Consumption Betas and Backwardation in Commodity Markets
Author(s): Thomas B. Hazuka
Source: *The Journal of Finance*, Vol. 39, No. 3, Papers and Proceedings, Forty-Second Annual Meeting, American Finance Association, San Francisco, CA, December 28-30, 1983 (Jul., 1984), pp. 647-655
Published by: Blackwell Publishing for the American Finance Association
Stable URL: http://www.jstor.org/stable/2327923
Accessed: 27/11/2009 07:44

THE JOURNAL OF FINANCE • VOL. XXXIX, NO. 3 • JULY 1984

FUTURES MARKETS

Presiding: ROBERT H. LITZENBERGER
Stanford University

Consumption Betas and Backwardation in Commodity Markets

THOMAS B. HAZUKA*

ABSTRACT

This paper examines the relationship between commodity consumption betas and realized commodity futures contract risk premiums. A linear relationship between risk premiums and consumption betas is developed based on a consumption oriented CAPM. The parameters of this linear model are estimated using fourteen commodities.

JOHN MAYNARD KEYNES WAS among the first to assert the existence of risk premiums in commodity futures markets. Keynes (8) argued that, although futures prices can be above or below current spot prices, futures prices will be below expected future spot prices by the amount of the "normal backwardation". He was assuming that hedgers were always short and speculators long in the futures markets.

Cootner (4) provided empirical support that risk premiums did exist. He found that the risk premiums could be positive or negative, depending on whether net hedging pressure was short or long.

This classical view of the determinant of the relationship between the futures price and the expected future spot price does not take into account the transfer of risk in a general equilibrium setting. More recently a number of authors have developed models to examine the relationship between commodity betas and risk premiums. This approach will be examined in Section I.

In Section II, a linear model for predicting the rate of change in a commodity's spot price will be developed based on the work presented in Section I. If realized rates of change are unbiased estimates of expected rates of change in commodity spot prices, the slope coefficient of this model is a measure of the commodity risk premium associated with the commodity futures market. This coefficient is shown to be a function of the commodity's consumption beta.

* Wells Fargo Investment Advisors.

This paper is an extension of part of Chapter I of the author's doctoral dissertation at Stanford University. I would like to thank Robert Litzenberger, the chairman, and Douglas Breeden and Michael Gibbons, the committee members, for their help, comments and encouragement during the course of the original research. The current paper has benefited from the comments of Robert Litzenberger, Jeffrey Ricker and William Sharpe. Any remaining errors are my sole responsibility.

Section III provides econometric justification for the form of the linear predictive model and Section IV fits the model to fourteen commodities.

The relationship between the commodity risk premiums and consumption betas is empirically investigated in Section V, and conclusions are summarized in Section VI.

I. Synthesis of Past Studies

In recent years studies of the relationship between the futures price and the expected future spot price have treated futures contracts like any other risky asset and have analyzed them in a general equilibrium framework.

Dusak (5) and Black (1) analyzed the determinants of futures prices in the context of the Sharpe (11), Lintner (9) Mossin (10) capital asset pricing model (CAPM). They argue that differences between expected future spot prices and futures prices are proportional to commodity market betas.

There are several problems with their straightforward application of the capital asset pricing model to futures contracts. The capital asset pricing model was developed in the context of a single-period single-good framework. In a multiperiod multi-good framework the traditional CAPM holds under the assumptions of a stationary investment and consumption opportunity set. These assumptions imply constant relative prices and constant interest rates. Under these conditions a commodity futures contract would be a riskless asset.

Treating different contract months separately, Dusak estimated commodity betas for wheat, corn, and soybeans (e.g., separate betas were calculated for July, March, May, September, and December wheat). These betas were calculated by regressing commodity returns on the return on the Standard and Poors Index of 500 Common Stocks. This index was used as a proxy for the return on total wealth. Dusak's results indicated that none of the commodity contract betas were significantly different from zero, and she interprets these findings as indicating that the futures price is an unbiased estimator of the future spot price.

She made no attempt to account for any seasonality of betas. Cootner had observed indications of a positive risk premium during one part of the year and a negative risk premium during the other part; it may be that some commodity betas change sign during the year, producing an average beta of zero.

