THE JOURNAL OF FINANCE • VOL. XXXIX, NO. 3 • JULY 1984

VALUATION ANOMALIES—EMPIRICAL

Presiding: Richard Roll
University of California at Los Angeles

Anomalies in Security Returns and the Specification of the Market Model

STEPHEN J. BROWN[1] and CHRISTOPHER B. BARRY[2]

ABSTRACT

We examine the hypothesis originally advanced by Roll [12] that observed anomalies in excess returns can be explained by misspecification of the market model used to estimate systematic risk. We find substantial misspecifications in the model systematically related to size and period of listing of the securities in question. There is some evidence that these misspecifications are associated with systemic biases in measured betas used to construct excess returns.

I. Introduction

MUCH EVIDENCE HAS ACCUMULATED over the past few years of a significant size anomaly in security returns, (Banz [1], Reinganum [10], Brown, Kleidon and Marsh [4] *inter al.*). Recently, Barry and Brown [2] have documented a period of listing anomaly associated with but distinct from the size anomalies observed by others. Figure 1 gives a graphical representation of these size and listing anomalies and shows that excess returns are clearly associated with size and period of listing characteristics of the securities studied.[3]

Roll [12] conjectured that some of these apparent anomalies could be attributed to a misspecification in the market model commonly used to estimate systematic risk. He proposed that since small firms were likely to be less frequently traded than large firms, a nontrading adjustment suggested by Dimson [7] would suffice to explain the apparent anomalies. While Reinganum [11] sought to show that such an adjustment would not suffice to explain the observed anomalies, he left open the possibility that other misspecifications might account for the anomaly. In this paper we examine a possible misspecification due to nonstationarity of the market model during the period used to estimate beta.

We use a recursive residual procedure suggested by Brown, Durbin, and Evans

[1] Bell Communications Research Inc., New Jersey, 07974, currently visiting Yale University, New Haven, Connecticut, 06520.

[2] E. L. Cox School of Business, Southern Methodist University, Dallas, Texas 75275.

[3] See Table 2 and related text in Barry and Brown [2] for details of the determination of these excess return values.

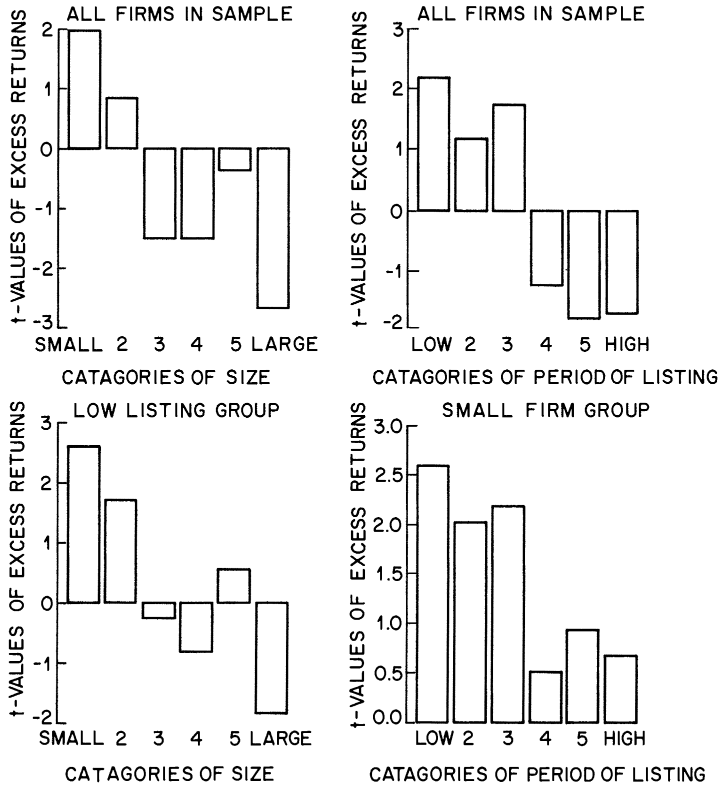


Figure 1. Excess Returns by Size and Period of Listing[a]

[5] adapted to the particular context of market model estimation, and we examine the association of potential misspecification with size and period of listing. We further examine the source of apparent misspecifications by looking at the time pattern of these residuals. By studying the correlation of one step ahead residuals with the market index, we obtain a measure of the extent to which the specification errors are associated with a bias in measured beta.

