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Author(s): James A. Ohlson
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The Structure of Asset Prices and Socially Useless/Useful Information

JAMES A. OHLSON*

ABSTRACT

This paper relates the value of additional information to asset prices in a pure exchange setting. The price structure of interest revolves around a "pricing-hypothesis": the prices in an economy with less information are unbiased estimators of the prices that would obtain in a more informative economy. Two basic results are developed. First, if the incremental information is useless then the pricing-hypothesis applies. Second, if the pricing hypothesis is assumed valid, then the information is valuable in a weak sense. The results are also considered in the context of empirical research. The case is made for viewing statistical tests of association between prices and signals as tests of the social value of information.

A SIGNIFICANT BODY OF theoretical research has examined the social utility of information in pure exchange (with one or two dates). The central message in the more recently published papers is that there is indeed a positive role for (additional) public information (see in particular Hakansson, Kunkel, and Ohlson [9] and Ohlson and Buckman [14] for a synthesis). Under "reasonable" assumptions about the structure of the economy, the analysis shows that individuals generally will be better off with more information, or, under slightly different assumptions, additional information does not in any event generate Pareto-inferior allocations. However, these kinds of analyses relate the value of information to investors' preferences and beliefs, and to characteristics of the market. No particular consideration is given to the structure of security prices. The present paper changes the perspective by focusing on the output of the economic system (prices) rather than on the input (preferences, beliefs, etc.). There are accordingly two questions one may want to address. First, given that the information is socially useless/useful, how would this be revealed in the structure of asset prices? Second, and, loosely speaking, this would be the converse question: can one ever *infer* usefulness or uselessness of information if prices (hypothetically) respond to the information in some specific fashion?

The outline of the paper, and the major results, can be summarized briefly as follows. The next section develops the model and certain crucial welfare concepts are reviewed. The setting is a two-dates pure exchange model, and the construction of the economic regime assures that all allocations attain constrained Pareto efficiency. One important implication of the latter, which can be

* Visiting Professor, Graduate School of Business, Columbia University. The author is indebted to an anonymous referee, Greg Buckman, Joel Demski, Robert Freeman, and Greg Kunkel for valuable discussions and comments.

inferred from Ohlson [13] and Ohlson and Buckman [14], is that more information is never Pareto-inferior to less information. Section II considers what happens if the information is presumed to be useless in the sense that all individuals are indifferent between having and not having the information. Useless information is shown to imply that the prices in the economy without (or with less) information are unbiased estimators of the prices that would obtain in the economy with (more) information. This particular structure of security prices maintains a key focus throughout the paper;[1] it is referred to as the "pricing-hypothesis." Section III proves the major theorem of the paper: if the pricing-hypothesis is *assumed* to hold, then there is always social value of (more) information in a weak sense. That is, uselessness is one possibility, but more generally, some individuals may be better off although nobody is ever worse off. The analysis further establishes that a strict welfare improvement is possible only if markets are "inadequate," and (more) information serves as a substitute for markets. However, it is also noted that there are, inevitably, dangers associated with the making of assumptions about variables (prices) which are fundamentally endogenous in a general equilibrium framework. This point is discussed in some detail, and the results are qualified accordingly.

The final section of the paper (Section IV) relates the analysis to empirical research in finance/accounting. Much of this research is concerned with various signals and their concurrent statistical impact on (changes in) asset prices and other capital market events like trading volume. The empirical signals considered can be those generated from financial reports (e.g., earnings releases) or other public information events such as dividends or stock-splits announcements. Generically, these kinds of studies are often referred to as "information content" studies. This paper delineates two points that would seem relevant for purposes of interpreting the statistical associations. First, Beaver's [3] discussion in his well-known study relates to the formal theory developed in this paper. Second, the current paper evaluates the idea that tests of the statistical association between prices and signals can be viewed as tests of the social value of information. A setting in which this notion becomes understood and meaningful is not difficult to develop. Hence, the Beaver-Dukes [4] contention about there being a relationship between APIs (the "abnormal performance index") and the social value of information is given conceptual support.

I. Preliminaries: Notation, Basic Concepts, and Economic Setting

The economy of concern will be a two-period pure exchange economy in which alternative sets of public information are provided to individuals. The notation used throughout the paper is as follows.[2] Let

η = a publicly available information structure partitioning the state space S. η may be interpreted as a function from

[1] This type of security-price behavior was derived and considered by Ohlson [12] in a dynamic partial equilibrium framework.

[2] Much of what is said in this section, including the notation, is also found in Ohlson and Buckman [14].

states to messages denoted $y \in Y_\eta$. Thus, S is partitioned into $S_y = \{s \mid \eta(s) = y\}$ where $\cup_y S_y = S$, $S_y \cap S_{y'} = \phi$ for $y \neq y'$. Let $\mid Y_\eta \mid$ and $\mid S \mid$ denote the cardinality of the sets Y_η and S, respectively. Following Marschak and Radner, η' is more informative than, or finer than, η if and only if the set $S_y \cap S_{y'}$ is either equal to $S_{y'}$ or is empty for any $y \in Y_\eta$ and $y' \in Y_{\eta'}$; $\mid S \mid$ is assumed finite;

$p_i(s, y)$ = individual i's probability measure defined on $S \times Y_\eta$ with $p_i(s, y) > 0$ for all $s \in S_y$, $y \in Y_\eta$, and $i \in H$; H denotes the set of all individuals and $\mid H \mid \geq 2$;

$c_i(0, y)$ = individual i's consumption of the current period good given signal $y \in Y_\eta$;

$c_i(s, y)$ = individual i's consumption of the end-of-period good given that state $s \in S$ occurs and message y has been received;

$u_{is}[c_i(0, y), c_i(s, y)]$ = utility of individual i's two-period consumption plan given state s. These functions satisfy concavity and other standard regularity conditions;

$a(f, s)$ = the gross payoff per share of security ("firm") f, $f \in F$, given state s, $s \in S$. The matrix $[a(f, s)] \equiv A$ is the securities-states tableau. The market will be said to be complete when the rank of A is full (i.e., equal to $\mid S \mid$); otherwise, it will be called incomplete;

$z_i(f, y)$ = number of shares held by individual i in firm f given signal y. By definition, $c_i(s, y) = \sum_{f \in F} z_i(f, y)a(f, s)$;

$\delta_i(\eta)$ = $\{c_i(0, y), z_i(f, y)\}_{f,y}$ = an allocation of the current consumption good and shares to individual i across different signals y;

$\{C(0), C(s)\}_s$ = exogenously determined aggregate supplies of goods across dates and states;

$Z(f)$ = aggregate supply of firm f shares; hence $C(s) = \sum_{f \in F} Z(f)a(f, s)$;

$\bar{z}_i(f)$ = the i^{th} individual's endowment in shares of firm f; $\sum_{i \in H} \bar{z}_i(f) = Z(f)$;

$\bar{c}_i(0)$ = the i^{th} individual's endowment in the current consumption good;

ϵ_i = $\{\bar{c}_i(0), \bar{z}_i(f)\}_f$ = the i^{th} individual's endowments;

$D(\eta)$ = the set of all feasible allocations of $\delta(\eta) \equiv \{\delta_i(\eta)\}_i$ satisfying the social total constraints ($C(0)$, $Z(f)$ all $f \in F$). The definition implicitly relates to some matrix A.

