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Contingent Claims Analysis of Corporate Capital Structures: an Empirical Investigation

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IN THEIR SEMINAL WORK Black and Scholes (1973) provide a significant insight which arguably is of more academic and practical value than their famous option pricing model. They demonstrate that corporate liabilities can be viewed as combinations of simple option contracts. This generalization of option pricing, as refined by Merton (1974, 1977), has become known as Contingent Claims Analysis (CCA). While CCA has subsequently been used by many researchers as a theoretical framework in which to view the pricing of corporate liabilities, its empirical validity remains an open question. Ingersoll (1976, 1977b) has tested the model's ability to predict prices for dual purpose funds and call policies for convertible bonds respectively. In a recent paper, JMR (1983), we tested CCA in one of its potentially most important applications, namely the valuation of debt in typical corporate capital structures. The objective of JMR (1983) was to test the predictive power of a prototypical model based on the usual set of assumptions in the CCA literature. The data base used in JMR (1983) was made up primarily of investment grade bonds, i.e. bond rating of BBB or higher. This paper extends that test of the prototypical model to a larger data base which includes a number of noninvestment grade, or "junk", bonds. In addition, this paper demonstrates that in the multiple bond problem the value of callable debt need not be a monotonic function of firm value.

Research attempting to explain corporate bond prices has had a long and varied history. One line of research (e.g. Sloane (1963) and Jaffee (1975)) can be described as "macro" in that relative bond prices, or yield spreads, are posited to be functions of the supply and demand of various assets, and/or the position of the economy in the business cycle. Another line of research pioneered by Fisher (1959) views relative bond prices, or yield spreads, as functions of firm specific, or "micro", characteristics such as financial or business risk. Contingent Claims Analysis is consistent with the approach of Fisher (1959) in that the model's inputs can be viewed as measures of financial and business risk. The advantage of CCA over the regression based analysis of Fisher (1959) is that CCA provides a specific functional relationship to be tested. In addition, given the structure of the CCA model, it is straightforward to infer firm values or other security values from the values of traded claims, and to price different convenant structures separately.

In Section 2 of the paper, a brief discussion of the CCA valuation problem for a firm with equity and multiple issues of callable non-covertible sinking fund

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coupon debt is presented. For such capital structures it is demonstrated that the value of callable debt need not be a monotonic function of firm value. The data and estimation procedures are discussed in Section 3. Section 4 is an analysis of the results. Finally, Section 5 concludes the paper.

2. Theory

The theoretical basis of CCA is developed in Black and Scholes (1973) and Merton (1974, 1977). The usual assumptions made in the CCA literature, e.g., Ingersoll (1976, 1977) are:

(A.1) Perfect markets: The capital markets are perfect with no transaction costs, no taxes, and equal access to information for all investors.

(A.2) Continuous trading.

(A.3) Ito dynamics: The value of the firm, V, satisfies the stochastic differential equation.

$$dV = (\alpha V - C)dt + \sigma V dz$$

where total cash outflow per unit time C is locally certain α and σ^2 are the instantaneous expected rate of return and variance of return on the underlying assets.

(A.4) Constant σ^2.

(A.5) Nonstochastic term structure: The instantaneous interest rate $r(t)$ is a known function of time.

(A.6) Shareholder wealth maximization: Management acts to maximize shareholder wealth.

(A.7) Perfect bankruptcy protection: Firms cannot file for protection from creditors except when they are unable to make required cash payments. In this case perfect priority rules govern the distribution of assets to claimants.

(A.8) Perfect antidilution protection: No new securities (other than additional common equity shares) can be issued until all existing non-equity claims are extinguished. Deals between equity and subsets of other claimants are prohibited.

(A.9) Perfect liquidity: Firms can sell assets as necessary to make cash payouts, with no loss in total value.

Under these assumptions it can be shown that corporate liabilities which are functions of the value of the firm and time obey a partial differential equation which depends on the known schedule of interest rates, $r(t)$, the variance rate, σ^2, of firm value and indenture provisions, e.g., payouts and covenants. More importantly, the CCA pricing of corporate liabilities is not dependent on the equilibrium structure of risk and return, i.e. α. Readers are referred to Black and Scholes (1973) and Merton (1974, 1977) for a derivation of the basic partial differential equation.

