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More on Estimation Risk and Simple Rules for Optimal Portfolio Selection

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ABSTRACT

For the risk-averse investor, consideration of estimation risk is important in selecting an expected-utility-maximizing portfolio. It has previously been shown that the composition of the tangency portfolio is unaffected by the recognition of estimation risk if the Full Covariance Model is used. Alternatively, if the Market Model is used, the composition of the tangency portfolio has been shown to be affected by the recognition of estimation risk. However, as is demonstrated in this paper, the effect will generally not be as substantive as previously believed and in many situations can be safely ignored.

IN ORDER TO DETERMINE the composition and location of an efficient frontier from which an optimal portfolio will be selected, the investor must employ estimates of expected returns, variances, and covariances for the set of securities under consideration. If the investor simply treats the estimated parameter values for the securities as true parameter values, then estimation risk will have been ignored and, as shown by Barry [1] and Klein and Bawa [14, 15], a suboptimal portfolio will most likely have been selected. Bawa, Brown, and Klein [6] and Klein and Bawa [14] discuss the effects of estimation risk on the selection of an optimal portfolio from a set of risky assets and note that the location, but not the composition, of the efficient frontier changes when estimation risk is recognized if security returns are generated by a stationary multivariate normal distribution for which the investor has a diffuse prior. Furthermore, Brown [8] has shown these results to be robust for severe misspecification of the underlying multivariate distribution of returns (e.g., assuming it to be normal when in actuality it is Student t with small degrees of freedom).

This paper is concerned with the effects of estimation risk on the composition of the tangency portfolio derived from a set of risky assets and a risk-free asset under the assumption that the investor has a diffuse prior.[1] Barry [2], Bawa and Brown [4], and Brown [7] have shown analytically that the composition of the tangency portfolio is unaffected by estimation risk if the Full Covariance Model is used. Chen and Brown [10] have expanded upon the work of Brown [9] and have argued that if security returns are generated by the Market Model, then the composition of the tangency portfolio *will be* substantively different when estimation risk is recognized.

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[1] It is also assumed that there is not differential information across securities. See Barry and Brown [3] for a discussion of differential information.

In this paper, it is shown that Chen and Brown [10] have erred in extending the work of Brown to the simple portfolio selection rules developed by Elton, Gruber, and Padberg [12]. The consequence of correcting their error is that the composition of the tangency portfolio *will not be* substantially different when estimation risk is recognized relative to when it is ignored, provided the parameter estimates are based on what is generally accepted as being an appropriate number of observations. Section I reviews the algorithm of Elton, Gruber, and Padberg for determining the composition of the tangency portfolio when estimation risk is ignored, and corrects the Bayesian version of the algorithm developed by Chen and Brown. Section II demonstrates its use with an example, and Section III presents the conclusions.

I. Estimation Risk and the Elton, Gruber, and Padberg Algorithm

A. The Elton, Gruber, and Padberg Algorithm for Lintnerian Short Sales

When the parameters of the Market Model take on known values, the expected return vector and covariance matrix can be expressed as:

$$E[\underline{R}] = \underline{\alpha} + \underline{\beta}\mu_m, \tag{1}$$

and

$$\text{Cov}[\underline{R}] = \sigma^2\underline{\Omega} + \sigma_m\underline{\beta}\,\underline{\beta}', \tag{2}$$

where $\underline{\alpha}$ and $\underline{\beta}$ are $(N \times 1)$ parameter vectors of the Market Model for the N risky assets, μ_m and σ_m^2 are the expected return and variance of the market portfolio, respectively, and $\sigma^2\underline{\Omega}$ represents the $(N \times N)$ variance-covariance matrix of error terms, $\varepsilon_1, \cdots, \varepsilon_N$. (Time subscripts are deleted for convenience.)

In order to identify the tangency portfolio by employing the algorithm of Elton, Gruber, and Padberg (henceforth, the EGP algorithm) [12], $\underline{\Omega}$ is assumed to be a diagonal matrix with each element on the diagonal expressed as $\sigma_{\varepsilon_i}^2/\sigma^2$. Hence, the off-diagonal elements, $\sigma_{\varepsilon_{ij}} (i \neq j)$, are assumed to be zero.