The expected utility associated with any signal-contingent holdings, $\delta_i(\eta)$, is then given by

$$V_i(\delta_i(\eta)) \equiv \sum_{s \in S} \sum_{y \in Y_\eta} p_i(s, y) u_{is}[c_i(0, y), \textstyle\sum_f a(f, s) z_i(f, y)].$$

The shares, and implicitly the signal-conditioned consumption plans, will be allocated in a neoclassical equilibrium framework. Thus, it is worthwhile to review some basic efficiency concepts.

Definition: Given a matrix A, an allocation $\delta(\eta) \in D(\eta)$ is said to be *constrained Pareto efficient* (*CPE*) *with respect to* η if there is no other feasible allocation $\delta'(\eta) \in D(\eta)$ such that

$$V_i(\delta_i'(\eta)) \geq V_i(\delta_i(\eta)), \qquad \text{all } i \in H,$$

and where the inequality is strict for at least one i.

It should be emphasized that CPE is defined with respect to a particular partition. Hence, if η'' is finer than η' then it is quite possible that an allocation $\delta(\eta')$ which is CPE with respect to η' does not remain CPE with respect to η''. Note that $\delta(\eta') \in D(\eta'')$ since $D(\eta') \subset D(\eta'')$ whenever η'' is finer than η'; the issue is therefore real.

A special case of CPE is *full Pareto efficiency* (FPE) which applies when one considers allocations of consumption plans $\{c_i(0, y), c_i(s, y)\}_{i,s,y}$ constrained only by social totals, $\{C(0), C(s)\}_s$, and not by the matrix, A itself. It follows that FPE is equivalent to CPE if $A = I$ or, more generally, A is complete. However, a CPE allocation is, in general, not FPE since constraints imposed on the mix of consumption plans by the matrix A may be binding. In Ohlson and Buckman [14] it is shown that if (A, η) is a *conditionally complete market* (CCM), then the CPE and FPE consumption sets are equivalent relative to that particular (A, η) pair.[3] With no homogeneity restrictions on $\{u_i, p_i\}_i$, the CCM condition is also necessary.

The concern of the paper pertains to competitive economies that attain CPE for every partition η and matrix A. The following trading arrangement ensures the final equilibrium allocation to be CPE.

Each investor maximizes his expected utility

$$\max_{\{\delta_i, w_i\}} V_i(\delta_i(\eta)).$$

In a prior market, he chooses $\{w_i(y)\}_y \equiv w_i$ subject to

$$\textstyle\sum_y \pi(y) w_i(y) \leq \sum_y \pi(y) \bar{w}_i(y),$$

and in a second round of trading, after some specific y has been observed, he selects $\delta_i(y) \equiv \{c_i(0, y), z_i(f, y)\}_f$ subject to

$$P(0, y) c_i(0, y) + \textstyle\sum_f P(f, y) z_i(f, y) \leq w_i(y)$$

where $\{\pi(y)\}_y$ and, for each y, $\{P(0, y), P(f, y)\}_f$ are price vectors. $\bar{w}_i(y)$ is defined by:

$$\bar{w}_i(y) \equiv P(0, y) \bar{c}_i(0) + \textstyle\sum_f P(f, y) \bar{z}_i(f).$$

The above economy is referred to as an Arrow-regime (A-regime) in Ohlson and Buckman [14]. In this regime every equilibrium is a CPE allocation; conversely, every CPE allocation is attainable through appropriate choice of endowments (ϵ) (see Ohlson and Buckman [14], Proposition IV). In what follows, these

[3] The definition of CCM is as follows: (A, η) is CCM if the rows in the $|F| \times |S_y|$ matrices $[a(f, s); f = 1, \cdots, |F|, s \in S_y]$ span a vector space of dimension $|S_y|$ for each $y \in Y_\eta$. For additional discussion of CCM, see Ohlson and Buckman [14].

are the only really important properties of the A-regime. Nevertheless, an equilibrium economy which yields a CPE allocation and is cast in the format of an A-regime permits a number of useful observations. First, the prior round of trading allows wealth transfers across signals. The signal-contingent endowed wealth is uncertain due to the uncertainty of the signal, but additional units of wealth in the event of signal y, $w_i(y) - \bar{w}_i(y)$, can be purchased (or sold) at a price $\pi(y)$ per unit. This trading opportunity is of course of great significance to accomplish CPE relative to the existing information structure. Without a prior round of trading, $w_i(y) = \bar{w}_i(y)$, and this is unlikely to lead to CPE allocation. (See Ohlson and Buckman [14], Section 4.4. The restriction implies that signal efficiency (SE) is attained; this level of efficiency is weak since SE is necessary but not sufficient to reach CPE.) Second, the model suggests that it is not particularly important whether the $w_i(y)$'s are chosen directly or indirectly. The essence is simply the efficient sharing of risks across signals. Under appropriate conditions, CPE-efficiency occurs also if there is sequential trading, i.e., an initial round of trading in the securities $\{c_i(0), z_i(f)\}_f$ is followed by a signal-contingent second round of trading in $\{c_i(0, y), z_i(f, y)\}_f$ for each y. (Conditions under which sequential trading models yield CPE can be inferred from Kreps [10], Ohlson and Buckman [14], and Townsend [17].) The results developed in this paper are equally applicable for these "more realistic" settings, provided that CPE is attained.

The reasons for studying CPE economies will become more apparent subsequently. Here it suffices to mention that there are no obvious alternatives. Any other substantially different arrangements would, at least under some conditions, have to allow for non-CPE allocations. That would leave us in the somewhat awkward position of analyzing a fundamentally flawed economy, while at the same time there would be no rationale as to why—and possibly also precisely how—it is flawed.

