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A Theoretic Framework for the Analysis of Credit Union Decision Making

DONALD J. SMITH*

ABSTRACT

This paper presents a formal theoretic framework to analyze credit union interest rates on loans and savings deposits. The unique motivational and institutional features of a credit union, in particular its structure as a financial service cooperative, are used to develop the objective function. This is based on a comparison of the credit union's rates to alternatively available market rates and includes parameters to recognize the possibility of borrower-saver conflict. The principal result is that the optimal rates and reactions to exogenous changes depend critically on the preference of the organization toward financial gain to the borrowing and saving members.

CREDIT UNIONS (CUs) have recently been the subject of a number of empirical investigations based on an assumption that CUs minimize cost given an exogenously determined level of output. For instance, see the work of Fry et al. [5], Koot [7], Murray and White [8], [9], Navratil [10], and Wolken and Navratil [17]. This assumption is usually defended by the recognition of a CU as a not-for-profit, service-oriented cooperative in competition with other financial institutions. Since it would be incongruous to model a CU as maximizing profit or the return on equity (since members' share deposits in the CU cannot appreciate in value or be publicly traded), cost minimization is deemed to be the appropriate objective function. However, this theoretic foundation is deficient for a general analysis of CU decision making for several reasons.

First, the fundamental motivation of a CU is to provide financial services to the membership, in particular a depository for savings and access to consumer and mortgage credit. Moreover, these offered services should be at least as attractive with respect to their price and nonprice characteristics as those available from other institutions—otherwise there would be no economic rationale to organize the CU or for members to participate. Therefore, the key decisions made by management are the types of loan and savings accounts to offer, and the prices and/or quantities of those accounts. Naturally, once those decisions are made, the CU would seek to combine labor and physical capital resources in the most cost-efficient manner.

Second, the output (however defined) of a CU can be taken as exogenously

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determined only in the broadest sense. Since member entry into the CU is ultimately constrained by the common bond specified in the CU's charter, there is some upper bound to potential output.[1] Aside from that, output is quite endogenous. The rates charged on loans and paid on savings deposits, the types of accounts offered, and promotional and advertising activities all influence the level of output, or the total asset size, of the CU. See Baltensperger [1] for a critique of the exogeneity of output assumption in models of banking behavior.

Cost minimization, therefore, is relevant and of importance but does not get to the heart of the objective of a CU. What is needed is a theoretic formulation that directly addresses the unique institutional and motivational features of a CU as a financial intermediary. Such a framework is developed in Section I of this paper. The optimization problem that follows from the framework, and the comparative static properties of the solution, are discussed in Section II. Section III summarizes the results of this analysis.

I. A CU Objective Function and Its Constraints

A. The Objective Function

Since the purpose of a CU is to provide financial services to its members, the objective function should be specified in terms of the flow of services during the relevant time period. The problem is to provide a measure of these service flows. The framework to be presented here will follow the approaches of Walker and Chandler [16] and Smith et al. [12]. A market rate comparison is used as the appropriate measure of benefits derived from CU patronage. For the j^{th} member of the CU, let the Net Gain on Loans (NGL^j) and the Net Gain on Savings (NGS^j) be:

$$\text{NGL}^j = (r_{LM}^j - r_L)L^j \tag{1}$$

$$\text{NGS}^j = (r_S - r_{SM}^j)S^j \tag{2}$$

where r_{LM}^j is the best interest rate at a competing institution available to the member for an equivalent loan, r_L is the CU's rate, L^j is the amount of the member's loan, r_S is the CU's dividend rate on a savings account, r_{SM}^j is the best rate at an alternative institution for the same type of transaction, and S^j is the amount of the savings involved.[2]

There are several assumptions implicit in this formulation. Notice that r_L and

[1] These common bonds are specified in the CU's charter and are usually occupational, but can also be associational or residential. Also, many CUs have once-a-member-always-a-member clauses and allow family members, further raising the upper limit on size.

[2] CUs have traditionally been able to offer their members positive Net Gains because of preferential regulatory treatment (tax exemption, higher than Regulation Q ceilings on savings rates), sponsor subsidies (payroll deductions, on-site locations), volunteer labor, and informational cost advantages by operating only within the common bond. Moreover, these common bond restrictions on member eligibility allow a CU to persist in offering attractive rates without encountering the usual competitive market equalization process associated with free entry. See Black and Dugger [2] and Brockschmidt [3] for thorough institutional descriptions of CUs.

r_S do not carry a superscript for the j^{th} member. CUs generally offer all of their members the same rate on a transaction made at the same time. Different degrees of potential loss on loans are managed with additional collateral or co-signers rather than explicit risk premiums on the loan rate. Also, it is assumed that an alternative rate always exists, thus the member can always borrow or save at another institution in the absence of the CU. In addition, these measures of the service flow capture only pecuniary gain. Other aspects of the transaction such as waiting time, convenience, courtesy, and information disclosure are neglected. Thus, the pecuniary measure of Net Gains could be zero, or even negative, and the member might still desire to transact at the CU if these other aspects were sufficiently valued.