Grauer (6) calculated betas for a larger number of commodities but examined fewer contract months. All his regressions were done in terms of real (price level adjusted) returns. Using Dusak's sampling technique, he concluded that there was no relation between commodity returns and the return on the S&P 500. Grauer then calculated betas holding time to contract maturity constant (3, 6, and 9 months). This procedure effectively allows one to test for seasonal changes in betas. Grauer's results indicated that betas were seasonal for at least some of the commodities and that the real betas of oats, wheat, silver, lumber and sugar were significantly different from zero.

More recently, Grauer and Litzenberger (7) and Breeden (2) have studied the valuation of capital asset prices under relative price uncertainty in a discrete time and in a continuous time framework respectively. In a two-period multigood model, Grauer and Litzenberger derived asset prices in terms of their convariances with the marginal utility of aggregate real wealth. Breeden demonstrated that in a multi-period and multigood world, real risk premiums are proportional to real consumption betas.

II. Derivation of Linear Predictive Model from a Consumption CAPM

While a futures contract does not require a commitment of present funds, the risk premium on a portfolio consisting of a futures contract and a nominal bond having a face value equal to the futures price is a convenient starting point for a study of the implications of a consumption-oriented CAPM model. The expected gross real return on this portfolio is $E(P_{i1}I^{-1})$, (where $\tilde{P}_{i1}$ is the ith commodity's futures spot price, $\tilde{I}$ is the price index and E is an expectations operator). The current market price of this portfolio is $f_{i1}/(1 + R_F)$, where f_{i1} is the i-th commodity's futures price for delivery in period one and R_F is the nominal interest rate). Therefore the consumption-oriented CAPM developed by Breeden implies

$$\frac{E(P_{i1}I^{-1})(1 + R_F)}{f_{i1}} - (1 + r_f) = [E(r_c) - r_f]\beta_{ic} \tag{1}$$

where: r_f = the real interest rate

$\tilde{r}_c$ = the real rate of return on the aggregate period one consumption portfolio,

r_i = $(P_{i1}I^{-1})(1 + R_F)/f_{i1}$, and

β_{ic} = $\mathrm{cov}(r_i, r_c)/\mathrm{var}(r_c)$, the real beta of the i-th commodity.

Dividing both sides of equation (1) by the current spot price of the i-th commodity and rearranging terms yields

$$\frac{E(P_{i1}I^{-1})}{P_{i0}} = \frac{1 + r_f}{1 + R_F}\frac{f_{i1}}{P_{i0}} + [E(r_c) - r_f]\beta_{ic}\frac{1}{1 + R_F}\frac{f_{i1}}{P_{i0}}. \tag{2}$$

Using the definition of a covariance, $E(P_{i1}I^{-1}) = E(P_{i1})E(I^{-1}) + \mathrm{cov}(P_{i1}, I^{-1})$ and rearranging equation (2) yields

$$\frac{E(P_{i1})}{P_{i0}} = a_i + b_i\frac{f_{i1}}{P_{i0}} \tag{3}$$

where: $a_i = -\mathrm{cov}(P_{i1}/P_{i0}, I^{-1})/E(I^{-1})$, and (3a)

$$b_i = \frac{1 + r_f + [E(r_c) - r_f]\beta_{ic}}{(1 + R_F)E(I^{-1})}. \tag{3b}$$

The above relationship holds at a given point in time. Under the assumption that the risk premium on the consumption portfolio is an intertemporal constant and that the consumption betas of commodity i and the nominally riskless assets are stationary, b_i is constant over time.

Under the assumption that the covariance of the rate of change of the price of commodity i with the percentage deviation of the reciprocal of the price index from its expected value is stationary, a_i is also constant over time. The subsequent estimation of a_i and b_i treats them as intertemporal constants.

The term a_i is opposite in sign to the covariance of the rate of change in the spot price with the reciprocal of unity plus the rate of change in the price index. The price changes of most commodities would be expected to have a positive association with changes in the general level of prices. Therefore, $\mathrm{cov}(P_{i1}/P_{i0}, I^{-1})$ would be expected to be negative and a_i positive for most commodities. The term b_i is equal to the ratio of (1) the required real rate of return on a portfolio consisting of a futures contract for the i-th commodity and a nominal discount bond having a face value of f_{i1}, and (2) the expected real rate of return on the nominal bond.

The expected real rate of return on a nominal bond would be above the real interest rate if, as recent empirical evidence indicates, the inflation rate were inversely related to the growth rate in real aggregate consumption (see Grauer and Litzenberger (7). Under these conditions b_i would be less than unity for a commodity whose real price changes are uncorrelated with the rate of growth in aggregate consumption.