In Section II we describe the data and empirical procedures used and present the results. Section III contains some concluding remarks.

II. The Empirical Evidence

In this section we examine one step ahead residuals from the market model. These residuals are computed on a monthly basis for NYSE securities from January 1931 to December 1980. By classifying these residuals according to market capitalization and period of listing, we are in a position to study the relationship between observed anomalies in security returns and the specification of the market model.

II.A Estimation

The data used in our study comprised the set of all securities traded on the New York Stock Exchange from December 1926 to December 1980, which were listed on that exchange for a period of at least 61 months and which had at least 21 months of data on the CRSP monthly returns file available to estimate the market model. On leaving the exchange, the security was assumed to have a return of minus one, or a return based on the price at which the security subsequently traded on another exchange.[4] We used the CRSP value weighted index as an index of market returns.[5]

We employ one step ahead residuals to examine the specification of the market model. In-sample residuals have mean zero by construction. Residuals estimated out of sample will not in general have mean zero. In fact, the extent of departure from mean zero can be used as a test of the specification of the model. Brown, Durbin and Evans [5] suggest that recursively reestimating one step ahead residuals, standardizing each by the standard deviation of prediction errors, provides a very useful diagnostic for model specification and stationarity. If these residuals significantly differ from zero, they indicate model misspecification. Since few writers in the empirical finance literature appear to believe that the market model is stationary over extended periods of time, we estimate the recursive residuals on the basis of a moving window of the past sixty months of data for each security in our sample.

For each month t, one step ahead residuals are computed in the following way. A market model specific to month t is estimated on the basis of available data[6] for the prior sixty months. This is done for each of the m_t securities in the sample as of month t. The market model estimates $\hat{\alpha}_{it}$ and $\hat{\beta}_{it}$ are obtained from the regression equation

$$R_{il} = \alpha_{it} + \beta_{it}R_{Ml} + \epsilon_{il} \begin{cases} l = t - 60, \ldots, t - 1 \\ i = 1, \ldots, m_t, \end{cases}$$

where R_{il} and R_{Ml} represent returns on the security and market respectively over the prior estimation period. The one step ahead residual for period t is then defined as

$$\hat{\epsilon}_{it} = R_{it} - \hat{\alpha}_{it} - \hat{\beta}_{it}R_{Mt}.$$

A recursive residual S_{it} is computed as

$$S_{it} = \hat{\epsilon}_{it}/SD_{it}$$

[4] This assumption maintains comparability with studies of Banz [1] Reinganum [11, 12] and Barry and Brown [2]. It might appear to underestimate returns in the month of delisting, since securities may leave the exchange by merger and acquisition as well as by bankruptcy. Excluding securities in the month they were delisted increased returns due to size and listing but, otherwise, the relative impact of these classifications was little affected by this change.

[5] The value weight index was used to maintain comparability of our results with those of Banz [1] and Reinganum [11, 12].

[6] If there were no price data for a particular month returns for that month and the subsequent month were excluded.

where SD_{it} is the standard deviation of prediction error[7]

$$SD_{it} = \hat{\sigma}_{it}\left[1 + \frac{1}{60} + \frac{(R_{Mt} - \bar{R}_{Mt})^2}{\sum_{l=t-61}^{t-1} (R_{Ml} - \bar{R}_{Mt})^2}\right]^{1/2},$$

where $\hat{\sigma}_{it}$ is the standard deviation of residuals over the estimation period, and $\bar{R}_{Mt}$ is the mean return on the market over the estimation period. This procedure is repeated for each month in the interval January 1931 through December 1980, with the market model parameters being reestimated each month on the basis of data for the prior sixty months. Thus time series of one step ahead residuals $\hat{\epsilon}_{it}$ and recursive residuals S_{it} are obtained. These recursive residuals have the property that they should have mean zero and variance one if the market model is well specified over the sixty one month estimation and testing periods.