Suppose the CU has J members, then the individual Net Gains can be aggregated over the membership.[3]

$$\text{NGL} = \textstyle\sum_{j=1}^{J} (r^j_{LM} - r_L)L^j \tag{3}$$

$$\text{NGS} = \textstyle\sum_{j=1}^{J} (r_S - r^j_{SM})S^j \tag{4}$$

Two further assumptions are useful to construct a practical CU objective function. First, assume that all members have the same alternatives, that is $r^j_{LM} = r_{LM}$ and $r^j_{SM} = r_{SM}$ for all J members. There is little problem with this for savings accounts since other financial institutions are generally open to all depositors. Loans are different because lending institutions necessarily consider the applicant's individual credit history and ability to repay. However, the common bond criterion for membership perhaps creates more homogeneity within a particular CU than across the general population. Another way to justify this equality of alternatives assumption is to suppose that the CU's management has a typical, or average, member in mind when making decisions. Then r_{LM} and r_{SM} could represent the options open to the typical member.

The second assumption is that the sum of the individual loan and savings balances can be written as a function of the CU and alternative market rates.

$$\textstyle\sum_{j=1}^{J} L^j = L(r_{LM}, r_L), \qquad \frac{\partial L}{\partial r_{LM}} > 0, \qquad \frac{\partial L}{\partial r_L} < 0 \tag{5}$$

$$\textstyle\sum_{j=1}^{J} S^j = S(r_{SM}, r_S), \qquad \frac{\partial S}{\partial r_{SM}} < 0, \qquad \frac{\partial S}{\partial r_S} > 0 \tag{6}$$

The signs to the partial derivatives indicate that a lower loan rate and a higher dividend rate will attract more loans and shares to the CU than otherwise. Moreover, better rates at alternative institutions will draw activity away from the CU. These loan demand and share supply schedules will probably not be infinitely elastic for a particular CU because members will have varying costs of search activity between institutions.

So far it has been implicitly assumed that all savings and loan transactions

[3] This implicitly assumes that there is only a single homogeneous class of loans and of savings accounts, for instance, personal loans and regular shares. This methodology can be easily extended to include multiple loan and deposit types, see Smith [11].

have a maturity of one period. In practice, however, CU loans and often savings deposits have maturities extending beyond the current period. Therefore, a comparative rate advantage on a transaction provides the member with an extended benefit. A way to model this issue in general terms is to let ϕ and ω represent for the loans and savings, respectively, the proportion of the balances inherited from the previous period that are retired during the current period. For example, if L is the amount of new loans issued this period, the declining balances for future periods will be $(1 - \phi)L$, $(1 - \phi)^2L$, and so on. Then if d is the rate such that the future gains are discounted by $(1 + d)$, $(1 + d)^2$, etc., to calculate the present value, the current period Net Gains need to be multiplied by $\frac{1 + d}{d + \phi}$ and $\frac{1 + d}{d + \omega}$, respectively.[4]

Another issue is the possibility of systematic preference by the CU to either its borrowers or savers.[5] Such a predisposition can be introduced by weights in the objective function, for instance β on NGL and $(1 - \beta)$ on NGS, where $0 \leq \beta \leq 1$. Then a CU totally oriented to the interests of its borrowers is represented by $\beta = 1$ or to its savers by $\beta = 0$. A neutral CU which places equal weight on gains to borrowers and savers is modelled by $\beta = ½$.[6]

A general objective function is to maximize the sum of NGL and NGS, the net pecuniary gains on loan and savings transactions. After substituting the equality of alternative assumptions ($r^j_{LM} = r_{LM}$ and $r^j_{SM} = r_{SM}$) and the descriptions of loan demand (5) and savings supply (6) into (3) and (4) and adding the future benefit discount factors and the behavioral preference weights, the following expression is obtained:

$$\beta \frac{1 + d}{d + \phi} (r_{LM} - r_L)L + (1 - \beta) \frac{1 + d}{d + \omega} (r_S - r_{SM})S \tag{7}$$

This represents the general form of the CU objective function. It can be maximized by choosing the loan and dividend rates subject to the balance sheet and operating statement constraints.

B. The Balance Sheet Constraint

Since a CU is an ongoing entity, it inherits a balance sheet at the beginning of the current accounting period. Then, depending on its current decisions and various exogenous factors, the end-of-period balance sheet is passed on to the

[4] The present value of the extended NGL and NGS would be calculated as follows:

$$\text{NGL} = (r_{LM} - r_L)L + \frac{(r_{LM} - r_L)(1 - \phi)L}{1 + d} + \frac{(r_{LM} - r_L)(1 - \phi)^2L}{(1 + d)^2} + \cdots$$

$$\text{NGS} = (r_S - r_{SM})S + \frac{(r_S - r_{SM})(1 - \omega)S}{1 + d} + \frac{(r_S - r_{SM})(1 - \omega)^2S}{(1 + d)^2} + \cdots$$

The terms in the text are derived from the sums of these convergent series.

