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Source: *The Journal of Finance*, Vol. 39, No. 3, Papers and Proceedings, Forty-Second Annual Meeting, American Finance Association, San Francisco, CA, December 28-30, 1983 (Jul., 1984), pp. 715-724
Published by: Blackwell Publishing for the American Finance Association
Stable URL: http://www.jstor.org/stable/2327931
Accessed: 27/11/2009 07:44

THE JOURNAL OF FINANCE • VOL. XXXIX, NO. 3 • JULY 1984

ISSUES IN INTERNATIONAL FINANCE

Presiding: DENNIS E. LOGUE, Dartmouth College

Purchasing Power Parity as a Trading Strategy

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Introduction

How much is purchasing power parity worth? In this note, I consider an individual with constant relative risk aversion who is engaged in currency speculation. The value of the PPP relationship is estimated by calculating the decline in the utility of the speculative strategy when the PPP relationship is withdrawn from the information set. This approach provides a different perspective for evaluating econometric tests of the relationship.

Since there are many expositions of the PPP doctrine, I begin with a definition of the version with which this paper will be concerned. The PPP model is defined by the following assertions.

A. The ratio of the consumer price index in one country to the consumer price index in another is proportional to the equilibrium exchange rate between the currencies of the two countries.

B. If the actual exchange rate deviates from the equilibrium rate, there will be a predictable tendency for the actual exchange rate to adjust towards the equilibrium rate.

These assertions are restated in equations (1) and (2).

$$z = k + p \tag{1}$$

$$Ds = a(z - s) + u \tag{2}$$

In these equations, p represents the log of relative prices (*/\$), s represents the exchange rate (*/\$), z represents the equilibrium exchange rate, k and a are parameters, and u is a random error.

While this model represents neither the absolute or relative versions of PPP, it does allow for absolute PPP as an equilibrium condition and it specifies that some of the adjustment to a divergence from PPP will be through the exchange rate, rather than through relative prices. For this reason, the model is explicitly a model for exchange rate forecasting, rather than a model of relative inflation

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This research is a part of the NBER Program in International Studies. The author is grateful to Jacob Frenkel, David Hsieh, and Michael Mussa for comments and advice. The views expressed are solely the responsibility of the author and are not necessarily shared by the sponsoring organizations.

rates. In the Dornbusch (1976) model, both the exchange rate and relative prices adjust to eliminate deviations from PPP.

The standard approach to testing the PPP model described above is to estimate equation (2) and test the hypothesis that the parameter 'a' is significantly different from zero. In addition, the model would be supported if the estimated equation accounted for a reasonable fraction of the variation in the exchange rate. I shall demonstrate below that this type of test does not give a very favourable impression of the PPP model. It is, however, unlikely that this type of test would give a favourable impression of other models of excess returns. Consider the analogy of testing for the small firm effect by regressing the rate of return on an individual firm's equity on its size. One problem with the approach is that the signal to noise ratio is very low at the individual asset level. A second probem is that standard significance levels may be too conservative for evaluating investment opportunities. A third problem is that a rational investor will select a portfolio of positions that will substantially reduce the apparent risk of the activity.

A. *The Speculative Strategy*

The trader selects positions in the three month forward exchange market in order to maximize the expected value of a utility function. The utility function is assumed to be of the following simple form.

$$U(E(R),\ V(R)) = E(R) - (\tfrac{1}{2}g)V(R) \tag{3}$$

In equation (3), $E(R)$ is the expected profit on the portfolio of positions, and $V(R)$ is the expected variance of profit. Maximizing (3) is equivalent to maximizing the expected value of a negative exponential utility function defined on the actual profits, R, from the portfolio. The utility function is expressed in terms of profits rather than the rate of return because the positions taken do not require the investment of capital. The positions may require the placement of a margin or certificate of deposit to ensure that obligations are met, but these instruments still bear interest which accrues to the investor rather than the institution holding the security deposit. The parameter 'g' defines the trader's degree of risk aversity.