If a_i and b_i are stationary over time, the following linear predictive model is indicated[1]:

$$\frac{\tilde{P}_{it}}{P_{i,t-1}} = a_i + b_i \frac{f_{it}}{P_{i,t-1}} + \tilde{e}_{it} \tag{4}$$

where: $\tilde{e}_{it} = \dfrac{\tilde{P}_{it}}{P_{i,t-1}} - \dfrac{E(P_{it})}{P_{i,t-1}}$.

Thus, the consumption-oriented CAPM combined with these stationarity assumptions provides an economic justification for a linear predictive model.

[1] A linear predictive model would still be valid if a_i were a linear function of $f_{it}/P_{i,t-1}$ and b_i were a hyperbolic function of $f_{it}/P_{i,t-1}$. That is if

$$a_i = \lambda_{i0} + \lambda_{i1} \frac{f_{it}}{P_{i,t-1}}.$$

and

$$b_i = \phi_{i0} + \phi_{i1}\left(\frac{f_{it}}{P_{i,t-1}}\right)^{-1}$$

the following linear predictive model is indicated:

$$\frac{P_{it}}{P_{i,t-1}} = A_i + B_i \frac{f_{it}}{P_{i,t-1}}$$

where

$$A_i = \lambda_{i0} + \phi_{ii1} \text{ and}$$

$$B_i = \phi_{i0} + \lambda_{i1}.$$

III. Derivation of the Form of a Linear Prediction Model Using the Futures Price as the Predictor of the Expected Spot Price

Assume that the futures price for commodity i set at period o for t periods hence (f_{it}) is equal to the expected spot price in period $t(E(P_{it})$ plus a risk premium (ζ_{it}).

$$f_{it} = E(P_{it}) + \zeta_{it} \tag{5}$$

Following the analysis of the previous section, the risk premium is broken down into two components. The first component, which is proportional to the current spot price, is equal to the covariance of the rate of change in the spot price with the reciprocal of unity plus the rate of change in the general price index. If this covariance is stationary over time, this component of the model is well specified.

The second component, which is proprortional to the futures price, is equal to the ratio of unity plus the required real rate of return on a portfolio consisting of a futures contract and a nominal discount bond having a face value of unity plus the expected real rate of return of the nominal bond. If the commodity beta and the parameters of the consumption CAPM were stationary over time this relationship would be stationary. Given these assumptions,

$$f_{it} = E(P_{it}) + \alpha_{i1}P_{i0} + \alpha_{i2}f_{it}. \tag{6}$$

The realization of the actual spot price at period is equal to its expected value plus a forecasting error:

$$\tilde{P}_{it} = E(P_{it}) + \tilde{\epsilon}_{it} \tag{7}$$

where ϵ_{it} is the error in forecasting from period o to period t.

Substituting (7) into (6) and rearranging terms gives the structural economimetric model:

$$\tilde{P}_{it} = a_iP_{io} + b_if_{it} + \tilde{\epsilon}_{it} \tag{8}$$

where

$$a_i = -\alpha_{i1} \text{ and } b_i = (1 - \alpha_{i2})$$

It is assumed that the standard deviation of $\tilde{\epsilon}_{it}$ is proportional to P_{i0}. To eliminate heteroscedasticity, both sides of equation (4) are deflated by P_{i0}, giving

$$\frac{\tilde{P}_{it}}{P_{i0}} = a_i + b_i\frac{f_{it}}{P_{i0}} + \frac{\tilde{\epsilon}_{it}}{P_{i0}}, \tag{9}$$

or

$$\frac{\tilde{P}_{it}}{P_{i0}} = a_i + b_i\frac{f_{it}}{P_{i0}} + \tilde{e}_{it}, \tag{10}$$

where

$$\tilde{e}_{it} = \frac{\tilde{\epsilon}_{it}}{P_{i0}}$$

If the futures price is a sufficient statistic with respect to the expected future spot price, and if the model of the risk premium is correctly specified, the error terms would be serially uncorrelated. This would not be the case with overlapping forecast periods. However, the forecasts examined are one month forecasts made at the beginning of each month. Under the assumptions that the disturbance term is homoscedastic and serially uncorrelated, the use of ordinary least squares is appropriate

IV. Empirical Test of Linear Predictive Model

The linear predictive model developed in the last section was fitted using fifteen commodities. The commodities were classified according to storage characteristics: nonstorable (live cattle, live hogs, eggs, iced broilers, pork bellies), seasonal storable (wheat, corn, oats, soybeans, soy meal, soy oil, cocoa, sugar), and non-seasonal storable (copper, silver). Only futures contracts with one month to expiration were utilized.