[5] Borrower and saver preference has been a key point in previous theoretic models of CU rate decisions, see Taylor [13], Flannery [4], and Walker and Chandler [16]. Smith [11] provides a review and critique of this literature.

[6] This description of a neutral CU was first introduced by Flannery [4].

succeeding period. The inherited balance sheet will be assumed to be the following:

$$L_0 + I_0 = S_0 + R_0 + U_0 \tag{8}$$

The principal assets are on the left-hand side, the sum of outstanding loans at the start of the period, L_0, and the investment account, I_0. CUs make money market investments, often in excess of their liquidity needs, such as deposits with other insured financial institutions and the purchase of Treasury and federal agency debt, typically in the form of common trust investments. Here these different types of securities are collected into just one category. Other asset types, such as buildings and office equipment, are neglected because they are usually a very small percent of total assets.

The principal liabilities, on the right-hand side, are the sum of the savings accounts, S_0, and the capital account. This capital account consists primarily of the regular reserves, R_0, which are only used to write off defaulted loans, and undivided earnings, U_0.[7] The use of undivided earnings is discretionary and can be to supplement current period income for the payment of dividends, among other uses. A CU can also borrow funds from other financial institutions. This debt will be modelled as an offset to the investment account, so it is possible that $I_0 < 0$.

Assume that all of the members' savings and loan transactions for the current accounting period take place at the beginning of the period. Then, after those transactions, the asset/liability structure must also be in balance. Therefore,

$$L_1 + I_1 = S_1 + R_0 + U_0 \tag{9}$$

The capital account will remain the same during the period until the "closing of the books," at which time a portion of revenue will be transferred to the regular reserve and, possibly, to undivided earnings. Also, any defaults on loans will be written off against reserves. The subscript 1 indicates the balances during the period, whereas the subscript 0 refers to the beginning, pretransaction amounts.

In the construction of the objective function, it was assumed that the ϕ and ω proportions, where $0 < \phi \leq 1$ and $0 < \omega \leq 1$, of the inherited loans and savings are retired each period. Using these assumptions, the following relations are obtained:

$$L_1 = (1 - \phi)L_0 + L \tag{10}$$

$$S_1 = (1 - \omega)S_0 + S \tag{11}$$

L and S (without the time subscripts) represent the current period issue of loans and savings accounts. From (5) and (6), these amounts are functions of the current rates at the CU and competing institutions, whereas past amounts are not.

Substituting (10) and (11) into (9) and then subtracting (8) yields (12), which shows the portfolio changes needed to finance the issuance of new loans.

$$L = \phi L_0 + (S - \omega S_0) - (I_1 - I_0) \tag{12}$$

[7] Some CUs have additional reserve accounts for contingencies and losses on investments. These can be thought of as part of R_0.

New loans are financed by a combination of repaid loans, net increases in savings deposits (new deposits less the redemption of retired savings accounts), and a reduction in the investment account or an increase in the CU's borrowings.

C. The Operating Statement

The operating statement for the CU's current accounting period must net out to zero. That is, profit (π) equals zero after expenses, transfers to reserves and undivided earnings, the payment of dividends, and possibly an interest refund. For convenience, assume that all of these payments take place at the end of the current period. Then the operating statement can be summarized as follows:

$$\begin{aligned}\pi = {} & [r_{L0}(1 - \rho) - c_{L0}](1 - \phi)L_0 \\ & + [r_L(1 - \rho) - c_L]L \\ & - [r_{S0} + c_{S0}](1 - \omega)S_0 \\ & - [r_S + c_S]S \\ & + r_{IM}I_1 - \bar{C} - \mu U_0 = 0 \qquad (13)\end{aligned}$$

The first two terms represent net revenue from the inherited and currently issued loan portfolio. The rate r_{L0} is the weighted average of past period CU loan rates, whereby the weights reflect the composition of the inherited loans. The loans repaid at the start of the period (ϕL_0) provide no current interest income. CUs must transfer a fraction ρ, where $0 \leq \rho < 1$, of their gross revenue to the regular reserve each period.[8] The average cost of managing the old and new loan accounts is represented by c_{L0} and c_L, respectively. These costs would include issuing, servicing, and providing life insurance for the loans, and also the lost revenue due to delinquency and default. Since the CU might pay an interest refund, r_{L0} and r_L are the net interest rates on loans after any refund.

The third and fourth terms are the total cost of savings deposits, including those inherited from the previous period and those newly received. The dividend rates on these deposits are r_{S0} and r_S, respectively. Those savings withdrawn at the start of the period (ωS_0) earn no dividends. The average costs of maintaining, promoting, and insuring the savings accounts are c_{S0} and c_S.