The expected profit and the variance of profit are defined in equations (4) and (5).

$$E(R) = q'r \tag{4}$$

$$V(R) = q'Vq \tag{5}$$

q is an $N \times 1$ column vector whose typical element is the dollar value of the position taken in the n'th currency. Both long and short positions are allowed. r is an $N \times 1$ column vector whose typical element is the expected bias in the forward rate as a predictor of the future spot rate, expressed as a rate of return. Specifically,

$$r = E(s(+1)) - f, \tag{6}$$

where $E(s(+1))$ is the forecast of the log of the future spot rate, and f is the log of the current forward rate. V is the estimate of the covariance matrix of the forecast errors.

The trader's problem is to choose the elements of q to maximize the expected utility defined in equation (3). The solution for q is given by:

$$q = VIrg, \tag{7}$$

where VI represents the inverse of the covariance matrix, V. Equation (7) demonstrates that the risk aversion parameter, g, determines the scale of the position. The role of this parameter may be further explored by substituting the definition of q into the expected profit and variance of profit terms. We have:

$$E(R) = r'q = r'VIrg \tag{8}$$

and

$$V(R) = q'Vq = r'VIrg * g. \tag{9}$$

Substituting (8) into (9), and rearranging yields:

$$E(R) = t * tg, \tag{10}$$

where t is the ratio of expected profit to the standard deviation of profit. Equation (10) states that when this ratio is high, the strategy calls for a more leveraged position yielding a higher level of expected profit. Given the risk aversion parameter, the expected profit increases with the square of the ratio. The speculative strategy consequently includes both a portfolio selection rule and a timing rule. The timing rule is of particular importance in the assessment of the economic value of relationship like PPP, since it implies that the speculative positions will be large when the deviations from PPP are large.

In the following simulation, the risk aversion parameter, g, has been set equal to \$100. Equation (7) demonstrates that this implies that the trader will attempt to make \$100 when the expected profit is one standard deviation above zero. The choice is not consequential, since the model is homogenous of degree one in this parameter.

B. The Econometrics

In order to test the forecasting ability of the PPP model, it is useful to have some alternative models. Among the set of simple models, two alternatives appear to have particular appeal. The first is the foward parity model, which asserts that the forward exchange rate is a sufficient statistic for the future spot rate. The forward parity condition is based upon the joint assumptions of risk neutrality and market efficiency. The second is the random walk model, which asserts that the current spot rate is a sufficient statistic for the future spot rate. The random walk model does not have any particular theoretical foundation, but a number of empirical studies, including Bilson (1981) and Meese and Rogoff (1982), have found that the random walk model is superior to the forward parity model for exchange rate forecasting.

The two alternatives are combined with the PPP model in equation (11).

$$s(+1) - s = b1(k + p - s) + b2(f - s) + u \tag{11}$$

If the random walk model is correct, both regression coefficients, $b1$ and $b2$, should not be significantly different from zero. If the forward parity model is correct, $b1$ should not be significantly different from zero and $b2$ should not be significantly different from unity. Finally, the PPP model predicts that $b1$ would be between zero and unity, and that $b2$ would not be significantly different from zero. In addition, the parameter k has to be estimated.

In estimating equations like (11), attention must be paid to the fact that the predictable part of the change in the exchange rate is extremely small relative to the variance of the residuals. The low r-squared of equations like (11) leads, in turn, to imprecise estimates of the regression coefficients. For the speculative strategy, the low r-squared is not of particular concern since forecast errors may result in either profits or losses and will, in any event, tend to cancel out over time. However, it is very important to have precise estimates of the model parameters since the precision of the parameter estimates influences the variance of the *expected* returns.

