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## International Asset Pricing under Mild Segmentation: Theory and Test

VIHANG ERRUNZA and ETIENNE LOSQ\*

### ABSTRACT

This paper conducts a theoretical and empirical investigation of the pricing (and portfolio) implications of investment barriers in the context of international capital markets. The postulated market structure—labelled “mildly segmented”—leads to the existence of “super” risk premiums for a subset of securities and to a breakdown of the standard separation result. The empirical study uses an extended data base including LDC markets and provides tentative support for the mild segmentation hypothesis.

THE QUESTION AS TO whether the international capital market is integrated or segmented appears particularly elusive. Indeed, the difficulties surrounding this important issue abound, as was made vividly clear by Solnik [20].

At the risk of tackling too ambitious a task, we undertake here to build a model and develop an empirical methodology to provide at least a partial answer to the segmentation-integration issue. To do so, we follow one of Solnik's recommendations [20, p. 505]:

The efficient way to test for segmentation would seem to be to specify the type of imperfection which might create it and study its specific impact on portfolio optimality and asset pricing.

The specific imperfection we introduce relates to the assumed inability of a class of investors to trade in a subset of securities as a result of portfolio inflow restrictions imposed by some governments. From this starting point, we derive a valuation model and conduct an empirical analysis. As a whole, this paper has the following distinctive features:

1. Using a new concept of risk—conditional market risk—the model yields a closed-form solution for the equilibrium risk-return tradeoff in segmented markets.<sup>1</sup> Specifically, the securities inaccessible to a subset of investors

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<sup>1</sup> By way of contrast, such an explicit representation of the risk-return tradeoff is generally missing in most of the well-known models of market segmentation. For example, in a recent paper on international asset pricing with barriers to entry, Stulz [24] fails to specify the risk-return tradeoff for at least one class of assets (those which are not traded by all investors). As for the oft cited paper of Adler and Dumas [1], the purpose is not really to characterize the equilibrium relationship between

command a super risk premium that is proportional to the conditional market risk.

2. The model lends itself to the analysis of a continuum of market structures, with the two polar cases corresponding to complete (one-way) segmentation and complete integration, respectively. Although there exist a number of papers dealing with the segmentation-integration issue,<sup>2</sup> few accommodate intermediate—and more realistic—structures where the markets are neither completely segmented nor completely integrated. The paradigm which follows was designed to deal with this more realistic problem, and thus follows the lead of Black [3] and, more specifically, Stulz [24].<sup>3</sup>
3. The kind of imperfection which constitutes the source of segmentation in the model appears quite prevalent in the international arena.<sup>4</sup> Indeed, access to the capital markets of many countries is severely restricted for nonresidents; in contrast, intra-country restrictions on portfolio investment are typically much less stringent. A model of market segmentation would thus seem more appropriate in an international context than it would in a purely domestic setting.
4. Finally, this paper tests the segmentation hypothesis by using an expanded data base. This data base includes not only the standard U.S. securities but also the returns on selected common stocks heavily traded in Less Developed Countries (LDCs). On a priori grounds one would expect the degree of segmentation between LDC and U.S. markets to be higher than between European and U.S. markets. As a result, the ex ante probability of obtaining inconclusive results should be lower. The test results are not statistically inconsistent with the mild segmentation hypothesis.

The analytical derivation of the asset pricing model under conditions of market segmentation is presented in Section I. In Section II, this valuation model is transformed to derive empirically testable hypotheses. Finally, Section III presents the test methodology and reports empirical results. A brief conclusion follows.

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risk and return in markets which are neither fully segmented nor fully integrated. A similar comment can be made concerning the model of Lee and Sachdeva [12] where the focus is also more on the multinationals' investment policies than on asset pricing. In the paper of Stapleton and Subrahmanyam [21], asset prices are indeed determined for different market structures but only numerical, as opposed to analytical, solutions are provided. Finally, the model of market segmentation proposed by Glenn [10] in a domestic context also falls short of yielding a closed-form representation of the risk-return tradeoff since his pricing formula involves an endogenous variable (namely, the final wealth of those investors whose portfolio investments are restricted).

<sup>2</sup> See, for instance, Solnik [18], Grauer et al. [11], Stulz [23], and especially Stehle [22].

<sup>3</sup> The model of this paper can be seen as the limiting case of the more general framework developed by Stulz [24]. Indeed, while in Stulz the cost of investing abroad is finite, in this model it is prohibitively high, so that domestic (U.S.) investors trade primarily in U.S. securities. In this extreme case, the problem can be completely solved while in the more general case Stulz could only obtain partial results. Our model can also be contrasted to that of Mayers [15]; indeed, while Mayers focuses on the concept of nonmarketability, we introduce the more general notion of restricted marketability.

<sup>4</sup> In contrast, Black's model is premised on a type of imperfection which appears somewhat artificial. Indeed, one recalls that he assumes proportionality between the cost of investing abroad and the *net* foreign position, which amounts to assuming the existence of a subsidy for short-selling foreign securities.

## I. A Model of International Market Segmentation

### A. Model Set-Up

We consider an idealized representation of an international capital market characterized by the following:

- (i) *Unequal Access Assumption.* A subset of the investing population—the unrestricted investors—can trade in all the securities available; the others, labelled the restricted investors, can trade only in a subset of the securities, those which are termed eligible; the noneligible or ineligible securities can thus be held only by the unrestricted investors.
- (ii) *Perfect Capital Market Assumption.* The different national capital markets are perfect and frictionless (no taxes, no transaction costs, . . .).
- (iii) *Mean-Variance Assumption.* The expected utility of each investor can be represented as a function of the expected value and the variance of the *real* returns on the investment portfolio.<sup>5</sup>
- (iv) *Free Lending and Borrowing Assumption.* Each investor can freely lend and borrow at the same real rate of interest.<sup>6</sup>
- (v) *Normality Assumption.* The real returns are assumed to be normally distributed.<sup>7</sup>

For the sake of illustration, we may focus on a two-country capital market where country 1 investors are restricted, country 2 investors are unrestricted, country 1 securities are eligible and country 2 securities are ineligible (for country 1 investors). Specifically, portfolio inflow restrictions imposed by the government of country 2 prevent country 1 investors from holding country 2 securities; whereas no such controls are imposed by the government of country 1. We shall characterize such a market structure by the term “mild segmentation.”

### B. Definitions and Notation

Before turning to the main results of the model, we need to introduce several notations and definitions.

<sup>5</sup> Explicit treatment of exchange risk is thus made unnecessary by the set of assumptions which are adopted in this paper for the sake of tractability. If it had to be recognized that the prices of consumption goods may vastly differ from one country to the rest, or that nationals of different countries do not necessarily share the same consumption preferences, or that tax systems may be highly discriminatory, then the problem of exchange risk could not be so conveniently dismissed. For a discussion of firm valuation under exchange risk and differential taxation, see Senbet [17]. A more modest objective is pursued in this paper which deals with only one imperfection of the international capital market. Also note that we deal only with the demand side. An equilibrium analysis in a framework that allows supply adjustments under barriers to capital flows is discussed in Errunza and Senbet [8].