To implement the EGP algorithm, the parameter values in Equations (1) and (2) are typically replaced by Ordinary Least Squares estimates calculated from a sample of T observations. Accordingly, use of the EGP algorithm implies that any risky security i has the following expected return, variance, and covariances, respectively:

$$\hat{\bar{R}}_i = \hat{\alpha}_i + \hat{\beta}_i\hat{\bar{R}}_m, \tag{3}$$

$$\hat{\sigma}_i^2 = \hat{\beta}_i^2\hat{\sigma}_m^2 + \hat{\sigma}_{\varepsilon_i}^2, \tag{4}$$

and

$$\hat{\sigma}_{ij} = \hat{\beta}_i\hat{\beta}_j\hat{\sigma}_m^2, \qquad i \neq j. \tag{5}$$

Under the additional assumptions of a risk-free asset yielding R_f and Lintnerian short sales, EGP have shown that the tangency portfolio weights, $X_i (i = 1, \cdots, N)$, can be found by solving the following maximization problem,[2]

$$\underset{\{X_i\}}{\text{Max}}\ \theta = \frac{\sum_{i=1}^{N} X_i(\hat{\bar{R}}_i - R_f)}{[\sum_{i=1}^{N} (X_i\hat{\beta}_i)^2\hat{\sigma}_m^2 + \sum_{i=1}^{N} X_i^2\hat{\sigma}_{\varepsilon_i}^2]^{1/2}}, \tag{6a}$$

$$\text{subject to} \quad \sum_{i=1}^{N} |X_i| = 1. \tag{6b}$$

EGP demonstrate that the solution to this problem is,

$$X_i = Z_i/\sum_{j=1}^{N} |Z_j|, \qquad i = 1, \cdots, N \tag{7}$$

where

$$Z_i = (1/\hat{\sigma}_{\varepsilon_i}^2)[\hat{\bar{R}}_i - R_f - C\hat{\beta}_i] \tag{8}$$

and

$$C = \frac{\sum_{j=1}^{N} [(\hat{\bar{R}}_j - R_f)\hat{\beta}_j/\hat{\sigma}_{\varepsilon_j}^2]}{(1/\hat{\sigma}_m^2) + [\sum_{j=1}^{N} (\hat{\beta}_j^2/\hat{\sigma}_{\varepsilon_j}^2)]}. \tag{9}$$

B. The Elton, Gruber, and Padberg Algorithms Without Short Sales

If it is alternatively assumed that short selling of securities is prohibited, nonnegativity constraints must be imposed, i.e., $X_i \geq 0$ for $i = 1, \cdots, N$. Moreover, as an initial step, this version of the EGP algorithm requires ranking securities from highest ($i = 1$) to lowest ($i = N$) by their respective values of $(\hat{\bar{R}}_i - R_f)/\hat{\beta}_i$.[3] The solution to the maximization problem in Equations (6a) and (6b) becomes:

$$X_i = Z_i/\sum_{j=1}^{k} Z_j, \qquad i = 1, \cdots, k(\leq N) \tag{10}$$

where

$$Z_i = (1/\hat{\sigma}_{\varepsilon_i}^2)[\hat{\bar{R}}_i - R_f - C_k\hat{\beta}_i] \tag{11}$$

and

$$C_k = \frac{\sum_{j=1}^{k} [(\hat{\bar{R}}_j - R_f)\hat{\beta}_j/\hat{\sigma}_{\varepsilon_j}^2)]}{(1/\hat{\sigma}_m^2) + [\sum_{j=1}^{k} (\hat{\beta}_j^2/\hat{\sigma}_{\varepsilon_j}^2)]}. \tag{12}$$

Here k is set so that $(\hat{\bar{R}}_k - R_f)/\hat{\beta}_k > C_k$ and $(\hat{\bar{R}}_{k+1} - R_f)/\hat{\beta}_{k+1} < C_{k+1}$; if $(\hat{\bar{R}}_N - R_f)/\hat{\beta}_N > C_N$, then $k = N$. Note that securities $k + 1$ to N have solution weights of zero under this version of the EGP algorithm.