Two techniques are employed to increase the precision of the parameter estimates. First, Zellner's seemingly unrelated regression technique is used to take account of the correlation in the forecast errors. (Zellner (1962)) Second, the regression coefficients, $b1$ and $b2$, are constrained to be the same for each country in the sample. Of course the real exchange rates, the k's, must be estimated individually for each currency. The models are estimated for five currencies—the British pound, the French franc, the Deutsche mark, the Dutch guilder, and the Swiss franc—using quarterly data over the period from 1973:*I* to 1982:*IV*. The data was taken from the IFS tape distributed by the International Monetary Fund.

The SUR estimates are presented below.

$$s(+1) - s = \underset{(.0279)}{.0664}(k + p - s) - \underset{(.3186)}{.5287}(f - s) + u \qquad (12)$$

(Standard errors in parentheses)

BP: s.e. = 0.50 R-squared = .016 $b1 * k = -.04113$ (.025)
FF: s.e. = 0.57 R-squared = .0428 $b1 * k = .1109$ (.043)
DM: s.e. = .064 R-squared = .0174 $b1 * k = .0509$ (.027)
DG: s.e. = .060 R-squared = .0493 $b1 * k = .0556$ (.028)
SF: s.e. = .072 R-squared = .0745 $b1 * k = .0416$ (.029)

These results are certainly not too promising. While the first coefficient is statistically significant at the 5 per cent level, the t-statistic is certainly not large enough for one to be confident of the model in a post-sample experiment. The results for the forward parity model are even more disappointing since, in this case, the coefficient is not of the correct sign. This result reflects the peculiar relationship between interest rate differentials and exchange rates during the past few years. While the forward parity approach suggests that high interest rates should be associated with depreciating currencies, it has often been the case that high interest rates have been associated with appreciating currencies. The poor econometric results are summarized in the R-squared statistics. The highest value is .07 for the Swiss franc, and the values for two major currencies, the Deutsche mark and the British pound, are less than .02.

In the next section of the paper, these econometric results will be employed in an in-sample simulation of the trading strategy described in the previous section. There are, of course, many problems with in-sample tests of trading strategies and there are a variety of econometric procedures which ensure that the in-sample estimates of the parameters and the covariance matrix are not employed in estimating the optimal vector of positions. It is not possible to explore these issues in this note, but Bilson and Hsieh (1983) do provide post-sample portfolios for a more restricted econometric model. For present purposes, the trading simulation is presented as an alternative way of describing the results reported in the previous regression.

C. The Simulated Trading Strategy

The procedure for selecting the portfolio is straightforward. From the econometrics, one step ahead forecasts are generated for each exchange rate and the covariance matrix is estimated. Given the target profit of \$100, these statistics represent all of the information required to select the positions in the five currencies. In calculating the expected and actual returns, it is necessary to take account of both the forecasted change in the exchange rate and the forward premium or discount. Ex post, the return on a position is equal to $s(+1) - f$, which can be written as $(s(+1) - s) - (f - s)$. This consideration is important in assessing the importance of the forward premium or discount on the strategy. Although the econometric analysis demonstrated that the forward rate is not a sufficient and unbiased forecast of the future spot rate, the forward premium is an important determinant of the rate of return on the currency.

The positions taken by the portfolio model are described in Table 1. Because of the practice of quoting exchange rates in terms of the number of local currency units required to purchase one dollar, a negative position represents a long position in the foreign currency. A long position may be considered as equivalent to borrowing in U.S. dollars and lending in the foreign currency. All of these statistics are expressed in dollars and are scaled by the target profit variable.

