned} \tag{4}$$

The resultant method of cumulants estimators are explicitly given by

$$\begin{aligned} \underset{\sim}{\lambda} &= 25(\bar{K}_4^{(b)})^3/3(\bar{K}_6^{(b)})^2 \\ \underset{\sim}{\sigma}^2 &= \bar{K}_2^{(b)} - 5(\bar{K}_4^{(b)})^2/3\bar{K}_6^{(b)} \\ \underset{\sim}{\delta}^2 &= \bar{K}_6^{(b)}/5\bar{K}_4^{(b)} \\ \underset{\sim}{\alpha} &= \bar{K}_1^{(b)}. \end{aligned} \tag{5}$$

Applying this methodology to 47 NYSE listed common stocks, each with 500 daily return observations spanning the period September 15, 1975 through September 7, 1977, Beckers often obtains negative estimates of the variance parameters σ^2 and δ^2.

Ball and Torous [1] follow Beckers and, a priori, set the mean logarithmic jump size equal to zero, $\mu = 0$, guaranteeing a symmetric return distribution. A Bernoulli jump process is put forward as an appropriate model for information arrivals and, as such, stock price jumps. The distinguishing characteristic of the Bernoulli jump process is that over a fixed period of time either no information impacts upon the stock price, or at most one significant information arrival occurs. If jumps in stock prices correspond to the arrival of "abnormal" information, by the very definition the number of such information arrivals ought not to be very large. Furthermore, if returns were computed for finer time intervals, the Bernoulli model would converge to the Poisson model. The daily security return density becomes

$$f(x) = (1 - \lambda)\phi(x; \alpha, \sigma^2) + \lambda\phi(x; \alpha, \sigma^2 + \delta^2).$$

Due to this simplified return generating structure, Ball and Torous are able to successfully implement a maximum likelihood estimation procedure. They demonstrate the efficiency of this technique, and apply it to Beckers' sample of NYSE listed common stocks. Significantly, all variance estimates are positive and standard errors of the estimates are readily available.

Given this rather restrictive characterization of information arrivals, we now proceed to investigate the maximum likelihood estimation of the parameters of the more general Poisson jump-diffusion process for security returns. For comparative purposes, the mean logarithmic jump size is also set equal to zero, $\mu = 0$, to ensure a symmetric return distribution. The resultant Poisson jump-diffusion density, $b(x)$, should be contrasted with the corresponding Bernoulli jump-diffusion density, $f(x)$. Clearly, for small values of λ, we have that $f(x) \cong b(x)$. More importantly, we may view $f(x)$, up to order λ, as a truncation, after two terms, of $b(x)$.

Assume we have m daily security returns $\mathbf{x} = (x_1, x_2, \cdots, x_m)$ as data and denote $\boldsymbol{\gamma} = (\lambda, \sigma^2, \delta^2, \alpha)$. The logarithm of the corresponding likelihood function is

$$\ln L(\mathbf{x}; \boldsymbol{\gamma}) = \textstyle\sum_{i=1}^{m} \ln b(x_i; \boldsymbol{\gamma}). \tag{6}$$

Necessary conditions for the existence of maximum likelihood estimators $\boldsymbol{\gamma}^*$ are provided by

$$\frac{\partial \ln L(\mathbf{x}; \boldsymbol{\gamma})}{\partial \gamma_i} = 0, \qquad i = 1, 2, 3, 4$$

whereas corresponding sufficient conditions require the positive definiteness of $-H(\mathbf{x}; \boldsymbol{\gamma}^*)$, the 4×4 Hessian matrix $H(\mathbf{x}; \boldsymbol{\gamma})$ being defined by

$$H(\mathbf{x}; \boldsymbol{\gamma})_{ij} = \frac{\partial^2 \ln L(\mathbf{x}; \boldsymbol{\gamma})}{\partial \gamma_i \partial \gamma_j}, \qquad i, j = 1, 2, 3, 4.$$

First, we consider the numerical calculation of the infinite sum for a single density $b(x)$. Clearly, adequate approximation of $b(x)$ will imply adequate approximation of the likelihood function given by Equation (6). The infinite sum in $b(x)$ shall be truncated at N and the resultant approximation error denoted by $B(N)$. Since all terms are positive and

$$\sup_{x \in R, n \in N} \phi(x; \alpha, \sigma^2 + n\delta^2) \le (2\pi\sigma^2)^{-1/2},$$

it follows that

$$0 \le B(N) \le \textstyle\sum_{n=N+1}^{\infty} \dfrac{e^{-\lambda}\lambda^n}{n!} (2\pi\sigma^2)^{-1/2}.$$

Using the Taylor series expansion for the exponential function, and applying integration by parts N times sequentially, it is possible to show that

$$\textstyle\sum_{n=N+1}^{\infty} \dfrac{e^{-\lambda}\lambda^n}{n!} = e^{-\lambda} \displaystyle\int_{s=0}^{\lambda} \frac{(\lambda - s)^N}{N!} e^s \, ds.$$

However, for $0 \le s \le \lambda$, $e^s \le e^\lambda$ so

$$B(N) \le (2\pi\sigma^2)^{-1/2} \int_{s=0}^{\lambda} \frac{(\lambda - s)^N}{N!} ds.$$

Completing the integration yields the following bound on the truncation error:

$$B(N) \le (2\pi\sigma^2)^{-1/2} \frac{\lambda^{N+1}}{(N+1)!}.$$

With this error bound as a framework, a series of experiments were performed to select an optimal truncation point. Of course, the truncation error is a function of the parameter values. For plausible values of the parameters, truncation at $N = 10$ provided double precision computer accuracy. It must be pointed out, however, that for small values of λ, one could truncate at a much smaller value of N and still provide full computer accuracy. As an experiment, for large computed values of λ, the truncation point N was increased, but no evidence of truncation error was revealed. On the basis of the above calculations and the experimental evidence, we can for practical purposes provide the maximum likelihood estimates by maximizing

$$\ln L_N(\mathbf{x}; \boldsymbol{\gamma}) = \textstyle\sum_{i=1}^{m} \ln\left(\sum_{n=0}^{N} \dfrac{e^{-\lambda}\lambda^n}{n!} \phi(x_i; \alpha, \sigma^2 + n\delta^2)\right)$$

with $N = 10$. Necessary conditions for a maximum become

$$\frac{\partial \ln L_N(\mathbf{x}; \boldsymbol{\gamma})}{\partial \gamma_i} = 0, \qquad i = 1, 2, 3, 4$$

with analogous sufficient conditions for the truncated likelihood function.