II. Socially Useless Information and Its Impact on Price

Given that the economy generates CPE allocations for every partition η, more information is never Pareto-inferior to less information. In other words, if η'' is more informative (finer) than η' then there are only two possibilities:

$$V_i(\delta_i(\eta'')) > V_i(\delta_i(\eta')), \qquad \text{at least one } i \in H, \tag{1}$$

or

$$V_i(\delta_i(\eta'')) = V_i(\delta_i(\eta')), \qquad \text{all } i \in H. \tag{2}$$

The assertion is immediate since $D(\eta') \subset D(\eta'')$ and the equilibrium allocations, $\delta(\eta')$ and $\delta(\eta'')$, are CPE relative to their respective partitions. Since (1) and (2) are exhaustive, a reasonable terminology is to say that the additional information provided by η'', as compared to (a coarser partition) η', is *socially useless* if and only if (2) obtains. This does not mean that (1) is the case of socially useful information since the inequality can be reversed for some other individual. No direct ranking of η'' vs. η' is now possible, so useful information requires not only (1) to hold, but also that $V_i(\delta_i(\eta'')) \geq V_i(\delta_i(\eta'))$ for all $i \in H$; the condition of usefulness is therefore much stronger as compared to (1) alone. The usefulness is weak if it is only known that $V_i(\delta_i(\eta'')) \geq V_i(\delta_i(\eta'))$ for all i. Further note that (2), as a practical matter, occurs only if the consumption plans in the two economies are identical for all individuals, i.e., the additional information has absolutely no (nontrivial) impact on the consumption plans. The condition will, for short, be written as $\delta(\eta'') = \delta(\eta')$.

Having specified the nature of useless information, the first question one may want to address can then be phrased as follows: if η'' adds no socially useful information as compared to η', how will this outcome be reflected in the two sets of signal-contingent asset prices? The simplest case considers no information ($\eta = \eta^0$) as a reference point, and the problem becomes one of saying something about $\{P(f, y)\}_{f,y}$ as it relates to $\{P(f)\}_f$. To get some additional focus on the problem, $\{P(f, \tilde{y})\}_f$ can beneficially be thought of as a vector of random variables with a distribution related to that of $\{P(f)\}_f$. The perspective effectively implies that the signal-beliefs are assumed to be identical across individuals. By adding the usual strategically simplifying assumption of time-additive utilities, the following Lemma is obtained.

LEMMA I. *Consider the following assumptions:*

(i) *signal-beliefs are homogeneous ($p_i(y) = p_1(y)$ all y and i);*
(ii) *utilities are time-additive ($u_{is}[\cdot, \cdot] = u_i^1[\cdot] + u_{is}^2[\cdot]$); and*
(iii) *the final allocations are identical in the two economies η' and $\eta^0(\delta(\eta') = \delta(\eta^0))$.*

Then, for $f = 0, 1, \cdots, |F|$, one has

$$\textstyle\sum_{y \in Y_{\eta'}} p(y) \frac{P(f, y)}{P(0, y)} \equiv E\left[\frac{P(f, \tilde{y})}{P(0, \tilde{y})}\right] = \frac{P(f)}{P(0)} \tag{3}$$

where $\{P(f, y)\}_{f,y}$ and $\{P(f)\}_f$ are the prices in the two respective economies.

Proof: See the Appendix.

The result (3) is precise and has a straightforward interpretation: the (relative) prices in the economy without information are unbiased estimators of the (relative) prices which are to be *expected* in the economy with information. The prices of securities $f = 1, \cdots, |F|$ must be deflated by the prices of the numeraire commodity ($C(0)$). This should not be viewed as a limiting restriction, but rather as a requisite for a meaningful interpretation. Subsequently, the pricing-hypothesis (3) will be generalized, and at that point further interpretations will be provided.

Lemma I does not rely on an assumption that the allocation is CPE (or, alternatively, that the $\{w_i(y)\}_y$'s are optimal). However, such purported generality would seem deceptive since the final allocations were presumed identical in the two economies. In any general equilibrium model the final allocations are endogenous; Lemma I is therefore of "dubious quality" unless conditions leading to identical equilibrium allocations are identified. Moreover, these conditions must be consistent with the assumptions of Lemma I. A direct inspection of its proof suggests the need for relatively restrictive conditions.

LEMMA II. *Consider two economies with partitions η' and η^0, respectively, and suppose that:*

(i) *signal-beliefs are homogeneous;*
(ii) *utilities are time-additive;*
(iii) *the allocations obtained in the two economies are CPE relative to their respective partitions; and*
(iv) *the allocation $\delta(\eta^0)$ is fully Pareto-efficient relative to η^0.*

Then the allocations are identical in the two economies.

Proof: See Ohlson and Buckman [14], Corollary III.[4]

On a technical level, the result is fairly straightforward. Conditions (i), (ii), and (iv) imply identical sets of FPE allocations for the two partitions η' and η^0. Hence, since $\delta(\eta^0) \in D(\eta')$ and $\delta(\eta^0)$ is FPE relative to not only η^0 but also η', it follows that the CPE allocation $\delta(\eta')$ does not depend on y. From a somewhat different perspective, there are potentially two reasons why the information may not be useless: (a) the optimal allocation of $C(0)$ across individuals is contingent on signals, or (b) markets are not sufficiently complete and information serves as a substitute for markets. The first reason (a) is ruled out by condition (i) in conjunction with (ii), and the second, (b), by (iv). A more global statement is that *all* sets of FPE allocations, regardless of the partition, are identical and independent of signals if utilities are time-additive and beliefs are completely homogeneous (Ohlson [13], Theorem II).

Only two additional observations are needed in order to produce a fairly general result out of Lemmas I and II combined. First, the use of the null partition as a point of reference is not necessary, and no significant complications are introduced if instead η'' is compared to any coarser η'. Second, as Arrow [1] points out, there are $|Y_\eta| + 1$ numeraire commodities in the A-regime. Hence, there will be no loss of generality if all of the prices of the signal-conditioned time-zero commodities are put equal to one.

THEOREM I. *Suppose that utilities are time-additive, signal-beliefs are homogeneous, and the two economies η'' and η' generate CPE allocations. Suppose additionally that the allocation attained in the η'-economy is not only CPE but also FPE. Then there is social indifference with respect to any partition η'' which is finer than η'. One further obtains the pricing relationship*

$$E[P(f, \tilde{y}'') \mid \tilde{y}'] = P(f, \tilde{y}'), \qquad f \in F, \quad y' \in Y_{\eta'}; \tag{4}$$

with no loss of generality, the numeraire prices $\{P(0, y'')\}_{y''}$ and $\{P(0, y')\}_{y'}$ have all been put equal to one.