To illustrate the use of the PPP relationship in the trading strategy, consider the case of the British Pound in 1979 to 1981. The oil induced appreciation of the pound in 1978 pushed the exchange rate above the PPP level and the portfolio model begins to borrow sterling in 1979. This was unfortunate because the pound continued to appreciate against the dollar from around \$2 to \$2.5 and losses were realized in every quarter but one from 1978:*IV* to 1980:*III*. Over the eight quarters, the average sterling borrowing was \$520 and the average quarterly loss on the borrowing was \$12.49, or 2.37 per cent per quarter. In other words, by borrowing in pounds rather than dollars, the cost of borrowing was increased by approximately 10 per cent per year for the two years before 1980:*III*. At this point, with sterling at \$2.5, the model extends its borrowing to \$2054 in 1980:*IV* and realizes a profit of \$149 over the next quarter. This sum more than covers the losses over the previous eight quarters, and it is followed by additional profits of \$262 and \$185 in the next two quarters.

Table 1

Positions Taken by the Isolated Speculative Strategy
Purchases of US$ with the local currency

Date	BP	FF	DM	DG	SF	US$
73.01	−371	21	2870	−604	−1841	−75
73.02	137	1356	4834	−6611	316	−32
73.03	−488	1199	3054	−3300	−830	365
73.04	−490	1874	6303	−7542	−1078	933
74.01	−1277	523	1666	−556	−1330	974
74.02	190	−576	3781	−2545	−1163	313
74.03	−160	179	64	779	−1043	181
74.04	−1326	1101	311	194	−820	540
75.01	−530	1311	1672	−1890	−771	208
75.02	−981	1839	−3596	3576	−690	−148
75.03	−249	1650	−1297	917	−759	−262
75.04	−451	1830	963	1989	−545	192
76.01	−566	506	−1835	1941	−170	124
76.02	−1108	1149	6040	−7610	155	1374
76.03	−1993	1254	16109	−17725	−143	2498
76.04	−1893	−453	−119	1212	20	1233
77.01	−278	−233	279	355	−514	391
77.02	−339	−274	−3223	4597	−699	−62
77.03	622	99	−539	867	−566	−483
77.04	1487	−1010	4857	−4403	−362	−569
78.01	509	871	−575	−42	84	−847
78.02	−410	497	−731	865	136	−357
78.03	−37	157	7373	−8376	760	123
78.04	438	2184	6251	−8338	317	−852
79.01	566	1734	−1118	−392	429	−1219
79.02	131	1023	94	−1356	741	−633
79.03	835	1017	1347	−1907	−244	−1048
79.04	529	1677	3082	−4472	45	−861
80.01	731	1596	−1245	503	−79	−1506
80.02	33	1500	−910	−288	−31	−304
80.03	948	1631	−2073	688	121	−1315
80.04	2054	2127	−3910	2504	−468	−2307
81.01	1761	1947	−7714	5580	189	−1763
81.02	2124	−153	−3297	2509	96	−1279
81.03	1657	153	−699	236	−474	−873
81.04	997	831	−2706	1337	197	−656
82.01	1276	−919	−4696	4718	507	−886
82.02	1594	−966	−2438	2476	641	−1307
82.03	1278	−1432	−2363	2328	454	−265

Note: Because of the convention of quoting exchange rates in units of foreign currency per dollar, a positive entry in this table represents a long dollar position or a short position in the local currency. The US$ column is the negative of the sum of the other columns. A positive entry in this column implies dollar borrowing and foreign currency lending.

For an example of an interest rate play, consider the great guilder gambit in 1973:*III*. At this time, fears of a guilder devaluation against the mark increased short term Dutch interest rates to around 20 per cent per annum. Since the DG/DM rate was not out of line with PPP, the model borrowed $16,109 worth of Deutsche marks and placed $17,725 in a three month Dutch guilder deposit. The guilder gambit also illustrates the timing aspect of the portfolio strategy. In this period, the expected return was 2.58 standard deviations from zero and consequently the expected profit was set at $665 (=2.58 * 2.58 * 100). The high '*t*' ratio was a combination of a large interest rate differential, a small deviation from PPP, and the strong correlation (97 per cent) between the Deutsche mark and the Dutch guilder. The story does have a happy ending. The guilder remained wedded to the mark, and the portfolio emerged from the flutter with an additional $800 in realized profits.