In order to solve the above nonlinear system of equations, a multidimensional Newton-Raphson procedure is employed. This numerical technique is known to converge quickly (see Ortega and Rheinbolt [13]), provided the algorithm is initiated at values close to the final solution. Since the likelihood surface has a local maximum at $\lambda = 0$, corresponding to no jump structure, care must be taken in selecting the starting values for this numerical technique. As we remarked earlier, up to order λ, the Bernoulli jump-diffusion density is essentially a truncation of the Poisson jump-diffusion density after two terms. Not surprisingly then, the Bernoulli jump-diffusion maximum likelihood estimates provide excellent starting values for the Newton-Raphson iterative procedure. Except for a small percentage of cases, which required alternative starting values, satisfactory convergence was achieved within ten iterations. Both careful truncation of the likelihood function and prudent starting values contribute to the successful convergence of the numerical algorithm.

The data consist of 30 NYSE listed stocks, each with over 500 daily return observations spanning the period January 1, 1981 through December 31, 1982. The data source is the CRSP daily return file, wherein dividends are added back in computing returns and weekend returns are treated as overnight returns. The four parameters to be estimated are λ, the intensity of information arrival; σ^2, the instantaneous variance of the continuous component of the stock's return; δ^2, the variance of the logarithm of the jump; and α, the instantaneous expected return on the stock. Two sets of estimation methods are applied to the Poisson jump-diffusion process: method of cumulants and maximum likelihood estimation. The results are tabulated by security in Table I. For a given common stock, the first row displays method of cumulants estimates, while the second row provides maximum likelihood estimates. Table I also provides a test for the presence of a jump structure in the sampled common stock returns. We tabulate the likelihood ratio statistic

$$\Lambda = -2(\ln L(\mathbf{x}; \boldsymbol{\gamma}^*) - \ln L(\mathbf{x}; \boldsymbol{\gamma}^0))$$

where γ^* is the maximum likelihood estimate under the Poisson jump-diffusion specification, and γ^0 is the parameter vector estimate corresponding to the local maximum when $\lambda = 0$ and hence when no jump structure is present. Under the null hypothesis that security returns are consistent with a lognormal diffusion process without a jump structure, Λ is asymptotically χ^2-distributed with two degrees of freedom.

Unfortunately, using the cumulants method, an estimate of standard error is not readily available. However, evoking asymptotic theory, estimates of the standard errors of γ^* are obtained by examining the main diagonal of the inverse of the Hessian evaluated at γ^* (see Rao [15]). The standard errors of γ^* are introduced in Table I in parentheses under the associated parameter estimate.

Summarizing the results, we observe that the cumulants approach frequently results in negative estimates of variance. These estimates are inefficient and, furthermore, depend upon high-order sample cumulants. Erratic behavior of these high-order sample cumulants gives rise to negative variance estimates. By contrast, the maximum likelihood estimation procedure provides consistently positive estimates of variance. Moreover, based on the likelihood ratio test statistic, in the majority of cases we have evidence implying the existence of a jump structure in security returns. What implications arise for option pricing if an investor now erroneously neglects the existence of a jump structure in underlying security returns?

II. Errors in Option Pricing

The equilibrium valuation of a call option is contingent upon the underlying security return process specification. Assume that the returns to a security follow a lognormal diffusion process

$$\frac{dS}{S} = \alpha\, dt + \sigma\, dZ.$$

We wish to value a call option written on this security with exercise price K and term to expiration τ. Assume the risk-free rate of interest is constant and equal to r per unit time and assume an absence of dividend payouts. There exists a self-financing dynamic position in the underlying security and riskless bonds which replicates the call option's cash flows. An absence of arbitrage opportunities yields the Black-Scholes [5] equilibrium call option value

$$W(\sigma^2; S, K, r, \tau) = SN(d_1) - K\exp(-r\tau)N(d_2)$$

where

$$d_1 = [\ln(S/K) + (r + \sigma^2/2)\tau]/\sigma\sqrt{\tau}$$

$$d_2 = d_1 - \sigma\sqrt{\tau}$$

$N(\cdot) \equiv$ standard normal cumulative distribution function.

Alternatively, assume that the returns to the security follow a Poisson jump-

Table I

Parameter Estimates[a]