The term $r_{IM}I_1$ is the income earned on the investment account or the cost of borrowed funds, depending on the sign of I_1. It is assumed that the same market-determined rate, r_{IM}, applies for either case.[9] If $I_1 > 0$, one should really multiply this term by $(1 - \rho)$ since the transfer to the reserve account is from gross revenue, including investment income, but that is not done to keep the formulation reasonably simple. The next term, $\bar{C}$, is the fixed cost of operation and could include expenditure for nonfinancial member services such as investment counselling, estate planning, tax preparation, and social activities.

[8] This follows the regulations for federally chartered CUs. Some state-chartered CUs transfer a fraction of net income. That would require a different specification than that presented here, but the same approach can be used.

[9] Since I_1 is the amount of investment or borrowing during the period, r_{IM} can be thought of as the average money market rate for the period.

The last term is the positive or negative transfer to undivided earnings U_0. It is assumed that this transfer is some multiple μ, where $-1 \leq \mu < \infty$, of U_0. Since the transfer of funds out of undivided earnings to supplement current revenue is limited by the amount of U_0 itself, the lower bound on μ must be -1. In this formulation the determination of μ is not explicitly modelled. It can be assumed, however, that this surplus retention parameter is set optimally through some more general intertemporal maximization process and that only deviations from that factor are considered here.[10]

This operating statement establishes the closing adjustments that will determine the next period's inherited balance sheet. For instance, the required transfer to the regular reserve $[\rho r_{L0}(1 - \phi)L_0 + \rho r_L L]$ is added to R_0 and μU_0 is added to undivided earnings. The amount of charged-off loans will reduce both the loan account and regular reserve. Changes in the investment account will reflect these flows. A net increase in the capital account and dividends credited to savings accounts will raise the amount of invested funds.

The balance sheet constraint can now be combined with the operating statement to produce a single constraint for the optimization problem. Substituting (10) and (11) into (9) will yield an expression for I_1. This is then substituted into (13). The combined constraint is the following:

$$\begin{aligned}\pi = {} & [r_{L0}(1 - \rho) - c_{L0} - r_{IM}](1 - \phi)L_0 \\ & + [r_L(1 - \rho) - c_L - r_{IM}]L \\ & + [r_{IM} - c_{S0} - r_{S0}](1 - \omega)S_0 \\ & + [r_{IM} - c_S - r_S]S \\ & + r_{IM}R_0 + (r_{IM} - \mu)U_0 - \bar{C} = 0\end{aligned} \tag{14}$$

II. The CU Optimization Problem

A. *Without Regulatory Rate Ceilings*

The objective of the CU is to maximize the weighted sum of the Net Gain on Loans (NGL) and the Net Gain on Savings (NGS), whereby the weights are the behavioral preference parameters. Equation (7) above, which includes the effect of extended or multiperiod Net Gains, is the formalized objective function. The CU chooses the loan rate, r_L, and the dividend rate, r_S. The equality constraint is that the operating statement nets out to zero. This constraint includes the balance sheet identity and is specified by Equation (14).

It should be noted that this is in essence a single-period problem. The inherited balance sheet introduces past decisions and results but the optimization process is to maximize the present value of Net Gains accruing to CU members as a result of only current period rate decisions. It is assumed that all parameters, predetermined variables, exogenous market rates, and functional relationships are known with certainty.

[10] I thank the referee for this observation.

The first-order conditions to the constrained optimization problem are the following:

$$\beta \left(\frac{1+d}{d+\phi}\right)\left[(r_{LM} - r_L)\frac{\partial L}{\partial r_L} - L\right] + \lambda \frac{\partial \pi}{\partial r_L} = 0 \tag{15}$$

$$(1-\beta)\left(\frac{1+d}{d+\omega}\right)\left[(r_S - r_{SM})\frac{\partial S}{\partial r_S} + S\right] + \lambda \frac{\partial \pi}{\partial r_S} = 0 \tag{16}$$

$$\pi = 0 \tag{17}$$

where λ is the Lagrangian multiplier and

$$\frac{\partial \pi}{\partial r_L} = [r_L(1-\rho) - c_L - r_{IM}]\frac{\partial L}{\partial r_L} + (1-\rho)L \tag{18}$$

$$\frac{\partial \pi}{\partial r_S} = [r_{IM} - c_S - r_S]\frac{\partial S}{\partial r_S} - S \tag{19}$$