In the final column of Table 1, the net dollar positions are reported. A negative entry in this column indicates that the program recommends borrowing foreign currencies in order to invest in dollars. The net dollar positions are typically small in comparison to the spreads across the European currencies, but the program did begin to have substantial net investment in dollars in 1980 in response to the preceding devaluation of the dollar and the higher level of U.S. interest rates relative to European rates.

The performance of the speculative portfolio is summarized in Table 2. The 'expected safety' statistic reported in the second column is the level of expected profit divided by the expected standard deviation of profit. As was mentioned in the previous paragraph, the expected profit in any period is proportional to the square of the ratio. The third column indicates that there is substantial variation in expected profit over the sample period as a result of movements in exchange rates, relative prices and interest rate differentials. Because of the endogenous leveraging of the portfolio, all of the observations in the sample should not be weighted equally. The fourth column gives the actual profits from the trading strategy. In the sample of 39 observations, there were 12 losses and the average loss was $48.5; there were 27 profits and the average profit was $95. The largest loss, $169, was considerably smaller than the largest gain, $805. Overall, then, the speculative strategy gives a far more favourable impression of the value of PPP than does the standard regression analysis.

The ability of the model to predict the return on any individual position is small. Over the 195 observations, the model accounts for a little more than 5 per cent of the variation in the forward rate bias. This figure is itself a little higher than the *r*-squared statistics reported for the forecasting equations because it takes account of the interest rate differential, but it is still, by conventional standards, low. The model does considerably better, however, in predicting the safety ratio of the portfolios. In this case, the expected safety, defined as the expected profit divided by the estimated standard deviation of profit, accounts for 31 per cent of the variation in actual profits divided by the estimated standard deviation of profits. Furthermore, the portfolio strategy scales up the position when the safety is high. As a consequence, expected profits account for 74 per cent of the variation in actual profits.

Table 2

Summary of the Isolated Trading Performance

Date	Safety Ratio	Expected Profit	Actual Profit	Cumulative Profit
73.01	0.85	72	−169	0
73.02	0.93	88	289	−169
73.03	0.64	41	28	120
73.04	1.18	139	109	149
74.01	0.82	68	60	258
74.02	0.62	39	136	319
74.03	0.46	21	79	455
74.04	0.70	49	−13	535
75.01	0.54	30	−120	521
75.02	0.83	70	65	400
75.03	0.56	31	10	465
75.04	0.60	36	37	476
76.01	0.36	13	−16	513
76.02	1.18	140	226	496
76.03	2.58	666	805	723
76.04	0.81	66	87	1529
77.01	0.30	9	12	1617
77.02	0.68	47	68	1629
77.03	0.36	13	10	1698
77.04	0.87	76	184	1708
78.01	0.43	18	−32	1893
78.02	0.34	12	−16	1860
78.03	1.20	146	288	1844
78.04	1.32	176	128	2133
79.01	0.72	52	−20	2261
79.02	0.55	30	−32	2240
79.03	0.60	36	−19	2208
79.04	0.82	68	87	2186
80.01	0.75	56	−122	2276
80.02	0.43	18	−2	2154
80.03	0.73	53	7	2151
80.04	1.26	159	246	2158
81.01	1.36	187	338	2405
81.02	0.99	99	201	2743
81.03	0.70	49	−37	2945
81.04	0.61	37	63	2907
82.01	0.90	81	9	2971
82.02	0.84	71	7	2981
82.03	0.77	59	152	2989
82.04				3141

Notes: Safety Ratio: Expected Profit/Standard Deviation of Profit. Cumulative Profits: Cumulative Actual Profits. All profits are expressed in US$.