Stock	λ	$\sigma^2 \times 10^4$	$\delta^2 \times 10^4$	$\alpha \times 10^2$	Λ
Amdahl	0.997	22.564	−8.305	0.024	
	0.452	7.396	15.534	−0.158	18.00**
	(0.273)	(1.605)	(7.170)	(0.163)	
American Hospital	6.566	−2.466	0.775	0.049	
	0.297	1.428	4.066	0.050	31.20**
	(0.201)	(0.306)	(2.007)	(0.066)	
ASA	0.156	5.406	18.388	0.057	
	1.096	2.418	5.244	0.034	28.10**
	(0.664)	(1.315)	(2.250)	(0.118)	
Bethlehem Steel	1.364	−0.792	3.226	0.046	
	0.262	1.537	8.055	−0.101	36.80**
	(0.091)	(0.222)	(2.532)	(0.071)	
Boeing	1.500	−0.207	3.457	−0.061	
	0.190	2.981	10.628	−0.107	18.68**
	(0.125)	(0.498)	(5.251)	(0.091)	
Bristol	1.639	0.953	0.842	0.086	
	0.212	1.821	2.420	0.068	5.32
	(0.531)	(0.592)	(3.532)	(0.067)	
Burroughs	2.083	−0.366	1.742	−0.149	
	0.295	1.802	4.982	−0.178	14.92**
	(0.197)	(0.378)	(2.446)	(0.074)	
Citicorp	2.720	0.050	1.238	0.041	
	0.195	2.479	4.835	−0.011	6.00
	(0.315)	(0.657)	(4.876)	(0.081)	
Deere	19.579	10.388	−0.385	0.005	
	1.006	0.970	1.868	−0.008	9.35*
	(0.826)	(0.576)	(1.059)	(0.071)	
Delta	3.931	6.274	−0.758	0.061	
	0.402	2.226	2.676	0.080	6.28
	(0.616)	(0.717)	(2.593)	(0.080)	
Dow	0.031	2.900	29.028	−0.108	
	0.007	3.338	68.670	−0.051	38.36**
	(0.010)	(0.248)	(93.805)	(0.083)	
Dupont	0.093	2.269	9.619	0.014	
	0.045	2.531	14.129	0.006	14.04**
	(0.042)	(0.251)	(10.429)	(0.077)	
Esmark	0.141	1.319	18.453	0.180	
	0.593	0.641	5.010	0.035	156.18**
	(0.175)	(0.197)	(1.381)	(0.061)	
Exxon	0.011	2.032	−5.144	0.056	
	4.168	0.138	0.443	0.029	3.32
	(2.401)	(0.486)	(0.190)	(0.063)	
General Electric	0.387	1.346	1.013	0.049	
	0.100	1.534	2.046	0.043	2.34
	(0.354)	(0.360)	(4.016)	(0.059)	
General Motors	0.917	1.231	2.056	−0.036	
	0.303	1.987	3.734	−0.073	7.39*
	(0.313)	(0.490)	(2.590)	(0.077)	
Goodyear	0.965	0.185	2.528	0.107	
	0.210	1.425	5.759	0.049	47.84**
	(0.103)	(0.210)	(2.327)	(0.064)	

Table I—*Continued*

Stock	λ	$\sigma^2 \times 10^4$	$\delta^2 \times 10^4$	$\alpha \times 10^2$	Λ
Greyhound	1.042	0.361	2.875	0.044	
	0.374	1.446	5.142	0.005	47.92**
	(0.168)	(0.286)	(1.880)	(0.072)	
Honeywell	1.180	1.003	2.042	−0.013	
	0.609	1.668	2.861	−0.037	8.74*
	(0.661)	(0.760)	(2.030)	(0.079)	
IBM	0.233	1.579	4.163	−0.002	
	0.081	1.960	7.314	−0.049	21.90**
	(0.075)	(0.234)	(5.037)	(0.068)	
Kodak	1.750	4.357	−1.000	0.098	
	0.405	1.653	2.413	0.068	9.48*
	(0.507)	(0.512)	(1.959)	(0.070)	
McDonalds	11.661	6.882	−0.371	0.090	
	0.506	1.583	1.919	0.055	3.92
	(0.781)	(0.685)	(1.833)	(0.071)	
Merck	0.317	2.758	−1.837	0.042	
	2.440	0.251	0.793	0.000	12.08**
	(1.472)	(0.380)	(0.362)	(0.063)	
Northwest Airlines	7.222	−1.361	−1.361	0.013	
	1.902	2.720	2.720	−0.084	12.07**
	(0.699)	(0.835)	(0.835)	(0.097)	
J.C. Penney	0.276	1.950	7.429	0.040	
	0.174	2.416	9.105	−0.060	39.96**
	(0.111)	(0.375)	(4.591)	(0.082)	
Sperry	1.377	4.939	−1.314	−0.053	
	1.330	1.198	1.455	−0.046	9.34*
	(2.286)	(1.417)	(1.508)	(0.078)	
Syntex	1138.845	−75.015	0.069	0.099	
	0.745	1.547	2.841	0.047	19.34**
	(0.989)	(1.095)	(2.440)	(0.082)	
Texas Instruments	0.095	2.504	10.041	−0.009	
	1.010	0.956	2.450	−0.062	31.64**
	(0.671)	(0.584)	(1.161)	(0.075)	
Xerox	0.316	2.466	2.123	−0.063	
	0.033	2.889	7.494	−0.082	3.22
	(0.069)	(0.291)	(10.147)	(0.079)	
Zenith	22.866	37.913	−1.054	0.045	
	0.548	3.148	8.837	−0.125	32.40**
	(0.312)	(0.985)	(3.756)	(0.120)	

[a] For each security, the first row provides method of cumulants estimates, the second row provides maximum likelihood estimates, while the third row gives the standard errors of the maximum likelihood estimates in parentheses.

* Indicates significance at 5% level.