<sup>6</sup> Clearly, in today's capital markets there does not exist a security which would be riskless in real terms. However, if the investment horizon is comparatively short, then, because of the low degree of uncertainty surrounding the short-term inflation rate, Treasury Bills might not constitute too unreasonable proxies for the riskless asset. For empirical treatment, see Section III.

<sup>7</sup> In the continuous time version of the model, the instantaneous returns are assumed to follow a stationary diffusion process which implies that the security prices are lognormally distributed. Preliminary results in Errunza and Losq [7] suggest that stock prices in various countries fit this specification reasonably well.

*Notation*—The subscript  $e$  ( $i$ ) is used as a generic index to represent the eligible (ineligible) securities. The capital letter  $R$  stands for rate of return, the tilde denotes randomness, the prime the transposition operator, and the inferior bar a vector; thus  $R_i$  is the rate of interest and  $\tilde{R}$  the vector of rates of return on the risky securities. This vector  $\tilde{R}$ , the variance-covariance matrix  $V$ , and the vector of aggregate market values  $\underline{P}$  are partitioned as follows:

$$\tilde{R} = \begin{bmatrix} \tilde{R}_e \\ \tilde{R}_i \end{bmatrix}; \quad V = \begin{bmatrix} V_{ee} & V_{ei} \\ V_{ie} & V_{ii} \end{bmatrix}; \quad \underline{P} = \begin{bmatrix} \underline{P}_e \\ \underline{P}_i \end{bmatrix}$$

Because of the mildly segmented structure of the market, we also need to introduce three market portfolios:

- The World Market Portfolio (WMP); market value:  $M$ ; rate of return:  $\tilde{R}_M$ ; representative vector:  $\underline{WMP} = \underline{P}$ .
- The Market Portfolio of Ineligible Securities (MPIS); market value:  $M_I$ ; rate of return:  $\tilde{R}_I$ ; representative vector:  $\underline{MPIS} = \begin{bmatrix} 0 \\ \underline{P}_i \end{bmatrix}$ .
- The Market Portfolio of Eligible Securities (MPES); market value:  $M_E$ ; rate of return:  $\tilde{R}_E$ ; representative vector:  $\underline{MPES} = \begin{bmatrix} \underline{P}_e \\ 0 \end{bmatrix}$ .

Finally, the subscripts  $u$  and  $r$  denote the unrestricted and the restricted investors, respectively. Thus  $A_u$  ( $A_r$ ) stands for the absolute risk-aversion coefficient for the subset of unrestricted (restricted) investors. The capital letter  $A$  denotes the risk-aversion coefficient for the aggregate population of investors ( $A^{-1} \equiv A_u^{-1} + A_r^{-1}$ ) and the greek letter  $\mu$  the ratio  $A/A_r$ .

*Definition 1:* The *Diversification Portfolio* (DP) is a portfolio of eligible securities where the dollar amounts invested in the various securities are given by the vector  $\underline{DP} \equiv \begin{bmatrix} V_{ee}^{-1} V_{ei} \underline{P}_i \\ 0 \end{bmatrix}$ ; it can be shown (see, for example, Dhrymes, Theorem

2, [4, p. 24]) that DP corresponds to that portfolio of eligible securities which is most highly correlated with the MPIS; hence, the name “Diversification Portfolio.”

*Definition 2:* The *Hedged Portfolio* (HP) consists of a long position in MPIS and a short position in DP:  $\underline{HP} \equiv \underline{MPIS} - \underline{DP}$ ; using the aforementioned theorem in Dhrymes, it can be verified that HP bears no correlation with any of the eligible securities, hence the name “Hedged Portfolio.”

*Definition 3:* The *conditional market risk* of a security is defined as the conditional covariance between its return and the return on the market portfolio of all ineligible securities, the returns on all eligible securities being given. Thus, we characterize the conditional market risk as:  $\text{Cov}[\tilde{R}_i, \tilde{R}_I | \{\tilde{R}_1 = R_1, \dots, \tilde{R}_e = R_e, \dots\}]$ . For an eligible security, the risk is zero by construction; indeed,

$$\text{Cov}[\tilde{R}_e, \tilde{R}_I | \{\tilde{R}_1 = R_1, \dots, \tilde{R}_e = R_e, \dots\}] = \text{Cov}[R_e, \tilde{R}_I] = 0 \quad (1)$$

Further, we recall from statistical theory (see, for example, Dhrymes, Theorem

2, [4, p. 24]) that in the case of a multivariate normal distribution, the conditional covariance does not depend on the given values  $\{\underline{R}_e = \underline{R}_1, \dots, R_e, \dots\}$ ; hence, the more compact notation  $\text{Cov}[\tilde{R}_i, \tilde{R}_I | \tilde{R}_e]$ . It is also a simple matter to verify that the conditional market risk is proportional to the covariance with the return on HP.<sup>8</sup>

### C. Risk, Return, and Portfolio Composition under Mild Segmentation

**PROPOSITION 1.** *In a mildly segmented market under conditions (i) to (v) and at equilibrium:*

- (a) *the eligible securities are priced as if the market was not segmented:*

$$E(\tilde{R}_e - R_f) = (AM)\text{Cov}[\tilde{R}_e, \tilde{R}_M] \quad (2)$$

- (b) *the ineligible securities command a “super” risk premium which is proportional to the conditional market risk:*

$$E(\tilde{R}_i - R_f) - (AM)\text{Cov}[\tilde{R}_i, \tilde{R}_M] = (A_u - A)M_I\text{Cov}[(\tilde{R}_i, \tilde{R}_I) | \tilde{R}_e] \quad (3)$$

with  $(A_u - A)M_I \geq 0$ .

- (c) *the unrestricted investors hold the portfolio  $D_u$  defined by:  $\underline{D}_u \equiv (1 - \mu) \underline{\text{WMP}} + \mu \underline{\text{HP}}$ .*
- (d) *the restricted investors hold the portfolio  $D_r$  defined by:  $\underline{D}_r \equiv \mu(\underline{\text{MPES}} + \underline{\text{DP}})$ .*

*Proof:* Please refer to Appendix A.

### Interpretation

1. Under mild segmentation, the eligible securities are priced as if the markets were completely integrated; this property should be contrasted with the results of Stulz [24] where it is not generally true that the eligible securities plot on a security market line which corresponds to the Sharpe-Lintner relationship (see Stulz [24, p. 930]).