COROLLARY I. *If the pricing-hypothesis* (4) *holds with probability one (i.e., for all $y' \in Y_{\eta'}$),* then

$$E[P(f, \tilde{y}'') \mid P(f, \tilde{y}')] = P(f, \tilde{y}') \tag{5}$$

[4] The proof can be viewed as an extension of Lemma III in Hakansson, Kunkel, and Ohlson [9]. This Lemma implies that, given (i), (ii), and (iv), there exists no allocation $\delta(\eta') \in D(\eta')$ which strictly Pareto-dominates an allocation $\delta(\eta^0) \in D(\eta^0)$ when $\delta(\eta^0)$ is FPE relative to η^0.

$$\mathrm{Var}[P(f, \tilde{y}')] \leq \mathrm{Var}[P(f, \tilde{y}'')] \tag{6}$$

where the inequality is an equality if and only if $P(f, \tilde{y}') = P(f, \tilde{y}'')$ *when* $p(y'/y'') > 0$.[5]

The hypothesis (4) is a statement to the effect that the disclosure of additional information has a predictable impact on prices.[6] The prices which would obtain in a more informative economy will be greater for some signals and less for some other signals, in such a fashion that the expected price, given the coarser information, is simply the value implied by the coarser information. Loosely speaking, the additional information now "calibrates" values, as opposed to a more arbitrary and unpredictable change in the pricing structure. The relationship (4) can be modified into statement (5) which also has a direct interpretation: with probability one, prices in the coarser economy are unbiased estimators of the prices in the finer economy. This is, of course, a sharp implication of the (indirect) assumption that the additional information is socially useless.

Theorem I, as well as the first part of Corollary I, are in some respects generalizations of Rubinstein's [16] analysis of information. His model is a multiperiod (three dates) economy with "primitive" contingent claims securities. Although the three-dates dimension is distinctly different from the comparative analysis of this paper, there are structural similarities in the equilibrium conditions. Rubinstein shows that if beliefs are homogeneous (a special case of "nonspeculative" beliefs), then there is no need for portfolio revisions at the intermediate date, and the price revisions of certain contingent securities are precisely offset by probability revisions (see [16], the theorem on p. 816). The "equivalent" result in the present setting occurs in the case of Arrow-Debreu securities; if $a(f, s) = 1$ when $f = s$ and zero otherwise, then it is readily verified that $p(s/y)/p(s) = P(s, y)/P(s)$ (assuming the numeraire prices have all been put equal to one). This equation therefore represents the most simple version of the pricing-hypothesis. (Alternatively, $p(s/y)k_s = P(s, y)$ and $p(s)k_s = P(s)$, where k_s is independent of y and the same in both equations.) However, the significance of Arrow-Debreu securities is that they lead to FPE, so the direction needed for a generalization is fairly clear-cut. The equilibrium conditions developed by Rubinstein contain most of the clues required for Theorem I, in spite of the differences in models.

The second part of the corollary is of interest because incremental useless information leads to larger variability in prices. Since one can readily show that generally, $P(f, y'') \neq P(f, y')$ (given $p(y''/y') \neq 0$), the increase in variability is strict. The condition $P(f, y'') = P(f, y')$ is met only under highly special circumstances such as when $a(f, s)$ and y'', given y', are statistically independent and utilities are state independent. The incremental information is obviously socially useless in this setting because it is also irrelevant in partial equilibrium. The converse, however, is not valid: incremental information may well be relevant and thus affect equilibrium prices, while at the same time the additional infor-

[5] The proof of (5) is immediate, and (6) follows from Rao-Blackwell's inequality.

[6] It may be noted that (5) is a *bona fide* martingale hypothesis in a sequential trading model. That is, it can be shown that $\{P(f)\}$ are the prices in the prior round of trading, and $\{P(f, y)\}$ the second round prices.

mation is useless from a welfare point of view. Uselessness is not necessarily contradictory to the facts that payoffs and prices correlate with signals, a point not always well appreciated in the empirical research literature. It may also be mentioned that the incremental variability in itself does not add any risk, thereby directly or indirectly inducing uselessness. Any conception of the problem of usefulness vs. uselessness in these terms is misleading if not outright false. This point is clarified in the next section in which the converse question will be analyzed. The focus there will be on the derivation of welfare implications from the pricing-hypotheses (3) and (4).

III. The Pricing-Hypothesis and the Social Value of Information

In analyzing the welfare implications of the pricing-hypotheses (3) and (4), the basic structural assumptions of previous sections are retained. The probabilistic structure of the hypotheses requires implicitly homogeneous signal-beliefs. Further, consistent with previous analysis, let the utilities be time-additive and let the equilibria be CPE. As noted, the allocations of the numeraire commodity are independent of signals under these three conditions. (See also the Proof of Lemma III).

The basic assumptions therefore add considerable structure to the economy, and this is of course needed if any welfare conclusions are to be derived from the structure of prices.

The first result considers the welfare implication of the pricing-hypothesis when the null partition serves as a point of reference. The pricing-hypothesis (3) is shown to imply that information *has social value in a weak sense*; under the worst of circumstances nobody is worse off, and some individuals may even be strictly better off. The latter depends on whether or not the final allocations in the two economies are the same. More formally, the following holds.

LEMMA III. *Consider two economies with partitions η' and η^0, respectively, and suppose that:*

(i) *signal-beliefs are homogeneous;*
(ii) *utilities are time-additive;*
(iii) *the allocations $\delta(\eta')$ and $\delta(\eta^0)$ are CPE relative to their respective partitions; and*
(iv)
$$\Sigma_{y \in Y_{\eta'}} p(y) \frac{P(f, y)}{P(0, y)} \equiv E\left[\frac{P(f, \tilde{y})}{P(0, \tilde{y})}\right] = \frac{P(f)}{P(0)}$$

where $\{P(f, y)\}_{f,y}$ and $\{P(f)\}_f$ are the prices in the two respective economies.

Then

$$V_i(\delta_i(\eta')) \geq V_i(\delta_i(\eta^0)), \qquad \textit{for all } i,$$

where the inequality is strict if and only if $\delta_i(\eta')$ depends on y; i.e., $\delta_i(\eta') \neq \delta_i(\eta^0)$.

Proof: See the Appendix.

Again, the use of the null partition as a benchmark is basically arbitrary.

Modifying the hypothesis from (3) to (4) does not affect the conclusions. Likewise, there is no loss of generality if the numeraire prices $P(0, y'')$ and $P(0, y')$ are all put equal to one. The second theorem of this paper can now be stated as follows.

THEOREM II. *Suppose that utilities are time-additive, signal-beliefs are homogeneous, and that the economies η'' and η' generate CPE allocations. Without loss of generality, put $P(0, y'') = P(0, y') = 1$ for all $y'' \in Y_{\eta''}$ and $y' \in Y_{\eta'}$ and suppose further that*

$$E[P(f, \tilde{y}'') \mid \tilde{y}'] = P(f, \tilde{y}'), \qquad \text{for all } f \in F,$$

with probability one, and where η'' is more informative than η'. Then there is positive social value in a weak sense, and in a strong sense if and only if η'' results in an allocation different from that of η'.