[2] Lintnerian short sales refers to the situation where the investor not only must leave the short sales with his or her broker, but also must put up collateral for an equal amount. In return, the investor earns the risk-free rate on both the short sale proceeds and collateral, while also being affected by any dividends or capital gains/losses on the underlying stock. See EGP [12, p. 1343] for more details. EGP also consider the case (which will be discussed here) where short sales are prohibited. Furthermore, Elton and Gruber [11, ch. 7] have considered the case where short sales are unrestricted, meaning the investor gets to use the proceeds of the short sale.

[3] It is assumed here that all securities have positive betas.

C. Modifications in Order to Recognize Estimation Risk

In a recent article, Chen and Brown [10] modify the EGP algorithm to recognize estimation risk. The Chen and Brown paper is an extension of the work of Brown [9], who develops expressions for the mean vector and the variance-covariance matrix of the unconditional predictive distribution of returns when returns are generated by the Market Model. The conclusion of Chen and Brown is that the weight vector of the tangency portfolio is substantively different when estimation risk is recognized in comparison to when it is ignored. Chen and Brown indicate that this difference reflects the added risk of an optimal portfolio due to parameter uncertainty. However, in modifying EGP's algorithm to recognize estimation risk, Chen and Brown incorrectly interpreted the matrix $\underline{\Omega}$. Correcting this mistake leads to a modified objective function that is different than they specified. As a result, the solution weights also differ from Chen and Brown's, as will be shown below.

Under certain assumptions, Brown [9, p. 171] has shown that the unconditional predictive distribution of security returns will have a variance-covariance matrix equal to,

$$\Sigma = [v/(v-2)]\hat{s}^2[\sigma_m^{*2}K_2 + (1 + 1/T)]\ \underline{\Omega} + [\sigma_m^{*2}\underline{\hat{\beta}}\,\underline{\hat{\beta}}\,'], \tag{13}$$

where

$$v = (T-2)N, \tag{14}$$

$$\hat{s}^2 = \textstyle\sum_{i=1}^N \sum_{t=1}^T \hat{\varepsilon}_{it}^2/v, \tag{15}$$

$$\sigma_m^{*2} = [(T+1)(T-1)/T(T-3)]\hat{\sigma}_m^2, \tag{16}$$

$$K_2 = [1/(T-1)\hat{\sigma}_m^2], \tag{17}$$

and

$$\hat{\sigma}_m^2 = \{\textstyle\sum_{t=1}^T (R_{mt}^2) - [(\sum_{t=1}^T R_{mt})^2/T]\}/(T-1). \tag{18}$$

The mean return vector is identical to that used when estimation risk is ignored. According to Equations (16), (17), and (18), the middle bracketed term in Equation (13) simplifies to: $H = [(T+1)(T-2)/T(T-3)]$. To modify EGP's algorithm, the assumption must be maintained that $\underline{\Omega}$ is a diagonal matrix, with the elements on the diagonal being expressed (in estimator form) as $\hat{\sigma}_{\varepsilon_i}^2/\hat{s}^2$. Hence, the counterparts to Equations (4) and (5) when estimation risk is recognized are,

$$\hat{\sigma}_i^2 = \hat{\beta}_i^2\sigma_m^{*2} + K\hat{\sigma}_{\varepsilon_i}^2 \tag{19}$$

and

$$\hat{\sigma}_{ij} = \hat{\beta}_i\hat{\beta}_j\sigma_m^{*2}, \tag{20}$$

where for simplicity $K = vH/(v-2)$.

Under the assumption of Lintnerian short sales, the counterparts to Equations (6a) and (6b) become,

$$\underset{\{X_i^*\}}{\text{Max}}\ \theta^* = \frac{\sum_{i=1}^N X_i^*(\hat{R}_i - R_f)}{[\sum_{i=1}^N (X_i^*\hat{\beta}_i)^2\sigma_m^{*2} + K\sum_{i=1}^N X_i^{*2}\hat{\sigma}_{\varepsilon_i}^2]^{1/2}}, \tag{21a}$$

$$\text{subject to}\quad \textstyle\sum_{i=1}^N |X_i^*| = 1. \tag{21b}$$

Following EGP (as do Chen and Brown), the tangency portfolio has the following weights when estimation risk is recognized,