2. The ineligible securities command a super risk premium which, on average, is positive. By aggregating Equation (3) over the set of ineligible securities, we obtain:

$$E(\tilde{R}_I - R_f) - (AM)\text{Cov}[\tilde{R}_I, \tilde{R}_M] = (A_u - A)M_I\text{Var}[\tilde{R}_I | \tilde{R}_e] \geq 0 \quad (4)$$

Further, if we denote by  $\rho$  the multiple correlation coefficient between  $\tilde{R}_I$  and the random vector  $\tilde{R}_e$ , we can show (see Dhrymes, Theorem 2, [4, p. 24]):

$$\text{Var}[\tilde{R}_I | \tilde{R}_e] = (1 - \rho^2)\text{Var}[\tilde{R}_I] \quad (5)$$

$\rho$  can be interpreted here as the correlation coefficient between  $\tilde{R}_I$  and that portfolio of eligible securities which is most correlated with  $\tilde{R}_I$ , i.e., the “Diversification Portfolio.”

Equation (4) gives the super risk premium for the market portfolio of ineligible securities and thus provides us with a measure of the increase in required return which the ineligible securities must yield because of the segmented nature of the

<sup>8</sup> We are grateful to René Stulz for pointing out this interpretation to us.

market. From the viewpoint of firms, this super risk premium also measures the effect of segmentation on the cost of supplying risky securities in the ineligible segment of the market. As is made clear by Equations (4) and (5), the effect of market segmentation becomes more pronounced as the risk aversion of the unrestricted investors increases and as the correlation between the two segments of the market decreases.<sup>9</sup>

3. The super risk premiums vanish only in two limiting cases, which we discuss in turn.

*Limiting case no. 1:*  $A_r/A_u \rightarrow \infty$ . As the unrestricted investors become much less risk-averse than the restricted investors, the following holds:

- $\mu \equiv (1 + A_r/A_u)^{-1} \rightarrow 0$ . According to parts (c) and (d) of Proposition 1, the unrestricted investors tend to hold all of the risky securities and the restricted investors none of them.
- $(A_u - A)/A \equiv A_u/A_r \rightarrow 0$ . According to part (b) of Proposition 1, the “super” risk premium becomes negligible relative to the “normal” risk premium.

*Limiting case no. 2:*  $\rho \rightarrow 1$ . As the multiple correlation coefficient  $\rho$  between  $\tilde{R}_I$  and  $\tilde{R}_e$  tends towards one, first, the return on the “Diversification Portfolio” becomes perfectly correlated with  $\tilde{R}_I$ ; second, the “Hedged Portfolio” becomes riskless and, at equilibrium, yields the risk-free rate of interest  $R_f$ ; and, third, the conditional market risk becomes negligible for all securities, so that the super risk premiums vanish.

In such a limiting case, the eligible segment of the market offers the same diversification opportunities as those offered by the whole market; consequently, the a priori constraint that denies the restricted investors access to the ineligible segment becomes ineffective. By holding DP as a perfect substitute for MPIS, the restricted investors can acquire a portfolio  $D_r$ , the return on which is perfectly correlated with  $\tilde{R}_M$ . As for the unrestricted investors, they view HP and the riskless asset as perfect substitutes; at equilibrium, they are thus willing to hold the fraction  $\mu$  of HP and adjust their lending or borrowing accordingly. In any case, the return on their portfolio  $D_u$  is also perfectly correlated with  $\tilde{R}_M$ . Thus, it appears that the case where  $\rho = 1$  does not essentially differ from the perfect integration case.

4. Because of segmentation, the restricted investors cannot hold the ineligible securities and thus properly diversify their holdings. As a second best solution,

<sup>9</sup> The extreme form of market segmentation takes place when  $\rho = 0$ , i.e., when no correlation exists between  $\tilde{R}_I$  and the return on any eligible security. In this extreme case, the ineligible securities are priced as if there were two-way segmentation (no investment in either direction between the two segments); indeed, if  $\rho = 0$ , Equation (3) implies:

$$E(\tilde{R}_i - R_f) = (A_u M_I) \text{Cov}[\tilde{R}_i, \tilde{R}_I]$$

As for the eligible securities, however, they would still be priced as if the market was fully integrated.

In the two-country case, if the product and factor markets of the two countries were well integrated, we would expect some degree of correlation between the returns on the securities of the two countries; to that extent, we would thus not expect the extreme form of mild segmentation to prevail.

they hold the market portfolio of eligible securities plus a proxy for the market portfolio of ineligible securities—the portfolio DP—which is supplied to them by the unrestricted investors. The unrestricted investors thus play the role of financial intermediaries: they provide diversification services for which they receive an implicit remuneration; indeed, they supply securities for which no super risk premium exists, i.e., securities with a comparatively low return.

5. We have already seen that the super risk premium becomes negligible when the unrestricted investors have a comparatively small risk-aversion level or when a suitable proxy for MPIS exists in the eligible segment. Outside these two limiting cases (i.e., if  $\mu > 0$  and  $\rho < 1$ ), the super risk premiums must exist to induce the unrestricted, but risk-averse, investors to supply diversification services.

Indeed, in the absence of super risk premiums, the expected excess return on any security would be proportional to its covariance with the world market portfolio; in such a case, the risk-averse unrestricted investors would hold the fraction  $A/A_u \equiv 1 - \mu < 1$  of the WMP, at the exclusion of any other risky investment. Since, by hypothesis, the restricted investors cannot hold the ineligible securities, these would be in excess supply. Consequently, for equilibrium to prevail, super risk premiums must exist, so that unrestricted investors are induced to acquire the residual fraction  $\mu$  of MPIS (which the restricted investors would like to, but cannot, hold) and supply the same fraction  $\mu$  of DP.

6. To conclude this section, we present Table I which summarizes the main features of three paradigms which may be used in the two-country case: perfect integration (no barrier to investments), mild segmentation (barriers in one direction only), and complete segmentation (barriers in both directions).

## II. The Segmentation / Integration Issue: A Testable Hypothesis

The magnitude of the super risk premium depends on  $A_u - A$ , i.e., on risk-aversion coefficients that are not directly measurable. We must therefore rely on cross-sectional regressions in order to capture the degree of effective segmentation in the international capital market. We sketch below the specific procedure we have chosen to achieve that aim.

### A. Estimation of Security-Specific Risk Characteristics

Without specifying a particular return generating model, the estimation of the conditional market risk for each ineligible security would be a formidable task. Thus, for the sake of tractability, we assume the following two-factor return generating model:<sup>10</sup>

$$\tilde{R}_e = \alpha_e + \beta_e \tilde{R}_M + \gamma_e \tilde{V}_E + \tilde{\varepsilon}_e \quad \text{with} \quad \text{Cov}[\tilde{\varepsilon}_e, \tilde{R}_M] = \text{Cov}[\tilde{\varepsilon}_e, \tilde{V}_E] = 0 \quad (6)$$

<sup>10</sup> Even though multivariate normality was assumed in the previous section, the two-factor return generating model does not require any assumption about the process generating returns (see Stehle [22]). In view of the significant world factor reported by Lessard [14] and Solnik [19], for the empirical tests, we use world market return ( $\tilde{R}_W$ ) in place of  $\tilde{R}_M$ .