Since anomalies have previously been observed for size and period of listing categories, we applied the recursive residuals procedure to securities classified on the basis of size and period of listing. The procedure can thus be used to estimate the issue of whether market model estimates of systematic risk for securities within the same size and/or listing categories are systematically in error.

One step ahead residuals and recursive residuals were computed for categories comprising different size and different periods of listing[8] by aggregating the residuals computed for individual securities. For each month, securities in the sample were classified independently by size (total market value of equity outstanding) and by number of months of listing. Both size and period of listing were computed as of the previous month. Size was organized into six quantiles, the smallest sixth of all securities going into group 1, the next sixth going into group 2 and so forth. Period of listing was independently organized into six groups, where listing group 1 corresponded to the securities listed for the least amount of time and the sixth listing group corresponded to those securities listed for the longest period. The one step ahead residual for category I is estimated as

$$\hat{\epsilon}_{It} = \textstyle\sum_{i \in I} \epsilon_{it}/N_{It}$$

where N_{It} is the number of securities in category I in period t, and the recursive residual for category I is[9]

$$S_{It} = (\textstyle\sum_{i \in I} S_{it}/N_{It})/(1/\sqrt{N_{It}}).$$

[7] This formula applies where there is complete data available for the estimation period. With incomplete data, the number 60 is replaced by the number of available months of data, and the summation and $\bar{R}_{Mt}$ are defined over those months.

[8] The months of listing were determined for the month of first listing on the CRSP tape or, in the case of 459 securities in our sample trading in 1926, the month in which the security was first accepted by the Committee on the List of the New York Stock Exchange. That date was determined from the first listing statement on record in the archives of the NYSE. This is not entirely satisfactory, since the listing statements were preserved only back as far as June 1884. In 9 of 459 cases, the preferred stock was listed prior to capital or common stock issues. Since it was not obvious that there was not a prior capital stock issue listed on the exchange—the date of the prefrred issue was taken as the "first listing".

[9] This measure assumes the S_{it} measures are independent in the cross section of firms comprising category I. We considered dependence adjustment, further standardizing the S_{It} by the standard deviation estimated on the basis of the prior sixty months of I recursive residuals, with little effect on the reported results.

II.B Results

Table 1 reports the average of recursive residuals estimated for January 1931 to December 1980, standardized by the reciprocal of the square root of 600, the number of months in the sample. These numbers can be interpreted as t-values. They indicate a strong misspecification of the market model, a misspecification

Table 1

Recursive Residuals by size and period of listing for all months January 1931–December 1980, and for all months in that interval excluding the month of January[a]

a. Overall Period (Jan. 1931–Dec. 1980)

		Period of Listing[b]						Size	Average
		1	2	3	4	5	6	Marginal[c]	Beta[e]
Size[c]	1	12.03	8.80	7.69	4.31	3.84	4.26	16.94	1.36
	2	4.06	2.56	1.57	0.25	−0.44	0.97	4.12	1.29
	3	−2.42	−2.12	0.95	−4.51	−0.77	0.63	−3.42	1.23
	4	−6.14	−7.21	−3.87	−2.55	−3.04	−3.01	−10.56	1.20
	5	−5.35	−7.61	−4.63	−2.22	−1.74	−5.50	−11.00	1.12
	6	−7.42	−7.67	−8.71	−7.89	−9.65	−8.93	−20.40	1.02
Period of listing marginal[d]		−0.87	−4.97	−2.04	−4.92	−5.38	−6.22		
Average beta[e]		1.22	1.21	1.21	1.19	1.20	1.19		

b. Excluding January (Feb. 1931–Dec. 1980)

		Period of Listing[b]						Size
		1	2	3	4	5	6	Marginal[c]
Size[c]	1	4.12	1.53	−0.76	−3.61	−3.07	−2.60	−1.81
	2	−1.67	−2.53	−4.04	−5.02	−4.79	−3.79	−8.46
	3	−5.56	−5.19	−3.39	−7.99	−4.70	−3.74	−12.54
	4	−7.42	−9.42	−5.99	−5.57	−5.71	−6.96	−16.79
	5	−5.15	−7.64	−5.25	−3.31	−4.11	−8.47	−14.02
	6	−6.56	−6.84	−7.74	−6.40	−8.78	−10.72	−19.56
Period of listing marginal[d]		−8.50	−12.37	−10.71	−12.97	−12.96	−15.70	

[a] The numbers in the table represent the mean of recursive residuals (defined in the text) standardized by the square root of the reciprocal of the number of observations (600). These numbers are to be interpreted as t-values. A significant t-value is to be interpreted as a misspecification of the market model (estimated on the basis of a moving 60 month interval) for that size and listing category.