The second-order condition is that the bordered Hessian matrix be negative definite. That is,

$$|\Delta| = \begin{vmatrix} \Delta_{11} & 0 & \dfrac{\partial \pi}{\partial r_L} \\ 0 & \Delta_{22} & \dfrac{\partial \pi}{\partial r_S} \\ \dfrac{\partial \pi}{\partial r_L} & \dfrac{\partial \pi}{\partial r_S} & 0 \end{vmatrix} = -\left[\frac{\partial \pi}{\partial r_S}\right]^2 \Delta_{11} - \left[\frac{\partial \pi}{\partial r_L}\right]^2 \Delta_{22} > 0 \tag{20}$$

whereby,

$$\Delta_{11} = \beta \left(\frac{1+d}{d+\phi}\right)\left[(r_{LM} - r_L)\frac{\partial^2 L}{\partial r_L^2} - 2\frac{\partial L}{\partial r_L}\right] + \lambda \frac{\partial^2 \pi}{\partial r_L^2} \tag{21}$$

$$\Delta_{22} = (1-\beta)\left(\frac{1+d}{d+\omega}\right)\left[(r_S - r_{SM})\frac{\partial^2 S}{\partial r_S^2} + 2\frac{\partial S}{\partial r_S}\right] + \lambda \frac{\partial^2 \pi}{\partial r_S^2} \tag{22}$$

In the work that follows, it will be assumed that the second-order condition is satisfied. Moreover, it is assumed that the sufficient condition holds:

$$\Delta_{11} < 0 \quad \text{and} \quad \Delta_{22} < 0 \tag{23}$$

Since there is only a single equality constraint in this problem, the comparative static properties can be obtained by taking the total differential of the first-order conditions and then solving by means of Cramer's Rule. The following classification will be useful in evaluating these properties:

Complete Borrower Preference ($\beta = 1$) $\quad \dfrac{\partial \pi}{\partial r_L} > 0$ and $\dfrac{\partial \pi}{\partial r_S} = 0$

Neutrality ($\beta = 1/2$) $\quad \dfrac{\partial \pi}{\partial r_L} > 0$ and $\dfrac{\partial \pi}{\partial r_S} < 0$

$$\text{Complete Saver Preference } (\beta = 0) \qquad \frac{\partial \pi}{\partial r_L} = 0 \quad \text{and} \quad \frac{\partial \pi}{\partial r_S} < 0 \tag{24}$$

These signs are obtained by rearranging (15) and (16) and substituting in the given value for β. It is assumed that the Lagrangian multiplier, λ, is strictly positive. A sufficient condition for this classification is that $r_L^* \leq r_{LM}$ and $r_S^* \geq r_{SM}$. (* will hereafter refer to a result of the optimization process.) In this situation both NGL and NGS are nonnegative, therefore the CU is offering at least a competitive dividend rate on savings and interest rate on loans. It is for that purpose CUs were originally organized and that members are motivated to participate in the CU. Moreover, the extent to which a CU is able to confer benefit to both borrowers and savers is most interesting.

A sidelight to the classification scheme in (24) is to consider the actions of a hypothetical profit-maximizing CU. The description of profit (π) in Equation (14) would be the objective function instead of a constraint. Maximizing π with respect to r_S and r_L would then require

$$\frac{\partial \pi}{\partial r_L} = 0 \quad \text{and} \quad \frac{\partial \pi}{\partial r_S} = 0$$

Therefore complete borrower preference entails treating savers as if to maximize profit, then using that "profit" to "subsidize" the lowest possible loan rate. Likewise, saver preference implies setting a loan rate as if to maximize profit, then eliminating that profit by choosing the highest dividend rate consistent with the $\pi = 0$ constraint.

The effect that behavioral motivation has on the CU's rate decisions is found by evaluating a change in β on r_L^* and r_S^*. From the total differential of the first-order conditions the following expressions are obtained:

$$\frac{dr_L^*}{d\beta} = \frac{\left(\frac{1+d}{d+\omega}\right)\left[(r_S - r_{SM})\frac{\partial S}{\partial r_S} + S\right]\frac{\partial \pi}{\partial r_L}\frac{\partial \pi}{\partial r_S}}{|\Delta|} + \frac{\left(\frac{1+d}{d+\phi}\right)\left[(r_{LM} - r_L)\frac{\partial L}{\partial r_L} - L\right]\left[\frac{\partial \pi}{\partial r_L}\right]^2}{|\Delta|} < 0 \tag{25}$$

$$\frac{\partial r_S^*}{d\beta} = -\frac{\left(\frac{1+d}{d+\omega}\right)\left[(r_S - r_{SM})\frac{\partial S}{\partial r_S} + S\right]\left[\frac{\partial \pi}{\partial r_S}\right]^2}{|\Delta|} - \frac{\left(\frac{1+d}{d+\phi}\right)\left[(r_{LM} - r_L)\frac{\partial \pi}{\partial r_L} - L\right]\frac{\partial \pi}{\partial r_L}\frac{\partial \pi}{\partial r_S}}{|\Delta|} < 0 \tag{26}$$

As β increases, that is as the CU becomes more borrower oriented, both the loan rate and dividend rate on shares are lowered, which is a fairly obvious conclusion. The higher degree of borrower preference is reflected by the lower rate on loans and the induced higher level of loan activity. This indicates that the overall