Table I  
The Two-Country Case

| Paradigm                                        | Conditions                                                                           | Expected Excess Returns                    |                                                                                                 | Portfolio Composition for                                              |                                                                          |
|-------------------------------------------------|--------------------------------------------------------------------------------------|--------------------------------------------|-------------------------------------------------------------------------------------------------|------------------------------------------------------------------------|--------------------------------------------------------------------------|
|                                                 |                                                                                      | Eligible Securities                        | Ineligible Securities                                                                           | Restricted Investors                                                   | Unrestricted Investors                                                   |
| Perfect Integration                             | No barrier to investment or $\text{Var}[\hat{R}_I   \hat{R}_e] = 0$                  | $AM \text{Cov}[\hat{R}_e, \hat{R}_M]$      | $AM \text{Cov}[\hat{R}_i, \hat{R}_M]$                                                           | World market portfolio                                                 | World market portfolio                                                   |
| Mild Segmentation                               | One-way barrier to investment: $A_u > A$ and $\text{Var}[\hat{R}_I   \hat{R}_e] > 0$ | $AM \text{Cov}[\hat{R}_e, \hat{R}_M]$      | $AM \text{Cov}[\hat{R}_i, \hat{R}_M] + (A_u - A)M_I \text{Cov}[\hat{R}_i, \hat{R}_I \hat{R}_e]$ | Market portfolio of country 1 securities + “diversification” portfolio | World market portfolio + hedged market portfolio of country 2 securities |
| Special Case: Extreme Form of Mild Segmentation | $A_u > A$ and $\text{Cov}[\hat{R}_e, \hat{R}_I] = 0$                                 | $AM_E \text{Cov}[\hat{R}_e, \hat{R}_E]$    | $A_u M_I \text{Cov}[\hat{R}_i, \hat{R}_I]$                                                      | Market portfolio of country 1 securities                               | World market portfolio + market portfolio of country 2 securities        |
| Complete Segmentation                           | Two-way barriers to investment                                                       | $A_r M_E \text{Cov}[\hat{R}_e, \hat{R}_E]$ | $A_u M_I \text{Cov}[\hat{R}_i, \hat{R}_I]$                                                      | Market portfolio of country 1 securities                               | Market portfolio of country 2 securities                                 |

Note: The subscripts  $r$  and  $u$  denote the investors of countries 1 and 2, respectively. The subscripts  $e$  and  $i$  denote the securities of countries 1 and 2, respectively.

where the second factor,  $\tilde{V}_E$ , is orthogonal to  $\tilde{R}_M$  and defined by:

$$\tilde{R}_E = a_E + b_E \tilde{R}_M + \tilde{V}_E \quad \text{with} \quad \text{Cov}[\tilde{V}_E, \tilde{R}_M] = 0 \quad (7)$$

A similar return generating model is assumed to apply to the ineligible segment of the market:

$$\tilde{R}_i = \alpha_i + \beta_i \tilde{R}_M + \gamma_i \tilde{V}_I + \tilde{\varepsilon}_i \quad \text{with} \quad \text{Cov}[\tilde{\varepsilon}_i, \tilde{R}_M] = \text{Cov}[\tilde{\varepsilon}_i, \tilde{V}_I] = 0 \quad (8)$$

where the second factor,  $\tilde{V}_I$ , is orthogonal to  $\tilde{R}_M$  and defined by:

$$\tilde{R}_I = a_I + b_I \tilde{R}_M + \tilde{V}_I \quad \text{with} \quad \text{Cov}[\tilde{V}_I, \tilde{R}_M] = 0 \quad (9)$$

We further assume:

$$\text{Cov}[\tilde{\varepsilon}_i, \tilde{R}_e] = 0 \quad e = 1, \dots, E; \quad i = 1, \dots, I \quad (10)$$

an assumption which severely limits the sources of covariability between the returns of eligible and ineligible securities; for example, as in most empirical models of this type, industry effects are simply assumed away.

Given the above simplifying assumptions about the structure of returns in the two segments of the market, the unconditional and conditional market risks can be readily expressed in terms of the coefficients  $\beta$  and  $\gamma$ . In such a simplified context, the mild segmentation paradigm yields the following results:

**PROPOSITION 2.**

- (a) *The unconditional market risk of any security is proportional to its beta coefficient:*

$$\text{Cov}[\tilde{R}_j, \tilde{R}_M] = \beta_j \text{Var}[\tilde{R}_M] \quad j = i \text{ or } e \quad (11)$$

- (b) *The conditional market risk of any ineligible security is a linear function of the  $\beta$  and  $\gamma$  coefficients:*

$$\text{Cov}(\tilde{R}_i, \tilde{R}_I | \tilde{R}_e) = A\beta_i + B\gamma_i \quad \text{with} \quad A \text{ and } B > 0 \quad (12)$$

- (c) *As a consequence, the risk-return tradeoff takes the following simplified forms:*

- *for the eligible segment:*

$$E(\tilde{R}_e) = R_f + \lambda_E \beta_e + \theta_E \gamma_e \quad \text{with} \quad \lambda_E \geq 0 \quad \text{and} \quad \theta_E = 0 \quad (13)$$

- *for the ineligible segment:*

$$E(\tilde{R}_i) = R_f + \lambda_I \beta_i + \theta_I \gamma_i \quad \text{with} \quad \lambda_I \geq \lambda_E \geq 0 \quad \text{and} \quad \theta_I \geq 0 \quad (14)$$

*Proof:* Please refer to Appendix B.

### B. Testable Hypothesis

Assuming that the consensus expectations are unbiased, the expected return,  $E(\tilde{R})$ , can be proxied by the average historical return,  $\bar{R}$ ; similarly, the beta and gamma coefficients can be estimated by means of time-series regressions conforming to Equations (6) and (8). Two cross-sectional regressions can then be

run, one for each segment of the market:

- for the eligible segment:<sup>11</sup>

$$\bar{R}_e = a_E + \lambda_E \beta_e + \theta_E \gamma_e + u_e \quad (15)$$

- for the ineligible segment:

$$\bar{R}_i = a_I + \lambda_I \beta_i + \theta_I \gamma_i + u_i \quad (16)$$

*The Hypothesis: Mild Segmentation Versus. Alternate of Not Mildly Segmented Market*

If the mild segmentation paradigm holds, we can infer from Proposition 2:

$$\{a_E = a_I = R_f; \quad \lambda_I \geq \lambda_E \geq 0; \quad \theta_I \geq 0; \quad \theta_E = 0\}$$

Of course, the usual caveat applies: the empirical procedure cannot really test whether the markets are mildly segmented independently of the valuation model which was used to derive the test. Thus, if the mild segmentation hypothesis was to be rejected, it might mean either that the markets are not mildly segmented or that the valuation model does not apply. As usual, a joint hypothesis is involved; care should therefore be exercised in deriving implications from the results of the empirical tests which follow.

### III. Test of the Mild Segmentation Hypothesis

#### A. Data

The data base consists of heavily traded securities from nine less developed countries (LDCs), and a random sample from the U.S.<sup>12</sup> Markets and securities included in the sample have monthly total return data for the period 1976–1980. The LDC markets are comparable to many European markets in terms of size, turnover, and liquidity (Errunza [6]). Similarly, tests of the random walk hypothesis suggest these markets to be comparable to some of the smaller developed markets (Errunza and Losq [7]).