Models fitted using storable commodities would be expected to have the least explanatory ability since the spot and futures are linked by the cost of storage. Assuming the cost of storage is known, all information concerning the future spot price that is reflected in the futures price will also be immediately reflected in the current spot price. In this case the futures market would not play an added informational role.

Models fitted using non-storable commodities would be expected to have the most explanatory ability since the inventory levels of these commodities are low relative to current consumption. Small fluctuations in inventory levels result in large fluctuations in the spread between the spot and futures prices. Under these circumstances the futures price may contain information about the future spot price not embedded in the current spot price.

The regression statistics obtained from fitting these models are shown in Table 1. As expected, the models fitted using the five non-storable commodities had the best explanatory ability; the simple average of the adjusted R^2's was .136. For the models fitted using the eight seasonable and two storable commodities the explanatory ability was nil; the respective average R^2's were .011 and .001.

V. Empirical Test of the Relationship Between Market Risk Premiums and Consumption Betas

Breeden (3) has calculated consumption betas for a number of commodity futures contracts. His results for contracts that are nearest to expiration are presented in Table 2. The slope coefficients (b_i) that were obtained from the linear predictive models estimated in section IV can be compared to Breeden's consumption betas to test the relationship derived in Section II.

Recall that equation 3b related the slope coefficients of the linear predictive models to consumption betas:

$$b_i = \frac{1 + r_f + [E(r_c) - r_f]\beta_{ic}}{(1 + R_F)E(I^{-1})}$$

Table 1

Regression of the Rate of Change In the Spot Price on the Basis

$$\tilde{P}_{it}/P_{i0} = a_i + b_i f_{it}/P_{i0} + \tilde{e}_{it}$$

	a_i	$t(a_i)$	b_i	$t(b_i)$	Adj R^2	Number of Observations
Non-Storable						
Live Cattle	.0095	1.41	.392	2.87	.079	85
Live Hogs	.0029	.20	.593	4.06	.195	65
Broilers	.0484	3.04	.824	4.66	.176	98
Eggs	.0511	3.34	.517	4.36	.184	83
Pork Bellies	.0164	1.05	.363	2.07	.047	68
Seasonal Storable						
Corn	.0089	1.66	.248	1.49	.013	97
Wheat	.0042	.55	−.100	−.76	−.005	94
Oats	.0158	1.27	.390	2.10	.066	49
Soybeans	.0071	.86	.103	.59	.005	137
Soymeal	.0196	1.55	.496	3.05	.049	161
Soy Oil	.0134	1.19	.109	.56	−.004	162
Cocoa	.0105	.38	.039	−.16	−.014	70
Sugar	−.0082	.46	−.297	−.53	−.011	68
Non-Seasonal Storable						
Copper	.0039	1.78	.017	1.54	.009	158
Silver	.0279	2.47	.411	−.47	−.008	103

Table 2

Consumption Betas for Commodity Futures Contracts Nearest to Expiration

Commodity	Beta	t-value (absolute value)
Broilers	11.4	2.84
Cattle	9.4	3.40
Hogs	13.1	2.91
Pork Bellies	9.5	3.00
Copper	12.6	3.14
Silver	−2.4	0.83
Cocoa	2.9	0.52
Eggs	−4.5	1.22
Sugar	−31.6	3.66
Corn	−1.7	0.57
Oats	.9	0.30
Soybeans	−0.3	0.07
Soymeal	3.1	0.73
Soy Oil	−6.2	1.30
Wheat	−0.5	0.16

Source: Breeden (3)

Rearranging this equation, it is seen that b_i is a linear function of the consumption beta:

$$b_i = \gamma_0 + \gamma_1 \beta_{ic} \tag{11}$$

where:

$$\gamma_0 = \frac{1 + r_f}{(1 + R_F)E(I^{-1})}, \text{ and} \tag{11a}$$

$$\gamma_1 = \frac{E(r_c) - r_f}{(1 + R_F)E(I^{-1})}. \tag{11b}$$

The estimated correlation coefficient between the slope coefficients and consumption betas was .563. This is in the expected direction and is significantly different from zero at the 95 percent confidence level. An ordinary least squares regression of the slope coefficients on consumption betas was also performed. The regression statistics are presented in Table 3.