As the proof of Lemma III reveals, the reason there is social value of information in a weak sense is that the consumption opportunities have been expanded from *each individual's* perspective. The additional opportunities may or may not be worthwhile to exploit, but their exploitation occurs if and only if there is a strict improvement in expected utility. Evidently, the "calibration" of asset values enriches the set of available opportunities, and all of the strategies based on coarser information remain feasible. The reason the scenario can work is due partly to the CPE requirement. That particular assumption makes the uncertainty inherent in signals insurable, so that the introduction of more information does not result in "distributive risks."[7]

The above results are conceptually distinct from those developed by Hakansson, Kunkel, and Ohlson [9, Lemma II and Theorem I]. Their result of positive value of information is based on assumptions about endowments (ϵ) as opposed to assumptions about the structure of prices. This obviously differs from the present case since Theorem II makes assumptions about the structure of prices but is independent of the endowments.

The conditions of Theorem I and Theorem II are in many respects similar, although Theorem I implied social indifference to information in contrast to Theorem II which implied social indifference *or* usefulness. This suggests the possibility of identifying conditions consistent with those of Theorem II which nonetheless preclude strict positive social value of information. The relevant condition present in Theorem I but not in Theorem II is the requirement that the allocation in the coarser environment is FPE (rather than simply CPE). As was discussed previously, under this condition the CPE allocations in the finer economy are also FPE, and, given the assumptions on preferences/beliefs, are in fact identical to the FPE allocations in the coarser economy. This argument leads to the following Corollary.

COROLLARY II. *Given the assumptions of Theorem II, there is no strict positive social value of information if the allocation in the less informative economy, $\delta(\eta')$, is FPE relative to η'.*

Ohlson and Buckman [14] have observed that the equilibrium allocation $\delta(\eta')$ is generally FPE if and only if (A, η') is a conditionally complete market (CCM).

[7] See also the discussion on sequential trading models in footnote 6 and Section IV.

Hence, given the general assumptions about preferences and beliefs, and the pricing-hypothesis (4), information serves a strictly positive role only if the spanning space of the matrix A is effectively constraining in the coarser economy. Conversely, if (A, η') is not CCM but (A, η'') is—i.e., A is nonconstraining in the more informative environment (η'') but not in the coarser environment (η')—then information takes the place of securities and the absence of complete markets has no adverse impact on the sharing of risks in the η'' economy. The allocation $\delta(\eta'')$ is now necessarily FPE although the allocation $\delta(\eta')$ is not. It follows that $\delta(\eta'') \neq \delta(\eta')$ and, given the pricing-hypothesis, the finer information structure η'' is of strict positive social value as compared to η'. Additionally, in this context the final allocation $\delta(\eta'')$ is always FPE for all sufficiently fine η''.[8] As a consequence, if the pricing-hypothesis happens to hold for all partitions, then there is strict positive social value of more information up to the point where the information ensures FPE allocations.

The previous discussion must be interpreted carefully. In an equilibrium setting the structure of prices is endogenously determined, and thus there is no *general* reason why the pricing-hypothesis should prevail. The conditions of Theorem I are, of course, consistent with the pricing-hypothesis, but in that theorem the information is useless. The emergent question therefore is whether there are conditions such that the pricing-hypothesis obtains *and* the information is socially useful. The only apparent "general" case requires at least one individual, but not all, to be risk-neutral. As a consequence the valuation is risk-neutral, i.e., $P(f, y)/P(0, y)$ is proportional to $\sum_s a(s, f)p(s/y)$, obviously implying (4). (Since not all individuals are risk-neutral, information affects the risk-sharing positively if markets are incomplete.) A more interesting possibility would be the development of approximation results; this approach is considered in the next section.

The above comments can be elaborated upon from a slightly different perspective, thereby clarifying the results of the two theorems. Theorem I could be viewed as an extension of the Modigliani-Miller theorem, and Theorem II extends a little known, highly specialized "converse" of the Modigliani-Miller theorem observed in Hakansson [8].[9] Consider first the Modigliani-Miller theorem. In economies with a null partition, the essence of their result is simply that if two market matrices A and A' span the same space, then the two economies are fundamentally identical if the endowed consumption plans are also the same, i.e., $\bar{c}_i(0) = \bar{c}_i'(0)$ and $\sum_{f\in F} \bar{z}_i(f)a(f, s) = \sum_{f\in F} \bar{z}_i'(f)a'(f, s)$ for all i, s. Because the two economies are identical, this specifically means that the final allocations and the marginal rates of substitutions of consumption goods are identical. In turn, this implies that the two price-vectors of securities, P and P', have a direct relationship regardless of preferences/beliefs. More precisely, since A and A' span the same space, there exists an $|F| \times |F'|$ matrix B such that $A = BA'$; the equivalence of the marginal rates of substitution vectors ensures the existence of one implicit price vector, R, such that $P = AR$ and $P' = A'R$. It follows

[8] Specifically, consider the case of $\eta'' = \eta^* \equiv$ perfect information; it is now readily shown that, regardless of A, an allocation $\delta(\eta^*)$ is CPE with respect to η^* if and only if it is also FPE with respect to η^* (see Proposition I in Ohlson and Buckman [14]).

[9] This observation is not surprising in view of Ohlson and Buckman's paper. They emphasize that the theory of social value of public information is essentially an extension of the theory of markets.

directly that $P = BP'$. The flavor of this result is basically identical to that of Lemma II (or Theorem I). Corresponding to the spanning equivalence of A and A' in the Modigliani-Miller theorem are assumptions (i), (ii), and (iii) in Lemma II, since these assumptions ensure the sets of *efficient* (FPE) allocations to be the same for the two economies. Endowed consumption plans are obviously independent of the information structure. The economy is thus fundamentally the same whether or not there is information, leading to identical final allocations ($\delta(\eta') = \delta(\eta^0)$). But given this equivalence, one should immediately also suspect a relationship between the price-vectors of the two economies. And, indeed, this is precisely what the pricing-hypothesis (3) expresses.