Table 3

Estimates of the Fraction of Explained Variance

Actual return on expected return (195 obs.): *r-sq* = .05
Actual safety on expected safety: (39 obs.): *r-sq* = .31
Actual profit on expected profit: (39 obs.): *r-sq* = .74

Over the sample period, the trading strategy would have realized an average profit of approximately \$80 per quarter for a total profit of \$3141. Given the trading rule, the variance of profit in any period is defined by

$$V(R) = E(R) * g = (t * t) * (g * g). \tag{13}$$

Using this rule, the standard deviation of the sum of profits may be found to be \$560, so that the t-statistic on the sum of profits is approximately 5.6. It is consequently possible to reject the hypothesis that the sum of profits is zero at any reasonable level of statistical significance.

These are in-sample tests and the purpose of this exercise is not to investigate the profitability of currency speculation. The purpose of this analysis is to demonstrate the different perspective that can be obtained from a financial analysis of an econometric test. If the in-sample results are surprising given the econometric model that supported them, then the exercise has been worthwhile.

We now turn to the evaluation of the usefulness of the PPP relationship. If the definition of the variance of profits is substituted into the utility function, we find that

$$U(E(R),\ V(R)) = (1/2)E(R). \tag{14}$$

Equation (14) states that, given the optimal selection of the positions, the utility of the strategy is equal to one-half of the expected profit. This result justifies using the sum of the realized profits as an ex-post measure of utility. This result offers a method of measuring the economic value the PPP model. The method is to re-run the estimation and portfolio simulation without the PPP variable, and then calculate the decline in utility that results from the omission. From (14), it is clear that the standard deviation of utility is equal to one-half of the standard deviation of the sum of profits.

	Utility	*Discount*
PPP + Forward Premium	\$1571	
Forward Premium Alone	\$1253	(20%)
PPP Alone	\$1084	(31%)

Omitting the PPP relationship from the forecasting equation would have resulted in a 20% reduction in utility. If the forward parity model is omitted, utility is reduced by 31%. This result is interesting because the statistical significance of the forward premium term is below that of the PPP term. The explanation is almost entirely due to the guilder/mark spread discussed above. Given the \$800 profit realized on this position, the investor would have been willing to pay \$400 to have access to the model for this period alone.

Neither the omission of the PPP model or the forward parity model results in a decline in utility that is more than twice the estimated standard deviation. However, both restricted models do yield statistically significant levels of utility. This suggests that the positions selected by the models may be similar, so that there are relatively small gains from the combined strategy. A complete answer to this question requires, however, a more detailed investigation of the distribution of profits in a small sample.

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DISCUSSION

BRADFORD CORNELL*: John Bilson has written a provocative paper. By applying procedures developed in finance, he provides new insight into the role of purchasing power parity as a determinant of exchange rates.

Exchange rate returns (percentage changes in the dollar price of foreign exchange), like stock returns are very noisy. The problem can be summarized by writing the simple model,

$$R_t = B_t + u_t, \tag{1}$$

where,

R_t = the period t return,

B_t = the equilibrium return (which can be thought of as the return expected as of $t - 1$ plus a risk premium), and

u_t = the random deviation from the equilibrium return.

For both stock and foreign exchange, $E(B^2) \ll \mathrm{Var}(u)$, even for observations intervals of a month or more. This is true for all reasonable models of the equilibrium return, B. Under these circumstances, regressions of the form,

$$R_t = a_0 + a_1 X_{1,t-1} \cdots\cdots a_n X_{n,t-1} + v_t, \tag{2}$$

are certain to have little explanatory power no matter how the X's are defined. For this reason financial ecnomists rarely use regressions like (2) to study stock returns.

Nonetheless, traditional tests of PPP, and to a lesser extent forward rate parity, have been based on regressions similar to those given by (2). The poor fits that result are then interpreted to be evidence of the failure of PPP. Bilson argues that this conclusion is unwarranted. He notes that rejecting PPP on the basis of models like (2) is similar to concluding that there are no systematic factors which affect stock returns because they are difficult to discover from time series regressions for individual equities.

An alternative approach is to develop a trading strategy based on the information contained in the forward premium or the PPP relation. In this context

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