A starting point for the analysis of typical capital structures is the example of CCA applied to non-convertible corporate bonds, namely the formulation in Merton (1974) of a callable coupon bond with no sinking fund. Merton shows that the debt $D(V, t)$ in a firm with one issue of such debt obeys the following partial differential equation and boundary conditions

$$\begin{aligned}
&0 = 1/2\sigma^2 V^2 D_{VV} + (rV - cP - d)D_V + D_t - rD + cP \\
&D(0, t) = 0 \\
&D(V, t^*) = \min(V, P) \\
&D(\bar{V}, t) = k(t)P \\
&D_V(\bar{V}, t) = 0
\end{aligned} \tag{1}$$

where $P = P(t)$ is the outstanding bond principal at time t, c is the coupon rate per unit principal, $k(t)$ is the call price schedule per unit principal, $d = d(V, t)$ is the known dividend policy and t^* is the maturity date of the bond. The upper free boundary, $\bar{V}(t)$, corresponds to the optimal call barrier at or above which the firm will call the bonds. Translating the standard set of assumptions, A.1–A.9, into an explicit model for valuing claims in a typical capital structure is considerably more difficult than suggested by (1). A typical capital structure consists of equity and multiple issues of callable nonconvertible sinking fund coupon debt. This differs from the standard example of a single issue of nonconvertible debt because of both the sinking fund and multiple issue features.

Most issues of corporate debt specify the mandatory retirement of bonds via periodic sinking fund payments. Typically the firm is required to retire a specified fraction of the initial bonds each period. Generally the firm has the option to redeem these bonds through either of two mechanisms: (1) it can purchase the necessary bonds in the market and deliver them to the trustee, or (2) it can choose the necessary bonds by lot and retire them by paying the standard principal amount to their owners. Often the firm also has the option to double the number of bonds retired each period if it wishes. Hence the firm faces the following choices each period:

(1) Should the bonds be called.
(2) Assuming the bonds are not called, should the mandatory number of bonds be sunk at market or par? (If this option exists.)
(3) Assuming the bonds are not called, should the sinking fund payment be doubled? (If this option exists.)

Unfortunately, incorporating the option to double the sinking fund payment in a capital structure with numerous debt issues dramatically increases the dimensionality of the valuation problem. Therefore, the option to double is ignored in the numerical approximations. This will lead to the underpricing of equity and the overpricing of debt. The reader is referred to JMR (1983) for a full analytic treatment of how the formulation of the debt valuation problem, 1, is affected by the presence of sinking funds.

Another major problem in the Contingent Claims Analysis of typical capital structures is the presence of multiple debt issues. This characteristic of typical capital structures introduces interactions among bonds that are not present in the standard example of one debt issue. Of particular importance is the interaction among the optimal call policies of the different bonds. The major complexity is the fact that minimizing the market value of a particular bond via a call is not necessarily equivalent to maximizing the value of the equity. To see this, consider a firm with two discount bonds of the same maturity and seniority. Let the first bond, D', be noncallable and have a promised principal of B′. Let the second bond, D, be callable at K for just the current point in time (then non-callable) and have a promised principal of B where $K < Be^{-r\tau}$, is the maturity of the bond, and r is the risk-free rate of interest. Given the current firm value, V, and the decision *not* to call D, then the value of the two bonds would be

$$\begin{aligned} D &= Bg(V, \tau, B + B')/(B + B') \\ D' &= B'g(V, \tau, B + B')/(B + B') \end{aligned} \tag{2}$$

where $g(\cdot)$ is the risky discount bond function of Merton (1974).