Table I

Some Comparative Static Properties of the Model

	Neutrality $\beta = ½$	Complete Borrower Preference $\beta = 1$	Complete Saver Preference $\beta = 0$
$\frac{dr_L^*}{d\bar{C}}$	>0	>0	=0
$\frac{dr_S^*}{d\bar{C}}$	<0	=0	<0
$\frac{dr_L^*}{dc_{L0}}$	>0	>0	>0
$\frac{dr_S^*}{dc_{L0}}$	=?	=0	<0
$\frac{dr_L^*}{dc_L}$	>0	>0	>0
$\frac{dr_S^*}{dc_L}$	=?	=0	<0
$\frac{dr_L^*}{dc_{S0}}$	=?	>0	=0
$\frac{dr_S^*}{dc_{S0}}$	<0	<0	<0
$\frac{dr_L^*}{dc_S}$	=?	>0	=0
$\frac{dr_S^*}{dc_S}$	<0	<0	<0
$\frac{dr_L^*}{dR_0}$	<0	<0	=0
$\frac{dr_L^*}{dR_0}$	>0	=0	>0
$\frac{dr_L^*}{d\mu}$	>0	>0	=0
$\frac{dr_S^*}{d\mu}$	<0	=0	<0
$\frac{dr_L^*}{dU_0}$	$\gtreqless 0$ if $r_{IM} \lesseqgtr \mu$	$\gtreqless 0$ if $r_{IM} \lesseqgtr \mu$	=0
$\frac{dr_S^*}{dU_0}$	$\lesseqgtr 0$ if $r_{IM} \lesseqgtr \mu$	=0	$\lesseqgtr 0$ if $r_{IM} \lesseqgtr \mu$
$\frac{dr_L^*}{dr_{L0}}$	<0	<0	=0
$\frac{dr_S^*}{dr_{L0}}$	>0	=0	>0
$\frac{dr_L^*}{dL_0}$	$\gtreqless 0$ if $(1-\rho)r_{L0} - c_{L0} \lesseqgtr r_{IM}$	$\gtreqless 0$ if $(1-\rho)r_{L0} - c_{L0} \lesseqgtr r_{IM}$	=0
$\frac{dr_S^*}{dL_0}$	$\lesseqgtr 0$ if $(1-\rho)r_{L0} - c_{L0} \lesseqgtr r_{IM}$	=0	$\lesseqgtr 0$ if $(1-\rho)r_{L0} - c_{L0} \lesseqgtr r_{IM}$

Table I—*Continued*

	Neutrality $\beta = \frac{1}{2}$	Complete Borrower Preference $\beta = 1$	Complete Saver Preference $\beta = 0$
$\frac{dr_L^*}{dr_{S0}}$	>0	>0	=0
$\frac{dr_S^*}{dr_{S0}}$	<0	=0	<0
$\frac{dr_L^*}{dS_0}$	$\gtreqless 0$ if $r_{S0} + c_{S0} \gtreqless r_{IM}$	$\gtreqless 0$ if $r_{S0} + c_{S0} \gtreqless r_{IM}$	=0
$\frac{dr_S^*}{dS_0}$	$\gtreqless 0$ if $r_{S0} + c_{S0} \gtreqless r_{IM}$	=0	$\gtreqless 0$ if $r_{S0} + c_{S0} \gtreqless r_{IM}$

balance sheet of the CU depends on the behavioral orientation, since loan demand and savings supply are functions of the CU's rates.

Some of the remaining comparative static properties of the model are presented in summary form in Table I. The principal conclusion drawn from these calculations is that when group preference is evident, the preferred group itself tends to absorb both the positive and negative disturbances to the operating statement variables. For instance, if a CU is totally oriented to the interests of its borrowers, the optimal loan rate will shift in response to any of the parameter changes while the dividend rate on shares will usually remain the same. With saver preference the loan rate is mostly undisturbed whereas the dividend rate absorbs the fluctuation.

At first it might seem counterintuitive that the savers in a saver preference CU absorb both the gains and losses from disturbances to the operating statement. Clearly they would raise the dividend rate if net revenue goes up but why not force the borrowers to cover a shortfall in earnings? The answer goes back to how saver preference is manifested in the CU—that is, by the reduction or elimination of the weight on NGL in the objective function. Gains to borrowers are not considered so the savers seek in effect to maximize the dividend rate taking loan demand as given. The loan rate is set to maximize loan revenue, so a change in costs merely changes net revenue, hence the attainable dividend rate. The savers would not change the loan rate because that would imply less than maximal loan revenue.