[b] Period of Listing is defined as the number of prior months the security in question was listed on the CRSP tapes. In the case of 459 securities in our sample listed at any time in 1926, listing is defined relative to the month the security was accepted by the Committee on the List of the NYSE as recorded in the NYSE Archives. The average period of listing in categories 1 through 6 were 89.5, 146.4, 213.6, 291.2, 387.3 and 592.1 months respectively.

[c] Size was defined as the value of equity outstanding of the security in question as of the end of the previous month. The average size (in $\$M$) for categories 1 through 6 were 9.8, 24.9, 49.9, 99.0, 221.5 and 1,197.4 respectively.

[d] The size and period of listing marginals give average returns for portfolios constructed on the basis of size alone, and of listing alone respectively. Note that since the number of securities differ from cell to cell and from one period to the next, the marginals are not simple averages of the corresponding elements in the rows and columns of the table.

[e] Average value of OLS beta estimated on the basis of prior sixty months of data using a value weighted index.

that is systematic by size and period of listing categories. The model would appear to be most seriously misspecified for the smallest firms in the sample. The small firms for which the model is most seriously misspecified appear to be those firms listed for the least amount of time.

These results provide strong confirmation of Roll's conjecture [12] that observed anomalies in excess returns are strongly associated with misspecifications of the market model to estimate systematic risk. Given that the model is misspecified in this way, it is difficult to assign unambiguous economic meaning to the excess returns derived from the market model.[10]

It is interesting to probe further to uncover the source of the observed misspecifications. Roll [12] argued that a nontrading adjustment would correct the apparent anomalies in excess returns. This point is discussed by Reinganum [11] who finds that such an adjustment does not suffice to explain the anomaly. Another possibility is that the model is not stationary over the estimation period.

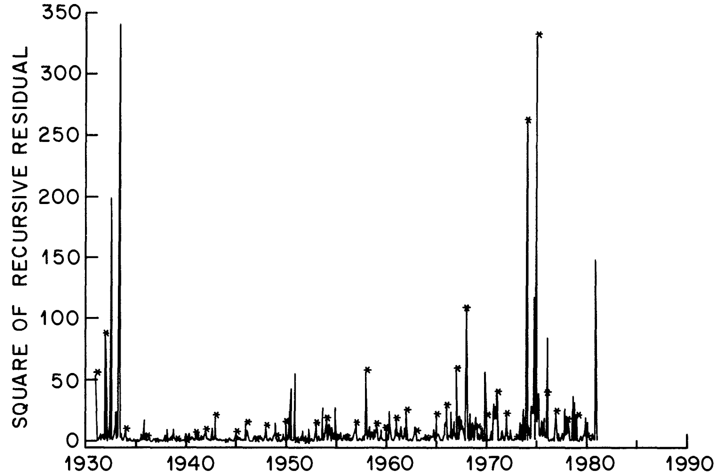
One source of nonstationarity is the January effect described by Keim [8] among others. This effect can also be examined by means of the recursive residuals. In Figure 2 we show the sqare of recursive residuals[11] as a function of time for the smallest firms listed the least amount of time. These should be identically distributed $F(1, 58)$ variates under the hypothesis of a stationary market model. The numbers show a distinct seasonal pattern consistent with a January-related nonstationarity in the model. However, as can be seen from the second panel of Table 1, the exclusion of January data did not remove the apparent misspecification of the model. Nevertheless, existence of such a systematic effect in the data would suggest that estimation of the market model be adjusted to account for it using for example, some form of January dummy variable.