Now consider that specific firm value, V^*, where the holder of the callable bond is indifferent to being called, i.e.,

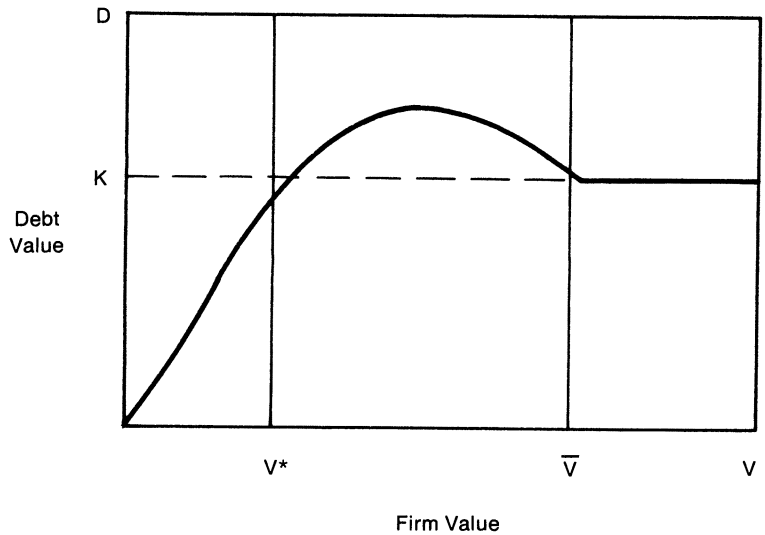
$$\begin{aligned} D(V^*, \tau) &= Bg(V^*, \tau, B + B')/(B + B') = K \\ D'(V^*, \tau) &= B'g(V^*, \tau, B + B')/(B + B') \end{aligned} \tag{3}$$

Should the callable bond be called when the firm value is V^*, then the value of the noncallable bond would be $D'(V^*, \tau) = g(V^* - K, \tau, B')$. The holder of the noncallable bond would strictly prefer the decision to call if $g(V^* - K, \tau, B') > B'g(V^*, \tau, B + B')/(B + B')$ which by the homogeneity of $g(\cdot)$ and the fact that $g_V > 0$ is equivalent to $V^* - K > V^*B'/(B + B')$. But, given (3), this is equivalent to $V^* > g(V^*, \tau, B + B')$ which is always true.

Since the holder of the noncallable bond would strictly prefer a call and, by construction, the holder of the callable bond is indifferent, then the equity would be strictly worse off by calling. Therefore, by continuity, the firm value, V, at which equity would call is greater than V^* and the value of the callable bond, $D(V, \tau)$, can exceed K for $V > V^*$. But this implies that $D(V, \tau)$ will not be monotonic in firm value. To see this consider a $V \gg V^*$. While it is true that the holder of the noncallable bond will always prefer a call for $V > V^*$, as V becomes large the value of the noncallable bond approaches $B'e^{-r\tau}$, the value of a risk-free bond. This restricts the extent to which the noncallable bond can benefit from a call of D and will, for a sufficiently large V, result in the equity capturing some of the benefit. Therefore, as depicted in Figure 1, the value of the callable bond will not be monotonic in firm value.

Thus, in order to identify the optimal call policies, it is necessary to systematically consider all possible capital structure states and to choose that state which will result in the value of equity being maximized. The reader is referred to JMR (1983) for a complete and detailed discussion of this issue. It is important to understand the dimensionality of the n callable bond case. First note that there are 2^n possible capital structure states, including the trivial state of an all-equity firm. Furthermore, there are a number of securities to be valued in each state. Given n callable bonds, there are $\binom{n}{n} = 1$ capital structure states corre-

Figure 1



sponding to 0 bonds having been called. In this one state there are $n + 1$ securities to be valued. There are $\binom{n}{n-1} = n$ capital structure states corresponding to 1 bond having been called. In each of these n states there are n securities to be value. Continuing in this way, we find that there are $\sum_{j=0}^{n-1} \binom{n}{n-j}(n + 1 - j)$ solutions in all. Hence one high priority line of research in terms of applying Contingent Claims Analysis to typical capital structures is the derivation of rational theorems which rule out various capital structure states—e.g., which show that certain kinds of bonds are always called first.