#### B. A Simple Test

If markets are fully integrated, then the risk (World beta) adjusted average returns should be similar across national markets. Table II reports monthly realized returns in US\$,  $\beta$ 's on GNP and market capitalization weighted world

<sup>11</sup> For the sake of notational simplicity, we do not make an explicit distinction between the actual and estimated values of the beta and gamma coefficients.

<sup>12</sup> Information on LDC securities is obtained from the International Finance Corporation emerging markets data bank. COMPUSTAT data tapes are used for U.S. securities. Use of official exchange rates as reported by International Financial statistics of the International Monetary Fund to convert LDC returns into U.S. \$ terms may induce bias. However, the problem does not appear to be serious in view of the extremely high correlations for U.S. \$ returns based on official exchange rates and purchasing power parity relationships as reported by Errunza and Losq [7]. The nine LDCs are: Argentina (22), Brazil (18), Chile (21), Greece (9), India (23), Korea (22), Mexico (21), Thailand (7), and Zimbabwe (10). The number of securities in the sample is indicated in parentheses after each market.

**Table II**  
**Realized Returns, Expected Returns, and the National Factor (1976–1980)**

|                           | GNP Wt. World Index                       |                                   | Mkt. Cap. Wt. World Index |                                   | Average Proportion of Variance Attributable to National Variations |      |
|---------------------------|-------------------------------------------|-----------------------------------|---------------------------|-----------------------------------|--------------------------------------------------------------------|------|
|                           | Average Monthly Realized Return in US\$ % | Monthly Expected Return in US\$ % | $\beta$                   | Monthly Expected Return in US\$ % |                                                                    |      |
|                           |                                           |                                   |                           |                                   |                                                                    |      |
| Argentina                 | 5.83                                      | -0.035                            | 0.49                      | -0.178                            | 0.41                                                               | 0.56 |
| Brazil                    | 0.39                                      | 0.392                             | 0.65                      | 0.162                             | 0.58                                                               | 0.43 |
| Chile                     | 6.9                                       | -0.591                            | 0.28                      | -0.623                            | 0.19                                                               | 0.50 |
| Greece                    | 0.26                                      | 0.213                             | 0.58                      | 0.139                             | 0.57                                                               | 0.41 |
| India                     | 2.45                                      | 0.742                             | 0.78                      | 0.637                             | 0.82                                                               | 0.42 |
| Korea                     | 2.23                                      | 0.876                             | 0.83                      | 0.753                             | 0.88                                                               | 0.40 |
| Mexico                    | 3.26                                      | 1.179                             | 0.95                      | 1.020                             | 1.01                                                               | 0.50 |
| Thailand                  | 1.64                                      | -0.359                            | 0.36                      | -0.621                            | 0.19                                                               | 0.60 |
| Zimbabwe                  | 2.24                                      | -0.049                            | 0.48                      | -0.024                            | 0.49                                                               | 0.32 |
| U.S.                      | 0.64                                      | 0.928                             | 0.85                      | 1.058                             | 1.03                                                               |      |
| GNP Wt. World Index       | 0.88                                      | 1.000                             |                           |                                   |                                                                    |      |
| Mkt. Cap. Wt. World Index | 1.00                                      |                                   |                           | 1.000                             |                                                                    |      |

Notes: 1. Assume  $R_f = 0.5\%$  per month in US\$.

2. Construction of world indices and national indices is described in Part E of Section III.

indices, as well as monthly expected returns in US\$. The realized returns in all cases are quite different from what one would anticipate under full integration. Further, for most LDCs, the ratio of realized to expected returns is substantially higher than that for the U.S. market, thereby providing some evidence of segmentation. Of course, this result should be construed as only the first step towards a formal asset pricing test of our model in view of the potential difficulties arising from country (sample) selection, use of official exchange rates, index construction procedure within each market, as well as the time period.

### *C. The National Factor*

Before proceeding with the test of the mild segmentation hypothesis in an international asset pricing model, it would be prudent to investigate the importance of the country factor for our sample of newly emerging markets. The results have implications for the design of the test methodology as discussed below. Table II provides average proportion of security risk explained by the national factor.<sup>13</sup> Country factor seems to be very strong in most cases.<sup>14</sup>

In view of the strength of country effect, the cross-sectional tests of the mild segmentation hypothesis can be approached in two ways:

1. Deal explicitly with the country factor using a two-country design. For example, U.S. securities can be characterized as the eligible set and say Argentinean securities constitute the ineligible set with U.S. investors restricted and Argentinean investors unrestricted. The primary problem with such a design is the small LDC samples which would necessarily lead to formation of overlapping and/or differing size portfolios within each LDC market and hence violate OLS assumptions as suggested by Stehle [22]:
2. Pool securities and construct portfolios that are diversified across countries within each (eligible and ineligible) set. The increased sample size will avoid overlapping and eliminate selection bias that results from constructing portfolios by country in the presence of strong country effects (Adler and Horesh [2]). We will use this approach in the tests which follow.

### *D. Security and Investor Characterization*<sup>15</sup>

Keeping in mind the existing capital flow controls and the patterns of portfolio investments which indirectly reflect perceptions of U.S. investors regarding feasibility or advisability of investments in LDC markets, the eligible (ineligible) segments will be further characterized as technically eligible (ineligible) or

<sup>13</sup> The proportions are average  $R^2$  for the regressions of security returns on equally weighted national indices based on our sample of highly traded securities on each market.

<sup>14</sup> This result is consistent with those reported by Errunza [5] and Lessard [12] for LDC markets and is quite similar to the European findings of Solnik [19].

<sup>15</sup> Since published information on capital flow controls relates primarily to direct foreign investments and not to portfolio flows, the characterizations are based on in-house research by members of the capital markets department of the International Finance Corporation (IFC), the World Bank group, as well as personal conversations with various LDC stock exchange authorities by the IFC personnel and one of the authors of this paper. Details on IFC research as well as contact persons for each LDC exchange can be obtained from the authors.

perceived eligible (ineligible). A market (security) is considered technically eligible if there are no formal capital controls on foreign portfolio investments, i.e., if, technically, foreigners can freely invest in these markets (securities). Thus, the securities of U.S., Thailand, Zimbabwe, and open Mexican firms constitute the technically eligible segment. A market (security) is considered perceived eligible if it is characterized by no formal capital controls as well as significant foreign portfolio investments. Since foreign investors are not active in either Thailand or Zimbabwe, the perceived eligible set includes U.S. and open Mexican firms.

|                                                |                                                                                                                             |
|------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------|
| <i>Technically Eligible Set:</i> <sup>16</sup> | 130 securities from U.S., Thailand, Zimbabwe, and Mexico (open firms).                                                      |
| <i>Technically Ineligible Set:</i>             | 130 securities from Argentina, Brazil, Chile, Greece, India, Korea, and Mexico (other than open firms).                     |
| <i>Perceived Eligible Set:</i> <sup>16</sup>   | 147 securities from U.S. and Mexico (open firms).                                                                           |
| <i>Perceived Ineligible Set:</i>               | 147 securities from Argentina, Brazil, Chile, Greece, India, Korea, Mexico (other than open firms), Thailand, and Zimbabwe. |

For our purpose, the nationals of markets constituting the eligible set(s) can trade only among each other and therefore are characterized as restricted investors, whereas the nationals of markets forming the ineligible set(s) can trade in all securities and hence are characterized as unrestricted investors. This nomenclature is confusing in view of our traditional perceptions of freewheeling U.S. investors and highly constrained LDC investors. A brief clarifying explanation would be valuable.