The coefficient of the consumption beta is, from equation 11a, an estimate of the excess real return on the consumption portfolio divided by unit plus the expected real return on a nominally risk free security. This coefficient was estimated as .0175, and was significantly different from zero at the 95 percent confidence level.

If an expected real return of .25 percent per month (3 percent per year) on the nominally risk free security is assumed, an estimate of the excess real return on the consumption portfolio can be obtained. The excess real return on the consumption portfolio implied by the regression slope coefficient is 1.7 percent per month (20.4 percent per year).

The intercept of the regression is, from equation 11b, an estimate of unity plus the real rate of interest divided by unity plus the expected real return on a nominally risk free security. For a one month horizon, the ratio would be expected to be very close to unity. The estimated value of the intercept was .195 and was significantly different ($t = 10.6$) from one. Although the sign of the intercept was as predicted, its magnitude was lower than predicted.

From equation 3a, the intercepts of the linear predictive models fitted in section V should be equal to minus the covariance of the rate of change of the commodity spot price with the expected reciprocal of the price index, divided by the expected reciprocal of the price index. The covariance term would be expected to be negative and the intercept of the linear predictive model to be positive for most commodities. The estimated intercepts were, in fact, positive for all commodities except sugar. Sugar also had a very large negative beta, as the theory would predict.

Table 3

Regression of Risk Premiums on Consumption Betas

$$b_i = \gamma_0 + \gamma_1 \beta_{ic} + e_i$$

γ_0	$t(\gamma_0)$	γ_1	$t(\gamma_1)$	Adjusted R^2	Number of Observations
.195	2.56	.0175	2.46	.265	14

VI. Conclusions

This paper investigated the relationship between commodity consumption betas and commodity futures risk premiums. A linear model to predict the rate of change in a commodity's spot price based on a consumption CAPM was developed. Econometric justification for the form of the model was presented and the model fit using fourteen commodities.

The relationship between the storage characteristics of the commodities and the informational content of the predictive models was examined; it was found that models fit using non-storable commodities had the greatest informational content and models fit using commodities having large inventory levels relative to consumption had the least informational content.

The slope coefficients of the linear predictive models were seen to be estimates of commodity futures risk premiums. The linear relationship between these risk premiums and consumption betas implied by the consumption CAPM was empirically investigated. Both the intercept and the slope coefficients were significantly positive, as the theory predicted; however, the magnitude of the intercept was smaller and that of the slope greater than predicted.

Finally, the fitted intercepts of the linear predictive models were compared with the theoretical prediction. All but one were positive as would be expected given the theory.

REFERENCES

1. Black, Fischer. "The Pricing of Commodity Contracts." *Journal of Financial Economics*, (January/ March 1976), 167–179.
2. Breeden, Douglas T. "An intertemporal asset pricing model with stochastic consumption and investment opportunities." *Journal of Financial Economics*, (September 1979), 265–296.
3. Breeden, Douglas T. "Consumption Risk in Futures Markets." *Journal of Finance*, (May 1980), 503–520.
4. Cootner, Paul H. "Returns to Speculators: Telser Versus Keynes." *Journal of Political Economy*, (August, 1960), 396–404.
5. Dusak, Katherine. "Futures Trading and Investor Returns: An Empirical Investigation of Commodity Risk Premiums." *Journal of Political Economy*, (November/December 1973), 1387–1406.
6. Grauer, F. L. A. *Equilibrium in Commodity Futures Markets: Theory and Tests.* Unpublished Ph.D. dissertation, Stanford University, 1977.
7. Grauer, F. L. A., and Robert H. Litzenberger. "The Pricing of Commodity Futures Contracts, Nominal Bonds and Other Risky Assets under Commodity Price Uncertainty." *Journal of Finance*, (March, 1979), 69–83.
8. Keynes, J. M. *Treatise on Money*, V. II. London: Macmillan, 1930.
9. Lintner, John. "The Valuation of risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets." *Review of Economics and Statistics*, (February 1965), 13–37.
10. Mossin, Jan. "Equilibrium in an Asset Market." *Econometrica*, (October 1966), 768–783.
11. Sharpe, William F. "Capital Asset Prices: A Theory of Market, Equilibrium under Conditions of Risk." *Journal of Finance*, (September 1964), 425–442.