** Indicates significance at 1% level.

diffusion process

$$\frac{dS}{S} = \alpha\, dt + \sigma\, dZ + dq$$

where the jump size Y has posited distribution $\ln Y \sim N(\mu, \delta^2)$. In order to value this call option, we now note that it is possible to form a hedge position in the call option, the underlying security, and riskless bonds whose only source of stochastic variation is the jump component of the underlying security's return. Assume that this jump component represents diversifiable risk. In equilibrium then, the expected return to this hedge position must equal the risk-free rate of interest, giving Merton's [11, 12] equilibrium call option value

$$F(\mu, \sigma^2\lambda, \delta^2; S, K, r, \tau) = \textstyle\sum_{n=0}^{\infty} \frac{e^{-\xi\tau}(\xi\tau)^n}{n!} W((\sigma^2 + n\delta^2/\tau);$$
$$S, K, (r + n(\mu + (1/2)\delta^2)/\tau - \lambda(e^{\mu+(1/2)\delta^2} - 1)), \tau)$$

where

$$\xi = \lambda e^{\mu+(1/2)\delta^2}.$$

Now suppose an investor believes that security price dynamics follow a lognormal diffusion process and, as such, the investor uses the Black-Scholes formula to value a call option, when security price dynamics actually follow a Poisson jump-diffusion process and the Merton formulation is appropriate. Assume that the investor has a sufficiently long time series of closing prices so that the resultant estimate of the variance per unit time, ν^2, corresponds to the actual Poisson jump-diffusion process' total variance, $\sigma^2 + \lambda\delta^2$:

$$\nu^2 = \sigma^2 + \lambda\delta^2.$$

The Black-Scholes value of the call option is then given by:

$$F_e(\nu^2; S, K, r, \tau) \equiv W(\nu^2; S, K, r, \tau).$$

In order to make the subsequent analysis tractable, we follow Merton [11, 12] and assume $\mu = -\delta^2/2$ or equivalently $E[Y] = 1$, implying that there will be no effect, on average, upon the underlying security's return from any particular jump. The Merton value of the call option now simplifies to

$$F(\sigma^2, \lambda, \delta^2; S, K, r, \tau) = \textstyle\sum_{n=0}^{\infty} \frac{e^{-\lambda\tau}(\lambda\tau)^n}{n!} F_e((\sigma^2 + n\delta^2/\tau); S, K, r, \tau).$$

How will the investor's appraised value of the call option, $F_e(\nu^2; S, K, r, \tau)$, compare with the appropriate value of the call option, $F(\sigma^2, \lambda, \delta^2; S, K, r, \tau)$?

We investigate the conditions on the actual security price process so that the Black-Scholes value diverges from the Merton value.

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$$\inf_{\substack{\sigma^2=\sigma_0^2\geq 0\\ \lambda\geq 0,\delta^2\geq 0\\ \sigma^2+\lambda\delta^2=\nu^2}} F(\sigma^2, \lambda, \delta^2; S, K, r, \tau) \leq F_e(\sigma_0^2; S, K, r, \tau).$$

Proof: Let $\sigma^2 = \sigma_0^2$ where $\nu^2 \geq \sigma_0^2 \geq 0$ and choose $\lambda = \varepsilon > 0$ and

$$\delta^2 = (\nu^2 - \sigma^2)/\varepsilon \geq 0.$$

Noting that

$$\sup_{\nu^2} F_e(\nu^2;\, S,\, K,\, r,\, \tau) = S,$$

we have

$$e^{-\varepsilon\tau}F_e(\sigma_0^2;\, S,\, K,\, r,\, \tau) \le F(\sigma_0^2,\, \lambda,\, \delta^2;\, S,\, K,\, r,\, \tau)$$
$$\le e^{-\varepsilon\tau}F_e(\sigma_0^2;\, S,\, K,\, r,\, \tau) + Se^{-\varepsilon\tau}(e^{\varepsilon\tau} - 1).$$

For this particular choice of $\lambda(\varepsilon)$ and $\delta^2(\varepsilon)$, then

$$\lim_{\varepsilon\to 0} F(\sigma_0^2,\, \lambda(\varepsilon),\, \delta^2(\varepsilon);\, S,\, K,\, r,\, \tau) = F_e(\sigma_0^2;\, S,\, K,\, r,\, \tau)$$

and it follows that

$$\inf_{\substack{\sigma^2=\sigma_0^2\ge 0\\ \lambda\ge 0,\delta^2\ge 0\\ \sigma^2+\lambda\delta^2=\nu^2}} F(\sigma^2,\, \lambda,\, \delta^2;\, S,\, K,\, r,\, \tau) \le F_e(\sigma_0^2;\, S,\, K,\, r,\, \tau). \qquad \text{Q.E.D.}$$

Intuitively, for $\sigma^2 = \sigma_0^2$ fixed, $F(\sigma_0^2, \lambda, \delta^2; S, K, r, \tau)$ becomes arbitrarily close to $F_e(\sigma_0^2; S, K, r, \tau)$ by choosing λ sufficiently small and δ^2 appropriately large so that $\lambda\delta^2 = \nu^2 - \sigma_0^2$. Under these conditions, noting that the Black-Scholes formulation is monotonically increasing in variance and that $\nu^2 \ge \sigma_0^2$, the investor's appraised value will overestimate the option's appropriate value. The potential for greatest discrepancy will occur when $\sigma_0^2 = 0$. That is, the underlying security price process actually follows a pure Poisson jump process with extremely large jumps which occur very infrequently.

How large must the variance of the Poisson jump process, δ^2, be and how infrequent the occurrence of these jumps, λ, for a given variance of the continuous process, σ_0^2, before the overpricing of the appraised Black-Scholes call value becomes operationally significant? The following numerical simulations provide some insights. Short-term ($\tau = 1/12$), medium-term ($\tau = 1/4$), and long-term ($\tau = 1/2$) call options are considered. For each, in-the-money ($K = 35$), at-the-money ($K = 40$), and out-of-the-money ($K = 45$) contracts are investigated. The risk-free rate of interest is fixed at $r = 0.10$ per unit time, and the values of S are chosen so that the call options are indeed at-the-money for $K = 40$, requiring that $S = \exp(-r\tau)K$. Throughout, the annualized total variance of the actual Poisson jump-diffusion process is assumed to be $\nu^2 = 0.100$. The corresponding annualized variance of the diffusion process is fixed at $\sigma_0^2 = 0.050$. Our results are presented in Tables II, III, and IV. Notice that the actual value of the call option, $F(0.050, \lambda, \delta^2; S, K, r, \tau)$, gets arbitrarily close to $F_e(0.050; S, K, r, \tau)$ for λ sufficiently small and δ^2 sufficiently large, although not monotonically. Further, for moderately small values of λ and moderately large values of δ^2, operationally significant differences prevail between the Merton value of the call option, $F(0.050, \lambda, \delta^2; S, K, r, \tau)$, and the appraised Black-Scholes value, $F_e(0.100; S, K, r, \tau)$.