$$X_i^* = Z_i^* / \sum_{j=1}^N |Z_j^*|, \qquad i = 1, \cdots, N \tag{22}$$

where

$$Z_i^* = (1/\hat{\sigma}_{\varepsilon_i}^2)[\hat{\bar{R}}_i - R_f - h(T)\hat{\beta}_i] \tag{23}$$

and

$$h(T) = \frac{\sum_{j=1}^N [(\hat{\bar{R}}_j - R_f)\hat{\beta}_j / \hat{\sigma}_{\varepsilon_j}^2]}{(K/\sigma_m^{*2}) + [\sum_{j=1}^N (\hat{\beta}_j^2 / \hat{\sigma}_{\varepsilon_j}^2)]}. \tag{24}$$

A comparison of Equations (6a)–(9) with Equations (21a)–(24) indicates that they are very similar. The only difference for determining the tangency portfolio's weights is that $1/\hat{\sigma}_m^2$ in Equation (9) is replaced by K/σ_m^{*2} in Equation (24). Taking note of Equation (16) and the fact that $K = vH/(v-2)$, it is clear that $h(T) \rightarrow C$ as $T \rightarrow \infty$, since $K/\sigma_m^{*2} = [v(T-2)/(v-2)(T-1)]/\hat{\sigma}_m^2$ and $[v(T-2)/(v-2)(T-1)] \rightarrow 1$ as $T \rightarrow \infty$. For example, using $T = 276$ and $N = 6$ (the values Chen and Brown use in an example), $K/\sigma_m^{*2} = 0.998/\hat{\sigma}_m^2$. Even at $T = 60$ and $N = 6$, $K/\sigma_m^{*2} = 0.989/\hat{\sigma}_m^2$. Consequently, under an assumption of Lintnerian short sales and when using at least (roughly) 60 observations, as is recommended by Fama [13, Ch. 4] for estimating β_i, the tangency portfolio should not have a composition when estimation risk is recognized that is substantively different from its composition when estimation risk is ignored.[4]

A similar conclusion can be drawn when recognizing estimation risk under an assumption of no short sales. In this case, the counterparts to Equations (10)–(12) become:

$$X_i^* = Z_i^* / \sum_{j=1}^k Z_j^*, \qquad i = 1, \cdots, k(\leq N) \tag{25}$$

where

$$Z_i^* = (1/\hat{\sigma}_{\varepsilon_i}^2)[\hat{\bar{R}}_i - R_f - \Phi_k\hat{\beta}_i] \tag{26}$$

and

$$\Phi_k = \frac{\sum_{j=1}^k [(\hat{\bar{R}}_j - R_f)\hat{\beta}_j / \hat{\sigma}_{\varepsilon_j}^2]}{(K_k/\sigma_m^{*2}) + [\sum_{j=1}^k (\hat{\beta}_j^2 / \hat{\sigma}_{\varepsilon_j}^2)]}, \tag{27}$$

and k is set so that $(\hat{\bar{R}}_k - R_f)/\hat{\beta}_k > \Phi_k$ and $(\hat{\bar{R}}_{k+1} - R_f)/\hat{\beta}_{k+1} < \Phi_{k+1}$. In Equation (27), $K_k = (T-2)kH/[(T-2)k-2]$, i.e., K_k is dependent on the number of securities (k) entering the tangency portfolio. The only difference between Equations (10)–(12) and (25)–(27) is that $1/\hat{\sigma}_m^2$ in Equation (12) is replaced by K_k/σ_m^{*2} in Equation (27). However, $\Phi_k \rightarrow C_k$ as $T \rightarrow \infty$, since $K_k/\sigma_m^{*2} = [(T-2)k(T-2)/((T-2)k-2)(T-1)]/\hat{\sigma}_m^2$ and $[(T-2)k(T-2)/((T-2)k-2)(T-1)] \rightarrow 1$ as $T \rightarrow \infty$, regardless of the value of k. For example, when $k = 1$ and

[4] Recognition of estimation risk will generally mean that the risk-averse investor will put a higher percentage of his or her funds into the risk-free asset, and a corresponding smaller percentage into the tangency portfolio. See Klein and Bawa [14].