Consider next the following "converse" of the Modigliani-Miller theorem. The analysis retains the assumption that the endowed consumption plans are the same in both economies. Further, assume that the implicit price systems of consumption plans can be equated, i.e., there exists a vector R such that $P = AR$ and $P' = A'R$. The latter is, in effect, an assumption about the relationship between the price-vectors P and P'. Given these two assumptions, if A, in addition, spans a *subspace* of A', then clearly nobody is worse off in the primed economy since each individual now enjoys expanded consumption opportunities. But, of course, there is generally no reason to expect that the two implicit price-systems can be equated, the obvious exception occurring when A and A' span the same space and the final allocations become the same. The reasoning is strikingly similar to that of Lemma III (Theorem II). The critical assumption in that lemma, (3), described a relationship between the two price-systems, and when combined with assumptions (i), (ii), and (iii), individuals' opportunities were expanded.

In the present analysis, as well as in Hakansson's, the presumed structure of prices permits all individuals to partake in the expanded consumption possibilities. Nobody could therefore be worse off. Aggregate allocation opportunities were expanded because $D(\eta') \subset D(\eta'')$; this corresponds to the assumption that A spans a subspace of A' in Hakansson's analysis. However, it is probably appropriate to, again, mention that there is no prior reason to expect the pricing-hypothesis to hold unless the added opportunity set, $D(\eta'') - D(\eta')$, contains only inefficient (non-CPE) allocations. The incremental information is then useless since these inefficient allocations are irrelevant in equilibrium.

IV. Some Implications for Empirical Research

As an empirical proposition, there appears to be no direct way to test the validity of the pricing-hypothesis (4) across two disclosure environments. Any direct empirical experiments are ruled out since the two environments cannot exist simultaneously. "Indirect" approaches would seem to involve considerable empirical difficulties in order to deal with the *ceteris paribus* problem.[10] The experimental setting would have to consider the price structure prior to and posterior to a change in the (mandated) disclosure environment (e.g., the require-

[10] Any hypothesis derived from "comparative" analysis suffers from this disadvantage. For example, this is one reason the estimation of demand elasticities is so difficult.

ment that corporations disclose inflation-adjusted financial data). In lieu of *ceteris paribus*, the experimental design must then identify variables, other than the "new" information, that could impact on the structure of prices. Dealing with these kinds of problems on a practical level most likely demands rather heroic assumptions. The implementation of a credible empirical test of the pricing-hypothesis therefore seems to be a complex if not impossible task.

The consideration of economies with sequential trading would not change the conceptual difficulties associated with the comparative nature of the welfare analysis. This is unfortunate since, as was mentioned in Section II, sequential trading models attaining CPE are structurally isomorphic, although "empirically distinct," to the model relied on throughout this paper. In a sequential trading regime, the pricing hypothesis (3) thus restricts the prices under the null partition to be unbiased estimators of the second round prices in the more informative economy. Given the assumptions on preferences/beliefs (assumptions (i) and (ii) in Lemma III), one can then analytically infer that the prices under the null partition are identical to the first round prices in the more informative economy. But, again, this proposition is not directly testable since the economies cannot exist simultaneously. Also, one cannot infer the pricing structure in the null economy from an empirical proposition that first round prices are unbiased estimators of the second round prices. In sum, the significance of the pricing-hypothesis is the nontestable idea that prices vary with the information (i.e., the signals), while the information *structure* is not directly relevant.

It may be more interesting and promising to consider the pricing-hypothesis as a maintained hypothesis, and test empirically for useful vs. useless (indifferent) social value of information. The natural approach in addressing this issue would be to focus on trading volume; the analysis has shown that uselessness occurs when the additional information has no impact on allocations, and, conversely, there is usefulness when allocations are affected. Trading volume takes on relevance as an empirical manifestation of the impact of information on risk-sharing. The maintained pricing-hypothesis, on the other hand, simply serves the function of excluding redistributive effects. Given that information is without cost, more information is now always weakly better than less, with strict improvement if and only if actions (i.e., allocations) are affected.

The idea that the trading volume is a variable of interest is, of course, hardly novel. Beaver [3], in an early study, examined the relationship between trading volume and the dissemination of financial reports. His research was motivated explicitly by the notion that trading volume reflects information content. This point is well taken, since a sufficient condition for the relevance of information is the response of individuals' trading decisions to information. However, the paper provides no direct and explicit link to the usefulness of information, although some of the language is quite suggestive. Beaver also tested for the relationship between the dissemination of information (earnings) and the variability of returns. On heuristic grounds this is appropriately justified by the idea that the increased variability occurs when prices correlate with concurrently released signals. But Beaver's analysis does not formalize the variability as an implication of the informativeness of the disclosures. Perhaps more significant, there is no suggestion that the analysis of usefulness must consider simultaneously the behavior of prices and the allocation of securities.

There exists a particularly straightforward model dealing with the structure of correlations between signals and payoffs, and yet it relates directly to the issue of usefulness vs. uselessness of incremental information. The approach is the simplest possible linear model of prices and signals in a mean-variance economy.

Suppose the price of every asset f, given any signal y and information η, equals:

$$P(f, y) = \sum_{i\in H} k_i E_i[\tilde{a}(f) \mid y, \eta] - \text{Risk}(f, \eta)$$

where

$\tilde{a}(f) \equiv a(f, \tilde{s})$ = the random gross payoff of security f,
$E_i[\tilde{a}(f) \mid y, \eta] \equiv \sum_s a(f, s) p_i(s/y)$ where $y \in Y_\eta$,
k_i = parameter pertaining to the i^{th} individual, and
$\text{Risk}(f, \eta)$ = a constant depending only on f and η.

The above equilibrium relationship is a generalized version of the CAPM allowing for heterogeneous beliefs. It holds under the specialized assumptions originally considered by Lintner [11] (the payoffs are normally distributed and utility is negative exponential). The k_i's relate to individuals' risk disposition. The risk measures are proportional to

$$\sum_{f'} \text{cov}[\tilde{a}(f), \tilde{a}(f') \mid \eta]$$

and are presumed independent of y. This independence follows if y is a vector of signals, and $E_i[\tilde{a}(f) \mid y, \eta]$ is linear in y for all i. A subscript i is unnecessary since all individuals are assumed to agree about the risk measure. That is, individuals disagree on means of payoffs but agree on variances-covariances.