Table II

Limiting Behaviour of Short-Term Merton Option Prices: Values of $F(0.050, \lambda, \delta^2; 39\frac{5}{8}, K, 0.10, \frac{1}{12})$ for alternative specifications of λ, δ^2, and K

		$\sigma_0^2 = 0.050$, $S = 39\frac{5}{8}$, $r = 0.10$, $\tau = \frac{1}{12}$		
		$K = 35$	$K = 40$	$K = 45$
λ	δ^2	$F_e(0.050) = 4.93308$ $F_e(0.100) = 5.02609$	$F_e(0.050) = 0.99908$ $F_e(0.100) = 1.42147$	$F_e(0.050) = 0.03480$ $F_e(0.100) = 0.17364$
1.0	0.050	5.02663	1.40654	0.17411
1.0×10^{-1}	0.050×10^{1}	5.03124	1.30057	0.17830
1.0×10^{-2}	0.050×10^{2}	5.03476	0.96955	0.19306
1.0×10^{-3}	0.050×10^{3}	4.78729	0.99608	0.03378
1.0×10^{-4}	0.050×10^{4}	4.91830	0.99878	0.03470
1.0×10^{-5}	0.050×10^{5}	4.93160	0.99905	0.03479
1.0×10^{-6}	0.050×10^{6}	4.93293	0.99907	0.03479
1.0×10^{-7}	0.050×10^{7}	4.93307	0.99907	0.03480
1.0×10^{-8}	0.050×10^{8}	4.93308	0.99908	0.03480
1.0×10^{-9}	0.050×10^{9}	4.93308	0.99908	0.03480

Table III

Limiting Behaviour of Medium-Term Merton Option Prices: Values of $F(0.050, \lambda, \delta^2; 39, K, 0.10, \frac{1}{4})$ for alternative specifications of λ, δ^2, and K

		$\sigma_0^2 = 0.050$, $S = 39$, $r = 0.10$ $\tau = \frac{1}{4}$		
		$K = 35$	$K = 40$	$K = 45$
λ	δ^2	$F_e(0.050) = 5.09653$ $F_e(0.100) = 5.50545$	$F_e(0.050) = 1.73214$ $F_e(0.100) = 2.45091$	$F_e(0.050) = 0.34510$ $F_e(0.100) = 0.86040$
1.0	0.050	5.50529	2.45005	0.86010
1.0×10^{-1}	0.050×10^{1}	5.50384	2.44236	0.85740
1.0×10^{-2}	0.050×10^{2}	5.48885	2.36975	0.83255
1.0×10^{-3}	0.050×10^{3}	4.65788	1.58306	0.31540
1.0×10^{-4}	0.050×10^{4}	5.05087	1.73059	0.34201
1.0×10^{-5}	0.050×10^{5}	5.09195	1.73199	0.34479
1.0×10^{-6}	0.050×10^{6}	5.09608	1.73213	0.34507
1.0×10^{-7}	0.050×10^{7}	5.09649	1.73214	0.34510
1.0×10^{-8}	0.050×10^{8}	5.09653	1.73214	0.34510
1.0×10^{-9}	0.050×10^{9}	5.09653	1.73214	0.34510

COROLLARY

$$\inf_{\substack{\sigma^2 \geq 0, \lambda \geq 0, \delta^2 \geq 0 \\ \sigma^2 + \lambda\delta^2 = \nu^2}} F(\sigma^2, \lambda, \delta^2; S, K, r, \tau) = F_e(0; S, K, r, \tau).$$

Proof: By the Lemma

$$\inf_{\substack{\sigma^2 = 0, \lambda \geq 0, \delta^2 \geq 0 \\ \sigma^2 + \lambda\delta^2 = \nu^2}} F(\sigma^2, \lambda, \delta^2; S, K, r, \tau) \leq F_e(0; S, K, r, \tau).$$

Clearly then,

$$\inf_{\substack{\sigma^2 \geq 0, \lambda \geq 0, \delta^2 \geq 0 \\ \sigma^2 + \lambda\delta^2 = \nu^2}} F(\sigma^2, \lambda, \delta^2; S, K, r, \tau) \leq F_e(0; S, K, r, \tau).$$

However, the actual value of the call option cannot be less than its corresponding intrinsic value, $F_e(0; S, K, r, \tau)$, requiring that

$$\inf_{\substack{\sigma^2 \geq 0, \lambda \geq 0, \delta^2 \geq 0 \\ \sigma^2 + \lambda\delta^2 = \nu^2}} F(\sigma^2, \lambda, \delta^2; S, K, r, \tau) = F_e(0; S, K, r, \tau). \qquad \text{Q.E.D.}$$

This result formalizes our intuition that the potential for the largest overpricing of the appraised Black-Scholes call value relative to the appropriate Merton call value occurs when the underlying security return process follows a pure Poisson jump process with very infrequent but very large jumps.