In practice, the U.S. investors can freely invest abroad but are not allowed to hold securities from the ineligible set. Thus, U.S. investors encounter portfolio inflow controls and could hold the World Market Portfolio (of eligible securities) excluding the ineligible set. In view of the restrictions placed on U.S. investors by the governments of ineligible set countries, we characterize them as restricted investors. On the other hand, LDC investors are not restricted by the governments of eligible set countries but cannot *easily* get funds out of their own countries. However, the official outflow controls on LDC investors do not appear to be prohibitive when one considers the participation of large (at times privileged) LDC investors in eligible markets (with or without home government knowledge). Thus, even though our characterization of LDC investors may not depict reality, it does approximate the current state of affairs as well as the implicit assumption of no outflow controls on LDC investors in our theoretical model. The assumption of free foreign portfolio investments among ineligible markets at both the theoretical and empirical levels is troublesome. An  $n$ -factor model to fully capture the pattern of international investment flows would be desirable but is left for further research because of the mathematical complications involved.

<sup>16</sup> The random U.S. (NYSE) sample size is chosen so as to match the appropriate eligible and ineligible sets.

### E. The Test Procedure

*Market Indices*—The LDC market returns are equally weighted for our sample firms, the U.S. market return is a capitalization weighted NYSE index, and the other market returns are market capitalization weighted indices reported by Capital International Perspective. Following Solnik [19], the various compound indices (world, technically (in)eligible, and perceived (in)eligible) are based on GNP weights.

*The Riskless Rate*—The problem of assuming a riskless rate in an international context is well known (Adler and Horesch [2]) and is especially troublesome for LDCs where a domestic market determined short-term rate similar to U.S. T-bills is generally not available. We reluctantly use 30-day U.S. T-bill rate as a proxy. Of course, in the capital asset pricing context, the various stock price indices are also proxies for market rates of return and hence have their own shortcomings (Roll [16], Solnik [20]).

*Portfolio Formation*—For each security in each of the four segments (technically (in)eligible and perceived (in)eligible),  $\beta_e$  ( $\beta_i$ ) and  $\gamma_e$  ( $\gamma_i$ ) are calculated using monthly returns for the period 1976–1977. The  $\beta_e$  ( $\beta_i$ ) and  $\gamma_e$  ( $\gamma_i$ ) are estimated directly to avoid the measurement error problem associated with estimation of  $V_E$  ( $V_I$ ) in an orthogonal design.<sup>17</sup> Following tradition, portfolio observations are used to minimize nonstationarity and measurement error problems. Since the optimal grouping procedure is not well defined in the case of three-variable linear regression and in view of limited data, we formed 16 nonoverlapping portfolios using a  $(4 \times 4)$  matrix design to maximize intergroup variation of  $\beta_e$  ( $\beta_i$ ) and  $\gamma_e$  ( $\gamma_i$ ) using two alternate portfolio construction procedures.<sup>18</sup> Thus, we have 16 nonoverlapping portfolios for each of the four segments and for each of the two portfolio construction procedures using 1976–1977 as the portfolio formation period. Next, monthly portfolio rates of return are calculated for each portfolio for the following 12-month period. The procedure is repeated with 1977–1978

<sup>17</sup> The  $\beta_e$  and  $\gamma_e$  are estimated using:

$$\hat{\beta}_e = \frac{\text{Cov}(\tilde{R}_e, \tilde{R}_w)}{\text{Var } \tilde{R}_w} = \frac{\sum_t (R_{et} - \bar{R}_e)(R_{wt} - \bar{R}_w)}{\sum_t (R_{wt} - \bar{R}_w)^2}$$

and

$$\hat{\gamma}_e = \frac{\text{Cov}(\tilde{R}_e, \tilde{V}_E)}{\text{Var}(\tilde{V}_E)} = \frac{\text{Cov}(\tilde{R}_e, \tilde{R}_E)\text{Var}(\tilde{R}_w) - \text{Cov}(\tilde{R}_E, \tilde{R}_w) \cdot \text{Cov}(\tilde{R}_e, \tilde{R}_w)}{\text{Var}(\tilde{R}_w)\text{Var}(\tilde{R}_E) - [\text{Cov}(\tilde{R}_E, \tilde{R}_w)]^2}$$

Similar expressions are used to calculate  $\beta_i$  and  $\gamma_i$ .

<sup>18</sup> Two portfolio constructions procedures were used.

$\beta - \gamma$  Portfolios. Within each segment and during each portfolio formation period, securities were ranked by  $\beta$  to form four  $\beta$ -portfolios. Next, securities within each  $\beta$ -portfolio were ranked by  $\gamma$  to obtain four  $\gamma$ -portfolios, thereby yielding a total of sixteen  $\beta - \gamma$  portfolios.

$\gamma - \beta$  Portfolios. Within each segment and during each portfolio formation period, securities were ranked by  $\gamma$  to form four  $\gamma$ -portfolios. Next, securities within each  $\gamma$ -portfolio were ranked by  $\beta$  to obtain four  $\beta$ -portfolios, thereby yielding a total of sixteen  $\gamma - \beta$  portfolios.

and 1978–1979 as the portfolio formation periods and portfolio returns calculated for years 1979 and 1980, respectively. This provides us with 36 monthly portfolio returns on 16 portfolios for each of the four segments and for each of the two portfolio construction procedures.

*Cross-Sectional Regressions*—As discussed in the previous section, we have  $\bar{r}_e = a_E + \lambda_E \hat{\beta}_e + \theta_E \hat{\gamma}_e + \mu_e$  for the eligible segments where  $r_{et}$  ( $\bar{r}_e$ ) is the ex post (mean) excess rate of return for a given eligible portfolio during month  $t$ ,  $t = 1, \dots, 36$ ; and  $\bar{r}_i = a_I + \lambda_I \hat{\beta}_i + \theta_I \hat{\gamma}_i + \mu_i$  for the ineligible segments where  $r_{it}$  ( $\bar{r}_i$ ) is the ex post (mean) excess rate of return for a given ineligible portfolio during month  $t$ ,  $t = 1, \dots, 36$ . Excess portfolio returns are used in cross-sectional regressions to obtain time series of estimated coefficients,  $a$ ,  $\lambda$ , and  $\theta$  for each of the four segments and for each of the two portfolio construction procedures. Following Stehle [22], the risk coefficients  $\beta$  and  $\gamma$  for period  $t$  are based on all portfolio returns excluding those during the portfolio formation period and the  $t^{\text{th}}$  month. Using 1976–1977 as the portfolio formation period and using mean monthly portfolio returns during 1978–1980 as the dependent variable in cross-sectional regressions did not result in significantly different findings. Generalized Least Squares (GLS) procedure was used since the residuals were heteroscedastic.