3. Data and Methodology

Data were collected for a total of 27 firms on a monthly basis from January 1975 through January 1981 when possible. The firms were selected according to a number of criteria at the beginning of 1975:

1. Simple Capital Structures (i.e. one class of stock, no convertible bonds, small number of debt issues, no preferred stock).
2. Small proportion of private debt to total capital.
3. Small proportion of short term notes payable or capitalized leases to total capital.
4. All publicly traded debt is rated.

Based on these criteria the following firms were selected:

Firm Name	Bond Rating Range
Allied Chemical	AA/A
Anheuser Busch	A
Braniff	BBB/CC
Brown Group	A
Bucyrus Erie	A
Champion Spark Plug	AA
Cities Service	A
CPC	AA/A
Crane	BBB
Food Fair	BB/B
Fuqua	B
General Cigar	BB/B
Kane Miller	B
MGM	BBB/B
National Tea	B
NVF	B
Procter and Gamble	AAA
Pullman	BBB
Rapid American	B/CC
Raytheon	AA/A
Republic Steel	A
Seagram	A
Sunbeam	A
Tandy	BBB/B
United Brands	B
Upjohn	AAA
Whittaker	BB/B

CCA requires three kinds of data in order to solve for prices of individual claims as functions of total firm value: (1) indenture data, (2) standard deviation data, and (3) interest rate data. The bond indentures define the boundary conditions and payout terms which constitute the economic description of various claims. For example, the following data were collected for each bond for each firm: principal, coupon rate, call price schedule, call protection period, sinking fund payments, seniority, and options to sink at market or par. The bond covenant data were collected from Moody's Bond Guide, except that sinking fund payments were collected from the monthly S & P Bond Guide. For purposes of testing the model, actual bond prices were also collected from the latter source.

Two procedures were used to estimate the standard deviation for each firm in the sample, as of each January from 1977 through 1981. The first procedure (hereafter referred to as Method I) was based on forming a monthly time series for the value of the firm using 24 trailing months of data. The value of the firm was estimated as the sum of the market value of equity, the market value of traded debt and the estimated market value of nontraded debt. The market value of the nontraded debt was estimated by assuming that the ratio of book to market was the same for traded and nontraded debt. The logarithmic total return on the value of the firm, including any cash payouts/payins, was calculated and the standard deviation of these returns determined.

The second procedure (hereafter referred to as Method II) is a maximum likelihood procedure based on the relationship between the standard deviations of the return to the firm and the equity. Given the assumptions of CCA, it follows from Ito's Lemma that the instantaneous standard deviation of equity, σ_E, is given by

$$\sigma_E = \sigma_V E_V V/E \tag{3}$$

where σ_V is the standard deviation of the return to the firm and E_V is the partial derivative of the value of equity with respect to the value of the firm. The Method II procedure was to run the model using the Method I estimate of standard deviation as a seed. The value of the firm, V, the value of equity, E, and the partial derivative of equity, E_V, with respect to the value of the firm which are implied by the observed total value of marketable claims are read from this first pass of the model. Then the standard deviation of return to the equity is calculated, using the CRSP daily returns data, for the three month period, October–December, immediately preceding the January test date. Given (3), a new estimate of σ_V is formed using σ_E, E, V, and E_V. The model is then rerun using this new estimate of σ_V. Table 1 summarizes the Method II estimates of the standard deviation of returns to the firm.

The standard assumption in CCA is that the future course of interest rates, $r(t)$, is known. Specifically, it is often assumed that the instantaneous rate of interest is constant through time, i.e., a flat term structure. The assumption of a flat term structure results in a fundamental problem for the empirical test of the contingent claims model in that the model will misprice riskless bonds. Therefore, the test of whether CCA can price risky bonds is systematically flawed. This problem is handled by the assumption that the future course of the one year rate of interest will be consistent with the one year forward interest rates implied by the current term structure. The following procedure was used to estimate implied one year forward interest rates as of each January from 1977 through 1981. First all par government bonds, adjusted for accrued interest, were identified as of that date. These data were gathered from the *Wall Street Journal*. If necessary linear interpolation was used to complete the yield curve. Then this yield curve was solved for implied one year forward rates. Hence the implied forward rates pertain to a par term structure and this procedure will result in the model correctly pricing par government bonds.