Most of the results shown in Table I are fairly obvious. An increase in operating costs ($\overline{C}$, c_L, c_{L0}, c_S, c_{S0}) will reduce net revenue. Since the $\pi = 0$ constraint must be maintained, the loan rate must be increased and/or (depending on the behavioral preference) the dividend rate must be lowered since $\frac{\partial \pi}{\partial r_L} \geq 0$ and $\frac{\partial \pi}{\partial r_S} \leq 0$. Similarly, an increase in the inherited reserve account, R_0, allows for a higher level of investments or a smaller amount of borrowed funds. That raises net revenue which is then passed on to members via a higher dividend and/or lower loan interest rate. A decrease in the transfer of current income to undivided earnings, μ, has the same effect. An increase in the inherited undivided earnings, U_0, has two impacts on net revenue. First, like R_0, it allows for more investment or less debt which raises π, but also for any μ a greater amount is transferred to undivided earnings this period. The combined effect depends on the difference between r_{IM} and μ.

An increase in the weighted average of previous loan rates, r_{L0}, explicitly raises current period revenue. This calls for a lower loan rate and/or a higher dividend rate on savings deposits, again depending on behavioral preference. The effect of an increase in the inherited loan portfolio, L_0, is more complex. Gross revenue from loans increases but also, on the balance sheet, the amount of borrowed funds would have to increase or the investment account to decrease. The combined effect depends on the net (after the cost of operations and the required transfer to reserves) return on inherited loans $[r_{L0}(1-\rho)-c_{L0}]$ compared to the cost of borrowed funds or the return on investments, r_{IM}. If the difference is positive, the extra earnings can be used to reduce the loan rate and/or raise the dividend rate. This result highlights the recent financial difficulty encountered by many thrift institutions, including CUs. The upward trend to money market rates led to r_{IM} surpassing the net return on the inherited portfolio. This did not allow the rate paid on savings to rise to counter disintermediation. Moreover, the losses incurred from the negative net return on the inherited loan portfolio put pressure on the capital account to total asset ratio. In the model, this means a low or even negative μ, the transfer to undivided earnings.

An increase in r_{S0}, the weighted average of inherited savings deposit rates, will clearly reduce net revenue. The CU will have to raise the loan rate and/or reduce the share dividend rate. The effect of an increase in S_0, the inherited level of deposits, depends on the overall cost of the deposits, $r_{S0}+c_{S0}$, compared to the money market rate, r_{IM}. The higher level of S_0 raises the investment account or reduces the need for borrowed funds. So if $r_{S0}+c_{S0}>r_{IM}$, the combined effect is to reduce the surplus, causing r_L^* to go up and/or r_S^* to go down. But if $r_{S0}+c_{S0} < r_{IM}$, the surplus increases with the opposite effect on the optimal rates. This demonstrates the advantage of "locking in" the rate on multiperiod deposits when the money market rate is high or expected to increase.

Upon examination, the comparative static results make intuitive sense, which serves as a check on the various assumptions made in the course of the analysis.

B. The Effects of Regulatory Rate Constraints

CUs face a variety of regulations, as do other financial intermediaries. In terms of the theoretic framework, the most important regulations in recent years have been the ceilings on allowable loan and dividend rates. Suppose these rate ceilings are $\bar{r}_L$ for loans and $\bar{r}_S$ for savings deposits. Clearly, if the optimal unconstrained rates are such that $r_L^* \le \bar{r}_L$ and $r_S^* \le \bar{r}_S$, the constraints are not binding.

First assume that r_L^* exceeds $\bar{r}_L$. The CU would have to charge a lower interest rate on loans than otherwise desired. This is the historic justification for imposing the usury ceiling—to ensure that CUs offer low cost credit—and it also is a historic rationale for preferential regulatory tax treatment. Moreover, lowering the loan rate induces a negative surplus on the operating statement, since in general $\dfrac{\partial \pi}{\partial r_L} > 0$. This deficit would have to be covered by a lower dividend rate or perhaps by a smaller transfer to undivided earnings.

Given the binding loan rate ceiling, the CU would have to impose rationing.[11] In the extreme, if the return on money market investments exceeds the constrained return on loans, the rationing could be total. The CU would become an "investment club." Therefore, the loan rate ceiling does not ensure that NGL will rise, it could even fall if the issuance of new loans drops significantly. Even if the CU continues to offer loans, rationing could be carried out by raising credit standards and emphasizing more secure loans, for instance car loans over personal loans. In terms of the theoretic framework, this means a lower loss rate from delinquency and default, which is included in c_L, and perhaps, then, less of a need to lower the dividend from its optimum.

Now consider that the dividend rate constraint is binding. Since $\frac{\partial \pi}{\partial r_S} < 0$, a positive surplus is obtained by limiting the dividend rate to $\bar{r}_S$ rather than r_S^*. This surplus could be eliminated so that $\pi = 0$ by lowering the loan rate since $\frac{\partial \pi}{\partial r_L} > 0$. Therefore, the ceiling on dividend payouts could have the same ultimate effect as the loan rate ceiling—an increase in NGL at the expense of NGS. However, a lower induced loan rate is not the only, nor the most likely, reaction to the rate constraint.