#### F. Results

Table III reports GLS parameter estimates from cross-sectional regressions. The table also reports the averages of the month-by-month GLS values of  $a_t$ ,  $\lambda_t$ , and  $\theta_t$ ;  $s(a)$ ,  $s(\lambda)$ , and  $s(\theta)$  the sample standard deviations of the monthly values of  $a_t$ ,  $\lambda_t$ , and  $\theta_t$ , respectively; relevant  $t$ -statistics; and the mean adjusted  $R^2$  and its standard deviation  $s(R^2)$  for month-by-month regressions of the 16 portfolio returns on the relevant risk measures. These statistics are reported for the four segments under two alternative portfolio construction methods.

The most striking feature of Table III is the consistency of results. Parameter estimates and their significance are quite similar across the two portfolio construction procedures. Further, the behavior of a given set (eligible or ineligible) across the differential characterization (perceived and technical) is also not very different.

The strength of the relationship between ex post returns and risk proxies is on average low and quite variable. Despite the use of a questionable proxy for the riskless return, the intercept estimates are not statistically significantly different from theoretical expectations. There are many months when  $\lambda_{Et}$ ,  $\lambda_{It}$ ,  $\theta_{Et}$ , and  $\theta_{It}$  are negative, indicating a negative relationship between return and risk during those months. This is neither very surprising nor contrary to the mild segmentation asset pricing model. Our theoretical model states the equilibrium relationship between expected return and risk. Thus, the postulated relationship should hold on average and not necessarily month by month. Following Fama [9], we use the average values and summary measures of the time-series properties of  $\lambda_{Et}$ ,  $\lambda_{It}$ ,  $\theta_{Et}$ , and  $\theta_{It}$  to test the mild segmentation hypothesis. We now discuss each of the four conditions associated with our hypothesis:

Table III  
GLS Parameter Estimates—Mild Segmentation Model

| Market Segment                                          | $\alpha_i$               |        |              | $\lambda_i$     |              |                    | $\theta_i$     |             |                   | $R_i^2$ |          |
|---------------------------------------------------------|--------------------------|--------|--------------|-----------------|--------------|--------------------|----------------|-------------|-------------------|---------|----------|
|                                                         | $\bar{a}$                | $s(a)$ | $t(\bar{a})$ | $\bar{\lambda}$ | $s(\lambda)$ | $t(\bar{\lambda})$ | $\bar{\theta}$ | $s(\theta)$ | $t(\bar{\theta})$ | Adj.    | $s(R^2)$ |
| $\beta - \gamma$ Portfolios<br>Perceived<br>Technically | { Eligible<br>Ineligible | -0.012 | 0.046        | -1.5            | 0.010        | 0.052              | 1.14           | 0.010       | 0.032             | 1.84    | 0.217    |
|                                                         |                          | 0.017  | 0.045        | 2.27            | -0.020       | 0.051              | -2.42          | 0.015       | 0.062             | 1.45    | 0.347    |
|                                                         | { Eligible<br>Ineligible | 0.002  | 0.036        | 0.33            | 0.019        | 0.062              | 1.87           | -0.013      | 0.037             | -2.18   | 0.335    |
|                                                         |                          | 0.023  | 0.078        | 1.9             | -0.019       | 0.077              | -1.46          | 0.011       | 0.071             | 0.95    | 0.382    |
| $\gamma - \beta$ Portfolios<br>Perceived<br>Technically | { Eligible<br>Ineligible | -0.011 | 0.050        | -1.35           | 0.011        | 0.058              | 1.13           | 0.009       | 0.036             | 1.45    | 0.241    |
|                                                         |                          | 0.017  | 0.044        | 2.32            | -0.013       | 0.044              | -1.83          | 0.013       | 0.063             | 1.20    | 0.338    |
|                                                         | { Eligible<br>Ineligible | 0.003  | 0.036        | 0.5             | 0.008        | 0.065              | 0.71           | -0.001      | 0.037             | -0.21   | 0.349    |
|                                                         |                          | 0.022  | 0.067        | 1.95            | -0.019       | 0.056              | -2.08          | 0.011       | 0.067             | 1.02    | 0.350    |

Notes: The  $t(\bar{a})$  values are calculated based on the theoretical expected value of zero. The  $t$ -statistics for  $\bar{\lambda}$  and  $\bar{\theta}$  test the hypothesis that  $E(\bar{\lambda}) = 0$  and  $E(\bar{\theta}) = 0$ , respectively.

- $\bar{a}_E = \bar{a}_I = 0$  Even though the intercept estimates are not statistically significantly different from their theoretical value of 0, the subcondition of  $\bar{a}_E = \bar{a}_I$  is rejected for the perceived set under  $\beta - \gamma$  portfolio construction procedure.
- $\bar{\lambda}_I \geq \bar{\lambda}_E \geq 0$  There is no evidence to support the hypothesis that  $\bar{\lambda}_E$  and  $\bar{\lambda}_I$  differ significantly.
- $\bar{\theta}_E = 0$  We cannot reject this condition in all cases. The almost significantly negative  $\bar{\theta}_E$  for technically eligible securities under  $\beta - \gamma$  portfolio construction procedure is troublesome.
- $\bar{\theta}_I \geq 0$  The  $\bar{\theta}_I$  are not statistically significantly different from zero. Thus, one could claim that the mildly segmented CAPM cannot be rejected; but the same is true for the standard CAPM applied to all securities indiscriminately ( $\bar{\theta}_I = 0$ ).<sup>19</sup> However, note that  $\bar{\theta}_I$  are consistently greater than zero and large in practical terms.<sup>20</sup>

In summary, the results are not statistically inconsistent with the mild segmentation hypothesis. The weak results can be attributed to the kinds of restrictions imposed in the real world, use of various proxies including the risk-free rate, and small sample size. Further empirical research based on more realistic world capital market structure, alternate proxies, and larger sample size is suggested.

#### IV. Conclusion

In this paper, a formal model of international capital asset pricing was developed. The primary distinguishing feature of the model is the unequal access assumption which approximates the reality of a mildly segmented world market. The incidence of mild segmentation does not affect required return on an eligible security whereas the required return on an ineligible security is different from what the standard CAPM would suggest. The ineligible securities would generally command a super risk premium which is proportional to the differential risk aversion and the conditional market risk.

We then conduct a cross-sectional test of the mild segmentation hypothesis. The overall results are not statistically inconsistent with theoretical expectations and thus lend tentative support to the mild segmentation hypothesis. Further empirical work based on a richer data set with longer time period and larger sample size is needed to improve the power of the test.