The method of Markov chains is used to approximate solutions to the partial differential equations. Parkinson (1977), Mason (1979) and Cox, Ross and Rubinstein (1979) use Markov chains to approximate solutions to valuation problems similar to the ones considered in this paper. The method of finite differences has been used by Brennan and Schwartz (1977a, 1977b) to treat similar contingent claims equations. The methods of Markov chains and finite differences are very similar, and in some cases identical, as demonstrated by

Table 1

Estimates of Annualized Standard Deviations of Returns to Firm: Method II

Name	1977	1978	1979	1980	1981
Allied Chemical	0.191	0.149	0.167	0.229	0.314
Anheuser Busch	0.343	0.276	0.219	0.184	0.221
Braniff	0.198	0.199	0.293	0.270	0.173
Brown Group	0.164	0.165	0.214	0.186	0.253
Bucyrus Erie	0.313	0.304	0.404	0.267	0.319
Champion Spark Plug	0.267	0.246	0.356	0.427	0.307
Cities Service	0.176	0.133	0.140	0.250	0.450
Crane	0.254	0.214	0.334	0.254	0.255
CTC	0.233	0.215	0.187	0.228	0.307
Food Fair	0.157	0.139	NA	NA	NA
Fugua	0.509	0.211	0.340	0.437	0.318
General Cigar	0.090	0.103	0.143	0.154	0.297
Kane Miller	0.136	0.121	0.146	0.169	0.167
MGM	0.176	0.240	0.535	0.464	0.610
National Tea	0.628	0.486	0.493	NA	NA
NVF	NA	0.264	0.421	0.663	0.474
Proctor & Gamble	0.171	0.142	0.191	0.168	0.235
Pullman	0.349	0.231	0.450	0.362	NA
Rapid American	0.224	0.117	0.133	0.124	0.052
Raytheon	0.253	0.270	0.369	0.222	0.369
Republic Steel	0.120	0.116	0.170	0.178	0.125
Seagram	0.233	0.169	0.292	0.384	NA
Sunbeam	0.372	0.240	0.351	0.365	0.324
Tandy	0.271	0.285	0.587	0.309	0.510
United Brands	0.177	0.176	0.276	0.170	0.215
Upjohn	0.247	NA	NA	NA	0.241
Whittaker	0.161	0.171	0.436	0.400	0.318

Brennan and Schwartz (1978) and Mason (1978). Readers are referred to these papers for background on numerical analysis techniques.

If all claims are publicly traded, then the value of the firm can be observed and prices for all claims, relative to the observed firm value, can be predicted. However, since all claims on the test firms are not publicly traded, an alternative approach had to be taken. Namely, the total value of all traded claims is used to infer the firm value. In other words, what firm value is consistent with the observed value of all traded claims? This implied firm value is then used to predict bond prices. Counting each year from 1977 through 1981, and counting each bond existing in each year for each of the 27 firms, we solved numerically for prices of 305 bonds.

4. *Results*

Tables 2–5 summarize the results. Percentage error is defined as the predicted price minus the actual price divided by the actual price. Absolute percentage errors and results from a naive model are also reported. The naive model essentially assumes that the value of the firm is sufficiently large to make all debt riskless. These results were obtained from the same runs of the model that produced the CCA estimates. Thus the naive model prices simply reflect the magnitude and timing of promised payments discounted back by the risk free interest rates, $r(t)$, plus the effects of the call provision and the sinking fund option to sink at the minimum of par or market. Incrementally, the CCA model prices attempt to capture the risk of default through the consideration of business risk, σ^2 and financial risk, i.e., finite firm value relative to promised payouts. In addition, the CCA model introduces the distinction between senior and junior debt as well as the presence of equity which complicates (relative to the naive model) the optimal call policy.