To what extent is the misspecification of the model associated with a nonstationarity in beta over the estimation period? Such nonstationarity can arise from a variety of sources. If the firms in the sample belong to industries that undergo significant changes in structure or conduct or if they change their debt structure, we should expect beta measured relative to the equity of such firms to change over the estimation period. If there are significant changes in the information set relating to the firm's future prospects we should also expect beta to change. Such events should not be expected to be independent of the size or period of listing of the firms in question.[12] Finally there is the problem of nontrading, addressed by Roll [12], and this problem which might be expected to be particu-

[10] Brown and Weinstein [6] suggest caution interpreting tests based on large numbers of observations. The small size/low listing group in Table 1 corresponds to 14,964 company months of data. A t-value of 2 would yield an odds ratio (cf. Klein and Brown [9]) of only .06 against a hypothesis of 'no effect', but one of 4 (or greater) yields 24.27. Thus these results are not an artefact of large samples.

[11] The use of the squared residual as a measure of model specification was suggested by Brown, Durbin, and Evans [5].

[12] Note that the categories of size and period of listing correspond to measures of *relative* size and listing which by their definition cannot be an increasing function of time for all firms in the sample. Barry and Brown [3] argue that relative size and listing can be considered as proxies for the relative information available about securities in the capital markets.



ASTERISKS (*) REPRESENT THE SQUARE OF THE RECURSIVE RESIDUAL FOR JANUARY

Figure 2

larly severe in the early years of our sample of monthly returns due to the relatively low volume of trades during those years.

One measure of the potential bias in measured beta is the degree of correlation between the one step ahead residuals and the market index. Suppose we estimate beta for category I as b_{It} using data for the prior sixty months. Then if this is a downward (upward) biassed measure of beta specific to the current month t, β_{It} we should expect a positive (negative) correlation between the market index R_{Mt} and the one step ahead residual

$$\hat{\epsilon}_{It} = (\alpha_{It} - a_{It}) + (\beta_{It} - b_{It})R_{Mt} + \xi_{It}.$$

Table 2 presents values of these correlations for the period January 1931 to December 1980. There would appear to be a substantial bias in beta of a magnitude and direction predicted by Roll [12] by size categories. Among the smallest firms in the sample, the positive bias is largest for those securities listed the least amount of time. The bias *within* the small firm class is not simply a further manifestation of the small firm bias since the average size of the smallest firms actually declines from $10.5 million in listing group 1 to $9.5 million in listing group 6.

There would appear to be no systematic difference in the downward bias in measured beta across period of listing groups other than that associated with size. However, upon closer examination, such a conclusion is not supported by Table 2. The size bias suggested by Roll [12] is most strongly indicated for those securities listed the least amount of time. However there is something of a reversal in the relative magnitude of the bias for the longest period of listing category. Further data analysis reveals that this reversal effect is limited to the pre second World War period of the data. In the prewar period, securities that had been listed for the longest period of time in the largest relative size categories tended to be the those issued by the railroads of our sample. The railroads were

Table 2

Correlation of One Step Ahead Residuals with market index for all months January 1931–December 1980[a]

		Period of Listing[b] 1	2	3	4	5	6	Size Marginal
Size	1	0.18	0.11	0.13	0.11	0.14	0.07	0.15
	2	0.04	0.14	−0.02	0.06	0.05	0.07	0.08
	3	0.15	0.14	0.04	0.12	0.15	0.14	0.16
	4	−0.03	0.08	0.03	0.07	0.14	0.14	0.12
	5	0.00	0.08	0.04	−0.09	0.14	0.27	0.12
	6	−0.11	−0.16	−0.06	−0.04	−0.08	−0.15	−0.08
Period of listing marginal[d]		0.11	0.14	0.06	0.12	0.14	0.16	

[a] Correlations in excess of .08 in absolute value are significant at the 5% level.

in financial distress, and they were precisely the firms for which we would expect to see a large increase in systematic risk over any particular sixty month estimation interval in that period.

These correlation results can only be considered suggestive. The pattern of correlations, like the size premium itself[13] is not particularly stable across subintervals of the data. In fact most correlations are negative for the period post 1945. There is a more serious problem. Since both true beta and measured beta have a weighted average of unity the bias, if any, has to have a weighted average of zero. The period of listing marginal results in Table 2 clearly violate this condition and call to question the interpretation of the correlations. For the constraint to be met we would require both the magnitude of the bias and relative value weights not to change through time. The data would appear to refute both conditions.