#### Appendix A: Proof of Proposition 1

The proof consists of three steps:

*Step 1:* Check that, given (c) and (d), no security is in excess supply or in excess demand. By construction, the aggregate demand vector  $\underline{D}_u + \underline{D}_r \equiv (1 - \mu)\underline{WMP} + \mu(\underline{MPIS} - \underline{DP} + \underline{MPES} + \underline{DP}) \equiv \underline{WMP}$  is indeed equal to the aggregate supply vector  $\underline{P} \equiv \underline{WMP}$ .

<sup>19</sup> We would like to thank the anonymous referee for pointing this out.

<sup>20</sup> In view of the major political risk factor associated with most of our sample LDCs during 1975–1980, the positive  $\theta$ 's may reveal political risk premia. We would like to thank the anonymous referee for this interpretation.

*Step 2:* Check the optimality of the portfolio  $D_r$  given (1) the constraint that restricted investors cannot acquire ineligible securities and (2) the postulated risk-return relationship in the eligible segment (Equation (2)).

This optimality condition is  $E(\tilde{R}_e - R_f) = A_r \text{Cov}[\tilde{R}_e, \underline{D}'_r \tilde{R}]$  for any arbitrary eligible security. Given the definition of  $\underline{D}_r$  in (d), we can write:

$$\text{Cov}[\tilde{R}_e, \underline{D}'_r \tilde{R}] = \mu \{M_E \text{Cov}[\tilde{R}_e, \tilde{R}_E] + V_{ee} V_{ee}^{-1} V_{ei} \underline{P}_i\} \quad (\text{A1})$$

that is,

$$\text{Cov}[\tilde{R}_e, \underline{D}'_r \tilde{R}] = \frac{A}{A_r} \text{Cov}[\tilde{R}_e, M_E \tilde{R}_E + M_I \tilde{R}_I] = \frac{AM}{A_r} \text{Cov}[\tilde{R}_e, \tilde{R}_M] \quad (\text{A2})$$

Thus, given part (a) of the proposition, the portfolio  $\underline{D}_r$  does realize the optimal compromise between risk and return.

*Step 3.* Check the optimality of the portfolio  $D_u$  given the postulated risk-return relationships in the ineligible and in the eligible segment (Equations (3) and (2)).

The first-order optimization condition can be written  $E(\tilde{R} - R_f) = A_u \text{Cov}[\tilde{R}, \underline{D}'_u \tilde{R}]$ . Given the definition of  $\underline{D}_u$  in (c), we can write:

$$\text{Cov}[\tilde{R}, \underline{D}'_u \tilde{R}] = (1 - \mu) \text{Cov}[\tilde{R}, M \tilde{R}_M] + \mu \text{Cov}[\tilde{R}, \underline{\text{HP}}' \tilde{R}] \quad (\text{A3})$$

that is, given that  $1 - \mu \equiv A/A_u$ :

$$\text{Cov}[\tilde{R}, \underline{D}'_u \tilde{R}] = \frac{AM}{A_u} \text{Cov}[\tilde{R}, \tilde{R}_M] + \mu \left[ \frac{\text{Cov}[\tilde{R}_e, \underline{\text{HP}}' \tilde{R}]}{\text{Cov}[\tilde{R}_i, \underline{\text{HP}}' \tilde{R}]} \right] \quad (\text{A4})$$

We have already noted that  $\text{Cov}[\tilde{R}_e, \underline{\text{HP}}' \tilde{R}] = 0$ ; we can also assert from the definition of HP that:

$$\text{Cov}[\tilde{R}_i, \underline{\text{HP}}' \tilde{R}] = \text{Cov}[\tilde{R}_i, \underline{P}'_i \tilde{R}_i - \underline{P}'_i V_{ie} V_{ee}^{-1} \tilde{R}_e] = (V_{ii} - V_{ie} V_{ee}^{-1} V_{ei}) \underline{P}_i \quad (\text{A5})$$

Consequently, given Equation (A4) and the identity  $\mu \equiv \frac{A_u - A}{A_u}$ , we have:

$$A_u \text{Cov}[\tilde{R}, \underline{D}'_u \tilde{R}] = (AM) \text{Cov}[\tilde{R}, \tilde{R}_M] + (A_u - A) M_I \text{Cov}[\tilde{R}, \tilde{R}_I | \tilde{R}_e] \quad (\text{A6})$$

so that the optimality condition is indeed satisfied by the portfolio  $D_u$  as long as Equations (2) and (3) represent the risk-return relationships in the two segments of the market.

## Appendix B: Proof of Proposition 2

Part (a) of the proposition being a standard result, we proceed directly to prove part (b). To that end, we use Equation (8) to express the conditional market risk as:

$$\begin{aligned} \text{Cov}[\tilde{R}_i, \tilde{R}_I | \tilde{R}_e] &= \beta_i \text{Cov}[\tilde{R}_M, \tilde{R}_I | \tilde{R}_e] \\ &+ \gamma_i \text{Cov}[\tilde{V}_I, \tilde{R}_I | \tilde{R}_e] + \text{Cov}[\tilde{\varepsilon}_i, \tilde{R}_I | \tilde{R}_e] \end{aligned} \quad (\text{B1})$$

The last covariance term in Equation (B1) is null, since, by assumption,

$\text{Cov}[\tilde{\varepsilon}_i, \tilde{R}_I] = \text{Cov}[\tilde{\varepsilon}_i, \tilde{R}_M]b_I + \text{Cov}[\tilde{\varepsilon}_i, \tilde{V}_I] = 0$  and  $\text{Cov}[\tilde{\varepsilon}_i, \tilde{R}_e] = 0$ . Furthermore, recalling that  $\tilde{R}_M \equiv (M_I/M)\tilde{R}_I + (M_E/M)\tilde{R}_E$ , the coefficient of  $\beta_i$  in Equation (B1) can be written:

$$A \equiv \text{Cov}[\tilde{R}_M, \tilde{R}/\tilde{R}_e] = (M_I/M)\text{Var}[\tilde{R}_I | \tilde{R}_e] \geq 0 \quad (\text{B2})$$

As for the coefficient of  $\gamma_i$  in Equation (B1), in view of Equation (9) in the text, it can be formulated as:

$$B \equiv \text{Cov}[\tilde{V}_I, \tilde{R}_I | \tilde{R}_e] = \text{Cov}[\tilde{R}_I - b_I[(M_I/M)\tilde{R}_I + (M_E/M)\tilde{R}_E], \tilde{R}_I | \tilde{R}_e] \quad (\text{B3})$$

or, equivalently,

$$B = [1 - b_I(M_I/M)]\text{Var}[\tilde{R}_I | \tilde{R}_e] \quad (\text{B4})$$

It is a simple matter to check that the expression  $[1 - b_I(M_I/M)]$  is positive as long as the returns on WMP and the returns on MPES are positively correlated, a condition which is assumed to hold. This condition thus ensures that the coefficient  $B$  is positive or null. Part (c) of Proposition 2 then follows directly.

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