Table 2 presents a simple sign test of the CCA model versus the naive model. The absolute pricing error for the two models are compared and the number of times are counted that the CCA model's error is less than the naive model's error. As is evident from Table 2, the CCA model significantly outperforms the naive model. This is not a surprising result since the naive model is perfectly nested within the CCA model and will (almost) always generate a higher predicted price (the exceptions having to do with the "cresting" of bond prices induced by the optimal call policy as depicted in Figure 1). Since the interest rates, $r(t)$, have been calibrated to replicate risk free bond prices and since most of the actual bond prices reflect some risk premium, then the CCA model should significantly outperform the naive model on a sign test.

Table 3 presents the pricing errors for the CCA and naive models for various subsamples. Results are reported for investment grade (bond rating of BBB or higher) and non-investment grade subsamples as well as the entire sample. All splits are at the 50 percentile for each sample. As is evident from inspection, the CCA and naive models are virtually indistinguishable for investment grade bonds. This can be interpreted as evidence that firm value risk is not playing a significant role in explaining investment grade bond prices. This also suggests that a stochastic interest rate model could be a better predictor of investment grade bond prices. There do appear to be significant differences between the CCA and naive models for non-investment grade bonds. However, there is little evidence in Table 3 that the CCA model performs significantly better (in an absolute sense) for any particular subsample of non-investment grade bonds.

Tables 4–5 present regression based results which are consistent with earlier tables. Specifically, Table 4 demonstrates that the CCA model is virtually indistinguishable from the naive model for investment grade bonds. However,

Table 2

Sign Test of CCA Model vs. Naive (Riskless) Model

	Number of Bonds	Number of Times that ^{S}CCA < ^{S}Naive
Entire Sample	305	256*
Investment Grade	176	139*
Non-Investment Grade	129	117*

^{S}CCA = | Actual Price − CCA Model Price |
^{S}Naive = | Actual Price − Naive Model Price |
* Significant at the 99% level or greater

Table 3

Pricing Results and Comparisons: CCA Model and Naive (Riskless) Model

	Number of Bonds	Percentage Error Error Mean (Standard Deviation)		Absolute Percentage Error Mean (Standard Deviation)	
		CCA Model	Naive Model	CCA Model	Naive Model
Overall Results					
Entire Sample	305	0.0452	0.0876	0.0845	0.1143
		(0.1003)	(0.1441)	(0.0705)	(0.1240)
Investment Grade	176	0.0047	0.0149	0.0587	0.0574
		(0.0727)	(0.0703)	(0.0432)	(0.0432)
Noninvestment Grade	129	0.1005	0.1867	0.1197	0.1919
		(0.1063)	(0.1590)	(0.0840)	(0.1528)
High Variance					
Entire Sample	149	0.0214	0.0644	0.0763	0.0942
(>0.055)		(0.0918)	(0.1055)	(0.0554)	(0.0800)
Investment Grade	85	−0.0101	0.0094	0.0603	0.0566
(>0.055)		(0.0738)	(0.0706)	(0.0438)	(0.0433)
Noninvestment Grade	64	0.0633	0.1375	0.0975	0.1443
(>0.055)		(0.0965)	(0.0996)	(0.0618)	(0.0896)
Low Variance					
Entire Sample	156	0.0679	0.1097	0.0923	0.1334
(≤0.055)		(0.1028)	(0.1701)	(0.0816)	(0.1522)
Investment Grade	91	0.0185	0.0200	0.0571	0.0581
(≤0.055)		(0.0688)	(0.0696)	(0.0425)	(0.0431)
Noninvestment Grade	65	0.1371	0.2352	0.1416	0.2388
(≤0.055)		(0.1027)	(0.1889)	(0.0963)	(0.1843)
High Financial Leverage					
Entire Sample	148	0.0857	0.1499	0.1061	0.1589
(>0.31)		(0.1049)	(0.1634)	(0.0843)	(0.1546)
Investment Grade	90	0.0072	0.0207	0.0656	0.0625
(>0.24)		(0.0806)	(0.0760)	(0.0475)	(0.0480)
Noninvestment Grade	63	0.1173	0.2375	0.1332	0.2427
(>0.55)		(0.1182)	(0.1951)	(0.1001)	(0.1886)
Low Financial Leverage					
Entire Sample	157	0.0070	0.0288	0.0641	0.0722
(≤0.31)		(0.0785)	(0.0897)	(0.0459)	(0.0606)
Investment Grade	86	0.0020	0.0088	0.0515	0.0521
(≤0.24)		(0.0632)	(0.0631)	(0.0367)	(0.0368)
Noninvestment Grade	66	0.0843	0.1382	0.1069	0.1434
(≤0.55)		(0.0906)	(0.0909)	(0.0624)	(0.0826)
Long Term					
Entire Sample	153	0.0479	0.0949	0.0821	0.1190
(>14 yrs.)		(0.0911)	(0.1417)	(0.0621)	(0.1222)
Investment Grade	88	0.0157	0.0265	0.0590	0.0599
(>15 yrs.)		(0.0716)	(0.0692)	(0.0434)	(0.0436)
Noninvestment Grade	66	0.1025	0.2066	0.1189	0.2070
(>12 yrs.)		(0.0962)	(0.1442)	(0.0750)	(0.1438)
Short Term					
Entire Sample	152	0.0424	0.0802	0.0869	0.1095
(≤14 yrs.)		(0.1087)	(0.1460)	(0.0779)	(0.1255)
Investment Grade	88	−0.0063	0.0032	0.0583	0.0548
(≤15 yrs.)		(0.0721)	(0.0694)	(0.0429)	(0.0427)