III. Conclusion

We have found evidence that observed anomalies in excess returns are associated with misspecifications in the market model used to estimate systematic risk. This evidence suggests that caution must be used in interpreting excess returns that result from betas estimated by such a model. We further study the source of such misspecification. The existence of the January anomaly suggests an adjustment to the market model to account for this source of nonstationarity. We find that the misspecification we observe is associated with a systematic bias in measured beta. However, like the size premium itself this bias is not particularly stable across subintervals of the data and is yet to be fully explained.

[13] C.f. Brown Kleidon and Marsh [4].

REFERENCES

1. Rolf W. Banz, "The relationship between return and market value of common stocks," *Journal of Financial Economics* 9, (1981), 3–18.
2. Christopher B. Barry, and Stephen J. Brown, "Differential information and the small firm effect" forthcoming, *Journal of Financial Economics* (1984).

3. Christopher B. Barry, and Stephen J. Brown, "Differential information and security market equilibrium" Working Paper 83-904 Southern Methodist University (1983).
4. Phillip Brown, Allan W. Kleidon, Terry A. Marsh, "New evidence on the nature of size related anomalies in stock prices" *Journal of Financial Economics* 12, (1983), 33–56.
5. R. L. Brown, J. Durbin and J. M. Evans, "Techniques for testing the constancy of regression relationships over time," *Journal of the Royal Statistical Society, Series B* 37, (1975), 149–192.
6. Stephen J. Brown and Mark I. Weinstein, "A new approach to testing asset pricing models: The bilinear paradigm," *Journal of Finance*, 38, (1983), 711–743.
7. Elroy Dimson, "Risk measurement when shares are subject to infrequent trading," *Journal of Financial Economics*, 7, (1979), 197–226.
8. Donald B. Keim, "Size related anomalies and stock market seasonality: Further empirical evidence," *Journal of Financial Economics*, 12 (1983), 13–32.
9. Roger W. Klein and Stephen J. Brown, "Model selection under minimal prior information," *Econometrica*, 52, (1984), forthcoming.
10. Marc R. Reinganum, "Misspecification of capital asset pricing: Empirical anomalies based on earnings' yields and market values," *Journal of Financial Economics*, 9, (1981), 19–46.
11. Marc R. Reinganum, "A direct test of Roll's conjecture on the firm size effect," *Journal of Finance* 37, (1982), 27–35.
12. Richard Roll, "A possible explanation of the small firm effect," *Journal of Finance* 36, (1981), 879–887.
13. Richard Roll, "Vas ist das? The turn of the year effect and the return premia of small firms," *Journal of Portfolio Management* 9, (1983), 18–28.

DISCUSSION

KENNETH R. FRENCH*: This paper presents more evidence about the small firm effect and the period of listing effect, which Barry and Brown (1983) document in their recent paper "Differential Information and the Small Firm Effect." Using a simple market model, the authors report standardized one-step-ahead forecast errors for firms grouped by both size and period of listing. Unfortunately, they provide little guidance in interpreting these errors.

It is useful to ask what one expects to observe. First, under the null hypothesis, each firm's returns are generated by a market model with absolutely constant parameters. This seems very unlikely. Almost any change in the projects undertaken by a firm will affect its covariance with the market. Moreover, Barry and Brown are almost certain to detect this misspecification. They compute monthly forecast errors between January 1931 and December 1980 for an average of 885 firms per month. Also, they assume that all of these firms in all of these months are independent. In other words, their results are based on 531,000 observations that are assumed to be independent. The average cell in their six by six matrix in Table 1 has over 15,000 observations. With this many observations it should be easy to detect small misspecifications in the market model.

Can we predict how these misspecifications should look when the firms are grouped by size and period of listing? The effect of differential information can be interpreted as mismeasurement in the market model estimate of β; the estimate for low information stocks is too low and, on average, the estimate for high information stocks is too high. If this mismeasurement and the expected return on the market are both constant, the expected value of the one-step-ahead

* Graduate School of Business, University of Chicago.