A CU might choose to pay implicit interest on savings in addition to the explicit constrained dividend rate. The extra services that could be attractive to savers include more employees for a shorter waiting time, longer office hours, more branch offices, automated tellers, investment and tax seminars, and so on. These all imply higher operating costs. The idea is that the positive surplus is eliminated by increasing expenditures and not by lowering the loan rate.

Another way that the CU could absorb the induced surplus is to increase μ, the transfer for current revenue to undivided earnings. A higher capital account could be desirable for savers because it provides security in addition to the NCUA or state agency deposit insurance programs. If loan revenue were to temporarily drop off because of a prolonged employee strike or layoff, these retained earnings from past periods could be used to allow for the payment of the customary, or anticipated, dividend on shares. Moreover, when rate ceilings are raised or eliminated, the accumulated surplus could be used for an immediate increase in the dividend rate to a new level.

Overall, the intention of a usury ceiling on loan rates and a Regulation Q type of ceiling on allowable dividend rates would seem to impart a pro-borrower bias to the CU. This conclusion parallels that of Walker and Chandler [16]. When binding, the rate ceilings tend to increase NGL and to reduce NGS compared to the unconstrained optimal rates. Nevertheless, CUs could react to the ceilings by rationing loans and switching funds to money market investments, paying implicit dividends, and increasing profit retention.

III. Summary

The theoretic framework developed in this paper is based on a CU objective function which measures the pecuniary gains to members by a market rate

[11] See Wolken and Navratil [18] for an empirical analysis of the CU usury ceiling.

comparison. The members enjoy positive Net Gains on Loans if the CU loan rate is less than alternative rates and positive Net Gains on Savings if the CU dividend rate exceeds rates available elsewhere on comparable accounts. The optimal CU loan and dividend rates are shown to depend critically on the inherited balance sheet portfolios, operating costs, regulatory constraints, and possible borrower-saver preference. A CU that is neutral between the interests of its borrowers and savers will change both its loan and dividend rates to reflect fluctuations in operating income. If borrower (saver) preference is evident, the loan (dividend) rate will adjust to absorb the disturbances.

REFERENCES

1. E. Baltensperger. "Alternative Approaches to the Theory of the Banking Firm." *Journal of Monetary Economics* 6 (January 1980), 1–37.
2. H. Black and R. Dugger. "Credit Union Structure, Growth and Regulatory Problems." *Journal of Finance* 36 (May 1981), 529–38.
3. P. Brockschmidt. "Credit Union Growth in Perspective." *Federal Reserve Bank of Kansas City Monthly Review* (February 1977), 3–13.
4. M. Flannery. "An Economic Evaluation of Credit Unions in the United States." *Federal Reserve Bank of Boston Research Report No. 54*, 1974.
5. C. Fry, C. Harper, and S. Stansell. "An Analysis of Credit Union Costs: A New Approach to Analyzing Costs of Financial Institutions." *Journal of Bank Research* 12 (Winter 1982), 239–49.
6. R. Koot. "The Demand for Credit Union Shares." *Journal of Financial and Quantitative Analysis* (March 1976), 133–41.
7. ———. "On Economies of Scale in Credit Unions." *Journal of Finance* 33 (September 1978), 1087–1094.
8. J. Murray and R. White. "Economies of Scale and Deposit-Taking Financial Institutions in Canada." *Journal of Money, Credit, and Banking* 12 (February 1980), 58–70.
9. ———. "Economies of Scale and Economies of Scope in Multiproduct Financial Institutions: A Study of British Columbia Credit Unions." *Journal of Finance* 38 (June 1983), 887–902.
10. F. Navratil. "An Aggregate Model of the Credit Union Industry." *Journal of Finance* 36 (May 1981), 539–49.
11. D. Smith. "On Variant Objective Functions in Credit Unions: Theory and Empirical Testing." Unpublished doctoral dissertation, University of California, Berkeley, 1981.
12. ———, T. Cargill, and R. Meyer. "An Economic Theory of a Credit Union." *Journal of Finance* 36 (May 1981), 519–28.
13. R. Taylor. "The Credit Union as a Cooperative Institution." *Review of Social Economy* 29 (September 1971), 207–17.
14. ———. "The Demand for Credit Union Shares: A Cross-sectional Analysis." *Journal of Financial and Quantitative Analysis* (June 1972), 1749–56.
15. ———. "Demand for Labour by Credit Unions." *Applied Economics* 11 (September 1979), 333–40.
16. M. Walker and G. Chandler. "On the Allocation of the Net Monetary Benefit of a Credit Union Membership." *Review of Social Economy* 35 (October 1977), 159–68.
17. J. Wolken and F. Navratil. "Economies of Scale in Credit Unions: Further Evidence." *Journal of Finance* 35 (June 1980), 769–77.
18. ———. "The Economic Impact of the Federal Credit Union Usury Ceiling." *Journal of Finance* 36 (December 1981), 1157–68.