However, the Black-Scholes estimate does not invariably overprice the appropriate Merton call value. In particular,

$$\sup_{\substack{\sigma^2 \geq 0, \lambda \geq 0, \delta^2 \geq 0 \\ \sigma^2 + \lambda\delta^2 = \nu^2}} F(\sigma^2, \lambda, \delta^2; S, K, r, \tau) \geq F_e(\nu^2; S, K, r, \tau).$$

Further, for all call option contracts there exist $\sigma^2 \geq 0$, $\lambda \geq 0$, $\delta^2 \geq 0$, such that $\sigma^2 + \lambda\delta^2 = \nu^2$, for which the actual value of the call is at least as large as its appraised value. This result obtains since (i) for all call options contracts by choosing $\sigma^2 = \nu^2$ and $\lambda = \delta^2 = 0$, it follows that

$$F(\sigma^2, \lambda, \delta^2; S, K, r, \tau) = F_e(\nu^2; S, K, r, \tau);$$

and (ii) for in-the-money and out-of-the-money call options, numerical evidence (Merton [12]) establishes that there exist $\sigma^2 \geq 0$, $\lambda \geq 0$, and $\delta^2 \geq 0$, $\sigma^2 + \lambda\delta^2 = \nu^2$, such that

$$F(\sigma^2, \lambda, \delta^2; S, K, r, \tau) > F_e(\nu^2; S, K, r, \tau).$$

Table IV

Limiting Behaviour of Long-Term Merton Option Prices: Values of $F(0.050, \lambda, \delta^2; 38, K, 0.10, ½)$ for alternative specifications of λ, δ^2, and K

		$\sigma_0^2 = 0.050$, $S = 38$, $r = 0.10$, $\tau = ½$		
		$K = 35$	$K = 40$	$K = 45$
λ	δ^2	$F_e(0.050) = 5.33908$ $F_e(0.100) = 6.06198$	$F_e(0.050) = 2.37076$ $F_e(0.100) = 3.35937$	$F_e(0.050) = 0.82985$ $F_e(0.100) = 1.68848$
1.0	0.050	6.06450	3.35834	1.68887
1.0×10^{-1}	0.050×10^{1}	6.05896	3.35345	1.68457
1.0×10^{-2}	0.050×10^{2}	6.03236	3.30090	1.65072
1.0×10^{-3}	0.050×10^{3}	5.71761	2.89329	1.34724
1.0×10^{-4}	0.050×10^{4}	5.24383	2.32847	0.81505
1.0×10^{-5}	0.050×10^{5}	5.32947	2.36649	0.82836
1.0×10^{-6}	0.050×10^{6}	5.33812	2.37033	0.82970
1.0×10^{-7}	0.050×10^{7}	5.33898	2.37072	0.82983
1.0×10^{-8}	0.050×10^{8}	5.33907	2.37075	0.82985
1.0×10^{-9}	0.050×10^{9}	5.33908	2.37076	0.82985

Numerical analysis is required to ascertain the magnitude of this underpricing by the Black-Scholes estimate. However, the Black-Scholes estimate cannot underprice the appropriate Merton call value if the call option is currently at-the-money, $S = \exp(-r\tau)K$. That is, for at-the-money call options our preceding result specializes to

$$\sup_{\substack{\sigma^2 \geq 0, \lambda \geq 0, \delta^2 \geq 0 \\ \sigma^2 + \lambda\delta^2 = \nu^2}} F(\sigma^2, \lambda, \delta^2; S, K, r, \tau) = F_e(\nu^2, S, K, r, \tau).$$

If the call option is at-the-money, then the Black-Scholes option price is strictly concave in variance. By Jensen's inequality, the appropriate value of the call cannot exceed its appraised value (Merton [11]).

In conclusion, the investor's appraised value of the call option, $F_e(\nu^2; S, K, r, \tau)$, may potentially overprice or underprice the appropriate Merton call value, $F(\sigma^2, \lambda, \delta^2; S, K, r, \tau)$. For all call option contracts, this overpricing will be significant if the underlying security return process is predominated by extremely large jumps which occur very infrequently. For at-the-money call options, the appraised Black-Scholes call value cannot underprice the appropriate Merton call value. However, numerical evidence demonstrates that the appraised Black-Scholes call value can underprice in-the-money and out-of-the-money call options, although the magnitude of this bias appears to be minimal. We graphically summarize these results in Figure 1, where the shaded region represents a subset of feasible Merton call values relative to the appraised Black-Scholes call values. Clearly, the investor's misspecification of the underlying security return process may potentially have a significant impact on call option pricing.

III. Empirical Call Option Valuation

Our previous empirical analysis provided statistical evidence consistent with the existence of lognormally distributed jumps in the sampled common stock returns,

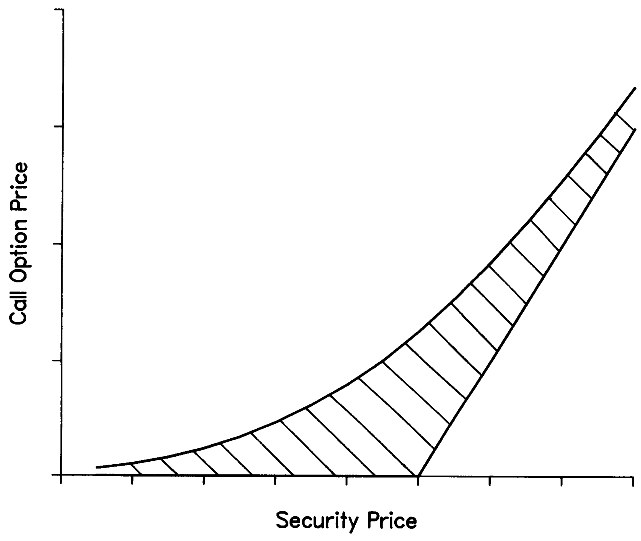

Figure 1. Feasible Region for Merton Call Values

the occurrence of these jumps being governed by a Poisson process. However, an investor ignoring this jump structure, and erroneously employing the Black-Scholes option pricing model, may potentially bring about significant errors in call option pricing. In order to empirically appraise the actual errors in call option pricing, we now proceed to implement corresponding Black-Scholes and Merton model prices of all call option contracts written on our sample of 30 NYSE listed common stocks. Whether the systematic biases of the Black-Scholes call option pricing model can be eliminated by the Merton call option pricing model is now explored.