Table 3 *Continued*

	Number of Bonds	Percentage Error Error Mean (Standard Deviation) CCA Model	Percentage Error Error Mean (Standard Deviation) Naive Model	Absolute Percentage Error Mean (Standard Deviation) CCA Model	Absolute Percentage Error Mean (Standard Deviation) Naive Model
Noninvestment Grade	63	0.0983	0.1659	0.1206	0.1761
(≤12 yrs.)		(0.1159)	(0.1707)	(0.0924)	(0.1601)
Senior Bonds					
Entire Sample	231	0.0327	0.0625	0.0780	0.0963
		(0.0949)	(0.1272)	(0.0633)	(0.1039)
Investment Grade	158	0.0004	0.0105	0.0573	0.0562
		(0.0707)	(0.0689)	(0.0414)	(0.0412)
Noninvestment Grade	73	0.1027	0.1752	0.1227	0.1831
		(0.1027)	(0.1494)	(0.0776)	(0.1396)
Junior Bonds					
Entire Sample	74	0.0841	0.1657	0.1048	0.1703
		(0.1066)	(0.1644)	(0.0862)	(0.1596)
Investment Grade	18	0.0422	0.0534	0.0708	0.0673
		(0.0790)	(0.0703)	(0.0549)	(0.0572)
Noninvestment Grade	56	0.0976	0.2017	0.1158	0.2033
		(0.1107)	(0.1696)	(0.0915)	(0.1677)
High Coupon					
Entire Sample	150	0.0600	0.1046	0.0886	0.1142
(>8.00%)		(0.0900)	(0.1015)	(0.0620)	(0.0906)
Investment Grade	87	0.0189	0.0351	0.0586	0.0575
(>7.75%)		(0.0702)	(0.0621)	(0.0430)	(0.0423)
Noninvestment Grade	68	0.0911	0.1581	0.1099	0.1584
(>9.25%)		(0.0885)	(0.0957)	(0.0636)	(0.0952)
Low Coupon					
Entire Sample	155	0.0309	0.0711	0.0805	0.1144
(≤8%)		(0.1074)	(0.1741)	(0.0776)	(0.1493)
Investment Grade	89	−0.0092	−0.0049	0.0587	0.0573
(≤7.75%)		(0.0724)	(0.0721)	(0.0433)	(0.0441)
Noninvestment Grade	61	0.1109	0.2186	0.1307	0.2292
(≤9.25%)		(0.1223)	(0.2033)	(0.1009)	(0.1913)
High Price					
Entire Sample	150	0.0273	0.0494	0.0672	0.0772
(<82.5)		(0.0811)	(0.0892)	(0.0530)	(0.0667)
Investment Grade	87	0.0186	0.0254	0.0535	0.0537
(>84.0)		(0.0650)	(0.0637)	(0.0413)	(0.0426)
Noninvestment Grade	64	0.0604	0.1206	0.0933	0.1310
(>78.5)		(0.0939)	(0.1081)	(0.0612)	(0.0952)
Low Price					
Entire Sample	155	0.0626	0.1245	0.1012	0.1502
(≤82.5)		(0.1132)	(0.1743)	(0.0805)	(0.1527)
Investment Grade	89	−0.0089	0.0046	0.0637	0.0609
(≤84.0)		(0.0771)	(0.0747)	(0.0443)	(0.0436)
Noninvestment Grade	65	0.1399	0.2519	0.1457	0.2519
(≤78.5)		(0.1029)	(0.1736)	(0.0946)	(0.1736)
High Current Yield					
Entire Sample	155	0.0752	0.1448	0.1025	0.1561
(>9.75%)		(0.1076)	(0.1620)	(0.0821)	(0.1511)