Consider next two different information structures, η'' and η', where η'' is strictly finer than η'. The signals provided are appropriately thought of as two vectors, $(\tilde{\underline{x}}_1, \tilde{\underline{x}}_2)$ and $\tilde{\underline{x}}_1$, respectively. Suppose further $\text{Risk}(f, \eta'')$ is identical to $\text{Risk}(f, \eta')$ for all f. The pricing-hypothesis (4) is then satisfied,

$$E[P(f, (\tilde{\underline{x}}_1, \tilde{\underline{x}}_2))] \mid \underline{x}_1] = P(f, \underline{x}_1)$$

since, by the law of conditional expectations,

$$E[E[\tilde{a}(f) \mid (\tilde{\underline{x}}_1, \tilde{\underline{x}}_2)] \mid \underline{x}_1] = E[\tilde{a}(f) \mid \underline{x}_1]$$

for all $\underline{x}_1$. Moreover, $P(f, (\underline{x}_1, \underline{x}_2))$ is independent of $\underline{x}_2$ if and only if this vector of incremental information is conditionally uncorrelated with $\tilde{a}(f)$ (i.e., $\text{cov}[\tilde{a}(f), \tilde{x}_{2j} \mid \underline{x}_1] = 0$ for all elements j). Under these circumstances, neither prices nor demand functions are affected by the outcomes of $\underline{x}_2$. The final consumption plans are therefore independent of $\underline{x}_2$, and thus the incremental information is socially useless. On the other hand, if the incremental information does correlate with $\tilde{a}(f)$ even though $\underline{x}_1$ is already available, then $P(f, (\tilde{\underline{x}}_1, \tilde{\underline{x}}_2))$ correlates conditionally with $\tilde{\underline{x}}_2$, i.e., $\text{cov}[P(f, (\tilde{\underline{x}}_1, \tilde{\underline{x}}_2)), \tilde{x}_{2j} \mid \underline{x}_1] \neq 0$ for some element j. The incremental information is also socially useful since the pricing hypothesis is satisfied, and, barring degenerate cases, equilibrium holdings depend on $\underline{x}_2$. In conclusion,

$$\text{cov}[P(f, (\tilde{\underline{x}}_1, \tilde{\underline{x}}_2)), \tilde{x}_{2j} \mid \underline{x}_1] \neq 0,$$

for some element j, is necessary and sufficient for the incremental information $\underline{x}_2$ to be socially useful.

The above provides a foundation for the well-known and not always appreciated[11] Beaver-Dukes [4] contention that statistical association metrics of returns and accounting signals bear on questions of the social choice of disclosure policies. Previous paragraphs delineate a set of "linear" conditions under which the contention is sensible. The general idea is formalized by the notion of partial correlations between signals and prices having a relationship to the social value of information. (Relative correlations ("more correlated than") are apparently not relevant. From that perspective, I find no validity in the Beaver-Dukes [4] analysis.)

This slightly modified version of the Beaver-Dukes contention is possibly important, so a summary of the underlying formal concepts seems worthwhile.

(a) In an (CPE-) efficient economy, more information has potential benefits since, with appropriate (re-) distribution of endowments, no individual need be harmed although all other individuals are at least as well off. The reason for this is simple: the set of total allocative opportunities is enlarged if more (public) information is available at no cost (i.e., $D(\eta') \subset D(\eta'')$ if η'' is finer than η').

(b) If, additionally, the pricing-hypothesis (4) holds, then all individuals' consumption opportunities are enhanced by more information independently of endowments. Every individual is therefore assured of not being harmed because of additional disclosure. (However, note that the conditions (i) and (ii) in Lemma III on preferences/beliefs are needed for this result.)

(c) If, additionally, individuals have state-independent utilities, then "information" that is independently distributed of the payoffs has no impact on final allocations and is socially useless.

(d) If, additionally, a specialized version of Lintner's heterogeneous beliefs CAPM holds, then the potential role of incremental information is to improve on predictions of expected payoffs. The prices are linear in the (conditional) expected payoffs, and the pricing-hypothesis is satisfied as a direct implication of the functional relationship between prices and signals. The identification of a positive welfare effect is therefore reduced to the presence of (conditional) contemporaneous correlation between signals and prices, provided that the individuals disagree about the signals' predictive implications.[12,13]

[11] See Gonedes and Dopuch [7] in particular.

[12] The reasons given here for supporting the claim that association metrics can be related to social value of information are more general (if not conceptually distinct) compared to Beaver and Dukes [4], or Beaver and Demski [6, footnote 11]. (These papers seemingly develop the special case of Theorem II in which $P(f, y') = P(f, y'')$ with probability one.) Furthermore, the ideas in these papers seem to be more related to statements about relative correlations ("more correlated than") rather than partial correlations ("$\tilde{x}_2$ correlates with payoffs, given $\tilde{x}_1$") Their pricing-hypothesis could also be interpreted as $P(f, \tilde{x}_2) = P(f, \tilde{x}_1)$ rather than $P(f, (\tilde{x}_1, \tilde{x}_2)) = P(f, \tilde{x}_1)$. As far as I know, there are no nontrivial assumptions leading to either of these results.

[13] Patell [15] identifies two types of association-metric tests: (a) information content, and (b) model evaluation. The distinction is important since it is implied that a test of (a) requires (b) as a maintained hypothesis and vice versa. (See Section 2 in Patell's paper.) Hence, a test of social value demands the specification of a model which relates signals to values; this is, of course, precisely what this section of the paper illustrates.

The careful reader must have noted that (d) in the above argument can only be approximate. As noted in the previous section, barring exceptional cases, one cannot generally expect the pricing-hypothesis to hold, and simultaneously, expect the incremental information to be of strict positive social value. The question, then, is: where is the approximation in the specialized version of the Lintner model? The answer is fairly straightforward: Since $a(f)$ (adjusted for noise) is linear in the signal-vector it follows that $\text{cov}[\tilde{a}(f), \tilde{a}(f') \mid \eta''] - \text{cov}[\tilde{a}(f), \tilde{a}(f') \mid \eta'] \neq 0$ generally (e.g., the difference is negative if $f = f'$), making the restriction $\text{Risk}(f, \eta'') = \text{Risk}(f, \eta')$ difficult if not impossible to derive from more basic assumptions. Rigor can be added, however, such that in the appropriate limit $\text{Risk}(f, \eta'')$ converges to $\text{Risk}(f, \eta')$ at a rate greater than the convergences of the conditional expectations. More specifically, a sequence of environments with decreasing "amounts" of additional information, indicated by Δ, results in[14]

$$\lim_{\Delta \to 0} \mid \text{Risk}_\Delta(f, \eta'') - \text{Risk}_\Delta(f, \eta') \mid / \mid E_{i\Delta}[\tilde{a}(f) \mid y''] - E_{i\Delta}[\tilde{a}(f) \mid y'] \mid = 0.$$

The development of a rigorous approximation in the context of the above model is not truly critical to justify the Beaver-Dukes perspective. The structure necessary for (a), (b), and (c) can be combined with assumptions other than those implied by (d), such that there is strict positive value if and only if there are nonzero partial correlations between security values and the additional information. (This is a more or less immediate consequence of Theorem I in Hakansson, Kunkel, and Ohlson [9].) The primary value of the model is structural simplicity. The economy is completely "linearized"; this means the pricing-hypothesis is met, and, further, the focus is entirely on how individuals perceive the expected payoff given the available information. Valuation is reduced to a linear relationship between signals and security values. This is instructive since the social value of information is then reflected in the underlying contemporaneous linear correlations. The nature of the model leads to a straightforward argument which is essentially consistent with the well-known Beaver-Dukes' contention.