The maximum likelihood estimation of the Poisson jump-diffusion model for security returns assumed a zero expected logarithmic jump size, $\mu = 0$. Given this particular parameter restriction on the underlying security return process, Merton's call option pricing model now becomes

$$F(\sigma^2, \lambda, \delta^2; S, K, r, \tau)$$
$$= \sum_{n=0}^{\infty} \frac{e^{-\xi\tau}(\xi\tau)}{n!} W((\sigma^2 + n\delta^2/\tau); S, K, (r + n((1/2)\delta^2)/\tau - \lambda(e^{(1/2)\delta^2} - 1)), \tau)$$

where

$$\xi = \lambda e^{(1/2)\delta^2}.$$

The corresponding Black-Scholes call option value remains

$$F_e(\nu^2; S, K, r, \tau) \equiv W(\nu^2; S, K, r, \tau).$$

Merton's call option pricing formulation requires that the jump component to an underlying security's return represent diversifiable risk, arising from firm-specific information. In equilibrium, this component of the security's return will not be rewarded with an expected return over and above the prevailing risk-free rate of interest. If the jump component of an individual security's return does represent nonsystematic risk, it should be eliminated within a well-diversified portfolio of individual securities. To empirically investigate this particular implication, daily returns to the CRSP value-weighted index over the period January 1, 1981 through December 31, 1982 were collected. The corresponding maximum likelihood estimates of the assumed Poisson-jump diffusion process' parameters are presented in Table V. The likelihood ratio test statistic provides insufficient evidence, at the 5% significance level, against the null hypothesis of an absence of jumps in the sampled daily returns of this well-diversified portfolio (see also Jarrow and Rosenfeld [7]).

Table V

Parameter Estimates: CRSP Value-Weighted Index[a]

λ	$\sigma^2 \times 10^4$	$\delta^2 \times 10^4$	$\alpha \times 10^2$	Λ
0.912	0.465	0.483	0.069	7.20
(2.230)	(0.469)	(0.694)	(0.042)	

[a] Standard errors of maximum likelihood estimates are in parentheses.

Dividends are paid to the sampled NYSE listed common stocks. It may be rational to prematurely exercise an American call option just prior to the underlying stock's ex-dividend date. This occurs when the value of the call if exercised exceeds the value of the call if held. Consequently, both the Black-Scholes and Merton call option formulae must be adjusted to take this premature exercise possibility into account. Given the actual dividends, if any, paid to the underlying common stock over the term to expiration of the call option and the corresponding ex-dividend dates, pseudo-American Black-Scholes and Merton call values are determined. The pseudo-American call model price is the maximum of the European call model price assuming no premature exercise and European call model prices with expiration dates corresponding to each of the underlying common stock's ex-dividend dates with appropriate ajustments in exercise price (Jarrow and Rudd [8]). While an approximation technique, pseudo-American valuation is necessitated since no analytical version of the Merton option pricing model exists for valuing American calls on dividend paying stocks. Pseudo-American valuation is currently employed in research and in practice. Further, it is with the pseudo-American version of the Black-Scholes option pricing model that the systematic biases have been observed. Although Whaley [17] finds that the American model dominates the pseudo-American version of the Black-Scholes model, in many cases the discrepancies are not too large.

Recall that a time series of daily returns on the sampled common stocks, January 1, 1981 through December 31, 1982, was employed to estimate the parameters of the Poisson jump-diffusion process and, alternatively, the lognormal diffusion process. The maximum likelihood parameter estimates are utilized to derive Black-Scholes and Merton model prices of all call option contracts written on our sample of 30 NYSE listed common stocks as of January 3, 1983. A total of 513 call option contracts existed. These contracts were traded either on the CBOE or the AMEX. The risk-free rate of interest employed corresponded to the return on a U.S. Treasury bill maturing as close as possible to the expiration date of the underlying option.

In the current analysis, our focus is not the comparison of model and market option prices but rather the extant differences between competing model prices. Actual market prices do differ from model prices. However, by contrasting these model prices we are able to ascertain whether such discrepancies should be attributed to the misspecification of the underlying security price process. Significant differences between these model prices would then warrant further comparison with market prices.

In Table VI we tabulate summary statistics on the percentage difference between Merton and Black-Scholes model prices, $(F - F_e)/F_e \times 100\%$, by expiration month, and then by whether the call option contracts were in-the-money, at-the-money, or out-of-the-money. In general, the mean percentage difference between Merton and Black-Scholes model prices is positive for in-the-money and out-of-the-money call options, and negative for at-the-money call options. The magnitude of this bias is minimal. For example, with reference to the May contracts, the mean percentage difference is 0.041% for out-of-the-money calls, −0.029% for at-the-money calls, and 0.001% for in-the-money calls.