$T = 60$, $K_k/\sigma_m^{*2} = 1.018/\hat{\sigma}_m^2$; when $k = 1$ and $T = 276$, $K_k/\hat{\sigma}_m^2 = 1.004/\hat{\sigma}_m^2$. When $k = 2$ and either $T = 60$ or $T = 276$, $K_k/\sigma_m^{*2} = 1.000/\hat{\sigma}_m^2$. Consequently, under an assumption of no short sales and when using at least (roughly) 60 observations, the tangency portfolio should not have a composition when estimation risk is recognized that is substantively different from its composition when estimation risk is ignored.

Chen and Brown conclude that the solution vectors will be substantively different and they provide an example to demonstrate this. However, as mentioned earlier, they have incorrectly interpreted the matrix $\underline{\Omega}$. Instead of having (in estimator form) $\hat{\sigma}_{\varepsilon_i}^2/\hat{s}^2$ on the diagonal, they interpret the diagonal elements to be merely $\hat{\sigma}_{\varepsilon_i}^2$. As a consequence, in the Lintnerian model Equations (21a), (23), and (24), they would have the term $\hat{\sigma}_{\varepsilon_i}^2\hat{s}^2$ everywhere instead of $\hat{\sigma}_{\varepsilon_i}^2$. They maintain that their model will still collapse to Equations (6a)–(9) as $T \to \infty$ because $\hat{s}^2$ approaches unity as T increases (see Chen and Brown [10, footnote 7]). However, this will never be true, as can be seen by noting the formula for $\hat{s}^2$ in Equation (15). A similar error was made in their versions of Equations (26) and (27) (see Chen and Brown [10, Equations (10) and (11)]).[5]

II. An Example

To demonstrate that the composition of the tangency portfolio is not substantively affected by the recognition of estimation risk, the example of Chen and Brown, presented in Table I, will be used. In Table II, the weights of the tangency portfolio recognizing estimation risk (labeled "Bayesian Versions") as calculated by Chen and Brown are presented along with the corrected weights and the weights that ignore estimation risk (labeled "Traditional Version"). These weights are presented for both the Lintnerian and the no-short-selling assumptions. The corrected Bayesian version produces optimal weights nearly identical to the traditional version in each case.

Table III presents the weights calculated according to the corrected Bayesian version under both short selling assumptions for different hypothetical sample sizes.[6] As should be the case, it can be seen that the $T = \infty$ weights are identical to the values obtained using the traditional versions as shown in Table II. Note that the weights diverge more and more from their $T = \infty$ values as T gets progressively smaller. Interestingly, these deviations are more pronounced for the no-short-sales case than for the Lintnerian-short-sales case. However, for values of $T \geq 10$, the weights are very close (within 0.01) to their respective values at $T = \infty$ for either short selling assumption. Even at $T = 5$ the difference in weights is not substantive.

[5] Note that the corrected Lintnerian version presented here in Equations (21a)–(24) does collapse to Equations (6a)–(9) when $T \to \infty$. In addition, the corrected no-short-selling version presented here in Equations (25)–(27) collapses to Equation (10)–(12) as $T \to \infty$.

[6] The analysis presented in Table III assumed that the estimates presented in Table I, except for σ_m^{*2}, would remain the same for all sample sizes. In the case of σ_m^{*2}, it was assumed that σ_m^2 was the same for all sample sizes (meaning that σ_m^{*2} will decrease as T increases). While not reported here, Table III was also calculated keeping σ_m^{*2} constant; the results were not substantively different from what is presented here.