Appendix

Proof of Lemma I: Given the information structure η', it is necessary that the equilibrium conditions

[14] In formal terms, the model of payoffs which yields the appropriate result is as follows. Let

$$\tilde{a}(f) = (\tilde{\underline{x}}_1, \underline{d}_1^i(f)) + \Delta(\tilde{\underline{x}}_2, \underline{d}_2^i(f)) + \tilde{\gamma}(f)$$

where $(\cdot, \cdot)$ denotes inner-product, and $\underline{d}_2^i(f)$, $\underline{d}_2^i(f)$ are vectors of constants ($i \in H$). The $\gamma(f)$'s are random terms uncorrelated with the $\tilde{\underline{x}}_1$, $\tilde{\underline{x}}_2$ vectors. The state-realization is given by $s \equiv (\underline{x}_1, \underline{x}_2, \gamma(1), \cdots, \gamma(|F|))$ and $y'' \equiv (\underline{x}_1, \underline{x}_2)$, $y' \equiv \underline{x}_1$; Δ is a "small" scalar. It is now readily shown that

$$E_i[\tilde{a}(f) \mid y''] - E_i[\tilde{a}(f) \mid y'] \sim \Delta$$

while

$$\text{cov}_i[\tilde{a}(f), \tilde{a}(f') \mid \eta''] - \text{cov}_i[\tilde{a}(f), \tilde{a}(f') \mid \eta'] \sim \Delta^2$$

for any $(\underline{x}_1, \underline{x}_2)$.

$$\textstyle\sum_{s\in S} p_i(s/y)m_{is}[c_i(0, y), c_i(s, y)]a(f, s) = \dfrac{P(f, y)}{P(0, y)}, \qquad f \in F,$$

are satisfied for all $y \in Y_{\eta'}$; using assumption (ii), in the above expression $p_i(s/y)m_{is}[\cdot, \cdot]$ is the y-conditioned marginal rates of substitution:

$$m_{is}(x_1, x_2) \equiv [\partial u_{is}^2(x_2)/\partial x_2]/[\partial u_i^1(x_1)/\partial x_1].$$

By assumption (iii), $\delta_i(\eta') = \delta_i(\eta^0)$, and this means that in the η'-economy $c_i(0, y)$ and $c_i(s, y)$ do not depend on y. Hence, multiplying the right-hand and left-hand sides of the equation by $p(y)$ and summing across y's one gets

$$\textstyle\sum_s p_i(s)m_{is}[c_i(0), c_i(s)]a(f, s) = \sum_y p(y) \dfrac{P(f, y)}{P(0, y)}, \qquad f \in F.$$

Note next that the left-hand side of the last expression constitutes the equilibrium condition of the η^0-economy and must therefore equal $\dfrac{P(f)}{P(0)}$. Hence, it follows indeed that

$$E\left[\frac{P(f, \tilde{y})}{P(0, \tilde{y})}\right] = \frac{P(f)}{P(0)}, \qquad f = 1, \cdots, |F|,$$

where the expectation is with respect to a common measure (assumption (i)).

Proof of Lemma III: It will be shown that a policy $\delta_i(\eta') = \delta_i(\eta^0)$ is in fact feasible in the η'-economy given assumptions (i)–(iv). The lemma then follows immediately.

The η'-economy is CPE with respect to η'; hence, without loss of generality, every individual i solves

$$\max_{\{\delta_i(\eta'), w_i\}} V_i(\delta_i(\eta'))$$

subject to:

$$\textstyle\sum_y w_i(y)\pi(y) \le \sum_y \bar{w}_i(y)\pi(y) \equiv \bar{\bar{w}}_i(\eta') \tag{A1}$$

and

$$P(0, y)c_i(0, y) + \textstyle\sum_f P(f, y)z_i(f, y) \le w_i(y) \tag{A2}$$

where

$$w_i \equiv \{w_i(y)\}_y, \; \delta_i(\eta) \equiv \{c_i(0, y), z_i(f, y)\}_{f,y}$$

and

$$\bar{w}_i(y) \equiv P(0, y)\bar{c}_i(0) + \textstyle\sum_f P(f, y)\bar{z}_i(f). \tag{A3}$$

Now, Equations (A1) and (A2) combine into

$$\textstyle\sum_y \pi(y)P(0, y)c_i(0, y) + \sum_y \sum_f \pi(y)P(f, y)z_i(f, y) \le \bar{\bar{w}}_i(\eta'). \tag{A4}$$

Next it is shown that given assumptions (i) and (ii), equilibrium in a (iii) economy implies that one can put $\pi(y)P(0, y) = p(y)$. To demonstrate this, first note that, in equilibrium, CPE implies $c_i(0, y)$ will never depend on y given (i) and (ii). The latter follows because

$$\sum_y p(y)u_i^1[c_i(0, y)] \le u_i^1[\hat{c}_i(0)] = (\sum_y p(y)u_i^1[\hat{c}_i(0)]), \qquad \text{all} \quad i \in H$$

where the inequality is Jensen's and $\hat{c}_i(0) \equiv \sum_y p(y)c_i(0, y)$ is a feasible allocation. Thus, a signal-contingent allocation of the time-zero good can always be Pareto-dominated by a signal-independent allocation, so the former type of allocation is not consistent with assumption (iii). Second, given that $c_i(0, y)$ is independent of y, and using Equation (A4), it is easily verified that assumption (iii) implies that $\pi(y)P(0, y) = p(y)$ is always consistent with the equilibrium conditions.

Equation (A4) now equals

$$\sum_y p(y)c_i(0, y) + \sum_y \sum_f p(y)[P(f, y)/P(0, y)]z_i(f, y) \le \bar{\bar{w}}_i(\eta')$$

Consider next any policy $\check{c}_i(0)$, $\check{z}_i(f)$ which does not depend on y; then, noting that $E[P(f, \tilde{y})/P(0, \tilde{y})] = P(f)/P(0)$, (A4) reduces to

$$\check{c}_i(0) + \sum_f \check{z}_i(f)[P(f)/P(0)] \le \bar{\bar{w}}_i(\eta').$$

It is readily verified that $\bar{\bar{w}}_i(\eta') = \bar{\bar{w}}_i(\eta^0)$ when $P(0) = 1$; hence, the policy $\delta_i(\eta^0)$, which is the equilibrium-allocation in the η^0-economy is also available in the η'-economy.

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