Table VI

Summary Statistics for Percentage Error in Black-Scholes Prices

Expiration Month	$\overline{S/K \exp(-r\tau)}$[a]	Number of Contracts	Min.	Max.	Mean	Standard Deviation
January	0.88	29	−0.557	100.000	13.713	25.410
	1.09	29	−1.025	−0.038	−0.124	0.247
	1.45	29	−0.038	0.012	−0.004	0.011
February	0.88	15	−0.257	5.365	0.940	1.643
	1.12	15	−0.273	0.016	−0.068	0.083
	1.58	15	−0.002	0.012	0.002	0.004
March	0.88	13	−0.385	1.794	0.165	0.548
	1.04	13	−0.560	0.079	−0.078	0.154
	1.30	13	−0.197	0.006	−0.014	0.055
April	0.90	29	−0.089	0.524	0.046	0.160
	1.12	29	−0.148	0.035	−0.022	0.041
	1.49	29	−0.003	0.046	0.009	0.015
May	0.88	15	−0.031	0.112	0.041	0.101
	1.15	15	−0.078	−0.004	−0.029	0.020
	1.69	15	−0.008	0.004	0.001	0.003
June	0.91	13	−0.170	0.172	−0.015	0.078
	1.08	13	−0.161	−0.003	−0.031	0.043
	1.34	13	−0.204	0.002	−0.017	0.056
July	0.93	29	−0.052	0.082	−0.002	0.031
	1.16	29	−0.070	0.036	−0.008	0.024
	1.53	29	−0.003	0.050	0.015	0.019
August	0.93	15	−0.040	0.026	−0.011	0.022
	1.19	15	−0.041	0.025	−0.009	0.021
	1.74	15	−0.007	0.034	0.007	0.016
September	0.94	13	−0.092	0.084	−0.002	0.047
	1.11	13	−0.096	0.029	−0.013	0.030
	1.39	13	−0.033	0.045	0.008	0.023

[a] Inness and outness of the money is measured by $S/K \exp(-r\tau)$. In this column we provide the mean of the first, second, and third quantile of the distribution of this measure for each month.

However, the bias for out-of-the-money calls tends to increase with decreasing term to expiration. For out-of-the-money January calls, the mean percentage difference between Merton and Black-Scholes model prices is 13.713%. Further, the bias for short term to expiration, out-of-the-money calls may be extremely significant. The maximum percentage bias for an out-of-the-money January call is 100%. Also, for a given expiration month the standard deviation of the percentage difference between the Merton and Black-Scholes model prices is highest for out-of-the-money calls and lowest for in-the-money calls. Again with reference to the May contracts, this standard deviation is 0.101 for out-of-the-money calls, 0.020 for at-the-money calls, and 0.003 for in-the-money calls. This tendency follows since the greatest potential for percentage mispricing by the Black-Scholes model arises in deep out-of-the-money call options.

Figure 2 graphically details for January, February, and March call options the relationship between the percentage bias, $(F - F_e)/F_e \times 100\%$, and the current common stock price denominated in units of the present value of the exercise

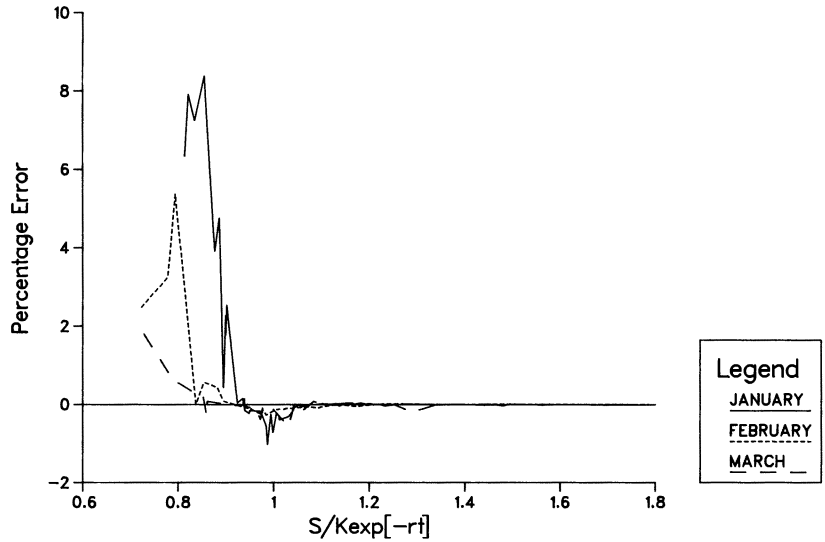

Figure 2. Percentage Bias in Option Price Misspecification

price, $S/K\ \exp(-r\tau)$. The percentage bias tends to be positive for out-of-the-money, negative for at-the-money, and negligible for in-the-money calls. This observed bias is more pronounced the shorter the term to expiration of the call option, and can be quite significant for deep out-of-the-money January calls. The somewhat erratic behaviour of this relationship reflects the fact that the observed percentage difference between the Merton and Black-Scholes model prices depends not only on whether the call option is in-the-money, at-the-money, or out-of-the-money, but also upon the volatility of the underlying security price process. This varies across call options having the same expiration date written on alternative securities.

IV. Summary and Conclusions

The Black-Scholes call option pricing model is subject to systematic empirical biases. These biases have been documented with respect to the option's exercise price, its time to expiration, and the underlying common stock's volatility. This paper has investigated whether Merton's call option pricing model can eliminate these systematic biases. Merton's specification explicitly admits jumps in the underlying common stock's return process. On the basis of a sample of daily returns to 30 NYSE listed common stocks, we indeed confirmed the presence of statistically significant jumps in a majority of these returns. However, there were no operationally significant differences between the Black-Scholes and Merton model prices of the call options written on this sample of common stocks.

Our analysis also established that significant discrepancies between Black-Scholes and Merton call prices may occur if the underlying common stock return process is predominated by large jumps which occur infrequently. Empirical evidence suggests that this is not the case, at least for our sample of common stock returns. The return to other financial securities, such as foreign exchange, may be more accurately modelled as a Poisson jump-diffusion process characterized by infrequent but very large jumps. However, statistical analysis will have difficulty discriminating between security return data characterized by a jump intensity parameter close to zero, when the Merton call formulation may be appropriate, and security return data characterized by a jump intensity parameter equal to zero, when the Black-Scholes call option pricing model obtains. So while theoretically appealing, discrepancies between Black-Scholes and Merton call prices may not be empirically demonstrable.

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