Table I

The Risk-Return Characteristics of Six Stocks

Security	$\hat{\bar{R}}_i$	$\hat{\beta}_i$	$\hat{\sigma}^2_{\epsilon_i}$	$\frac{(\hat{\bar{R}}_i - R_f)}{\hat{\sigma}^2_{\epsilon_i}}$
1	0.00937	1.175	0.00638	0.9201
2	0.00705	0.522	0.00179	1.9832
3	0.00944	1.160	0.00381	1.5591
4	0.00126	0.832	0.00386	−0.5803
5	0.01027	0.925	0.00197	3.4365
6	0.01019	0.724	0.00257	2.6031

$\hat{s}^2 = 0.00339$, $\sigma^{*2}_m = 0.00168$, $R_f = 0.0035$

Table II

Weights for the Tangency Portfolio

	Security					
	1	2	3	4	5	6
A. Lintnerian Short Sales:						
Bayesian Versions:						
Chen and Brown	−0.1818	0.0736	−0.0238	−0.3481	0.1664	0.2063
Corrected	0.0321	0.1408	0.0596	−0.2456	0.2683	0.2535
Traditional Version*	0.0322	0.1409	0.0597	−0.2454	0.2684	0.2535
B. No Short Sales:						
Bayesian Versions:						
Chen and Brown	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
Corrected	0.0218	0.1857	0.0469	0.0000	0.3689	0.3768
Traditional Version*	0.0219	0.1857	0.0471	0.0000	0.3688	0.3766

* The values reported here differ slightly (by 0.0004 or less) from those reported by Chen and Brown.

III. Conclusion

Consideration of estimation risk is important in the implementation of Modern Portfolio Theory. In the presence of a risk-free asset, the efficient frontier is known to consist of linear combinations of the risk-free asset and a tangency portfolio. If the Full Covariance Model is used, it has been shown elsewhere that the composition of the tangency portfolio when estimation risk is recognized is identical to its composition when estimation risk is ignored. If the Market Model is used, it is shown here that for all practical purposes (e.g., as long as T is 60 or larger and N is 6 or larger) Elton, Gruber, and Padberg's algorithm need not be modified to incorporate estimation risk in identifying the tangency portfolio. Instead, it may be used as originally presented since there is little substantive difference in the tangency portfolio's composition when estimation risk is recognized relative to when it is ignored. Of course, it should be kept in mind that the location of the efficient frontier will shift, even though the tangency portfolio's

Table III

Weights for the Tangency Portfolio for Different Hypothetical Sample Sizes Using the Corrected Bayesian Version

	Security					
Sample Size	1	2	3	4	5	6
A. Lintnerian Short Sales:						
5	0.0276	0.1374	0.0524	−0.2628	0.2646	0.2552
10	0.0302	0.1394	0.0565	−0.2531	0.2667	0.2542
20	0.0312	0.1402	0.0582	−0.2490	0.2676	0.2538
50	0.0318	0.1406	0.0591	−0.2468	0.2681	0.2536
100	0.0320	0.1407	0.0594	−0.2460	0.2682	0.2535
200	0.0321	0.1408	0.0596	−0.2457	0.2683	0.2535
∞	0.0322	0.1409	0.0597	−0.2454	0.2684	0.2535
B. No Short Sales:						
5	0.0125	0.1852	0.0327	0	0.3747	0.3928
10	0.0177	0.1855	0.0407	0	0.3714	0.3847
20	0.0199	0.1856	0.0441	0	0.3700	0.3804
50	0.0211	0.1856	0.0459	0	0.3692	0.3780
100	0.0215	0.1857	0.0465	0	0.3690	0.3773
200	0.0217	0.1857	0.0468	0	0.3689	0.3769
∞	0.0219	0.1857	0.0471	0	0.3688	0.3766

composition is unaffected, when estimation risk is recognized.[7] This means that the expected-utility-maximizing, risk-averse investor will generally invest more in the risk-free asset and less in the tangency portfolio when estimation risk is recognized relative to when it is ignored.

[7] The denominator of Equation (6a) represents the estimator of portfolio risk, $\hat{\sigma}_p$, under both short selling assumptions when estimation risk is ignored. The denominator of Equation (21a) represents the estimator of portfolio risk, $\hat{\sigma}_p^*$, under Lintnerian short sales when estimation risk is recognized. Under the no-short-sales assumption, the K in the denominator of Equation (21a) is replaced by K_k, recognizing that $X_i^* = 0$, for $i = k + 1$ to N, in calculating $\hat{\sigma}_p^*$. Accordingly, recognition of estimation risk will mean that the tangency portfolio will have shifted its location by a horizontal distance equal to $\hat{\sigma}_p^* - \hat{\sigma}_p$ (without any shift vertically).

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