Table 3 *Continued*

	Number of Bonds	Percentage Error Error Mean (Standard Deviation)		Absolute Percentage Error Mean (Standard Deviation)	
		CCA Model	Naive Model	CCA Model	Naive Model
Investment Grade	102	0.0125	0.0270	0.0616	0.0598
(>8.50%)		(0.0753)	(0.0703)	(0.0451)	(0.0457)
Noninvestment Grade	69	0.1074	0.2255	0.1207	0.2255
(>10.50%)		(0.0997)	(0.1690)	(0.0831)	(0.1690)
Low Current Yield					
Entire Sample	150	0.0142	0.0284	0.0659	0.0710
(≤9.75%)		(0.0813)	(0.0906)	(0.0496)	(0.0631)
Investment Grade	74	−0.0061	−0.0018	0.0546	0.0540
(≤8.50%)		(0.0674)	(0.0668)	(0.0400)	(0.0394)
Noninvestment Grade	60	0.0924	0.1421	0.1186	0.1532
(≤10.50%)		(0.1129)	(0.1335)	(0.0850)	(0.1206)

Table 4

Regression Results of Actual Prices vs. CCA Model and Naive Model

	Dependent Variable = Actual Price			
	Constant	Naive Model Price	CCA Model Price	R^2
Entire Sample	0.0056	0.0832	0.7781	0.977
	(0.0008)	(0.0301)	(0.0355)	
	0.0131	0.7243		0.942
	(0.0012)	(0.0103)		
	0.0050		0.8746	0.977
	(0.0008)		(0.0077)	
Investment Grade	0.0025	1.0022	−0.0637	0.985
	(0.0007)	(0.1780)	(0.1798)	
	0.0025	0.9392		0.985
	(0.0007)	(0.0087)		
	0.0027		0.9473	0.983
	(0.0007)		(0.0095)	
Non-Investment Grade	0.0043	0.1740	0.6541	0.983
	(0.0013)	(0.0359)	(0.0442)	
	0.0117	0.6911		0.953
	(0.0020)	(0.0136)		
	0.0029		0.8627	0.981
	(0.0014)		(0.0110)	

Standard Errors are in parenthesis below coefficient estimates.

the CCA model does appear to have incremental explanatory power for non-investment grade bonds. Table 5 presents the results of a multivariate regression of the percentage pricing error for the CCA model against the characteristics identified in Table 3 plus a year dummy variable. The presence of a year effect is indicated, with the effect being stronger for investment grade bonds. This

Table 5

Regression of CCA Model Errors

Dependent Variable = $\dfrac{\text{CCA Model Price} - \text{Actual Price}}{\text{Actual Price}}$

	Entire Sample		Investment Grade		Non-Investment Grade	
Independent Variables	Coefficient	Mean	Coefficient	Mean	Coefficient	Mean
Constant	0.0234		−0.0875		0.5397	
	(0.1351)		(0.2176)		(0.2281)	
Variance	−0.4599	0.084	−0.3465	0.078	−0.5987	0.092
	(0.0653)		(0.0797)		(0.1214)	
Maturity	−0.0028	14.260	−0.0021	15.120	−0.0041	13.100
	(0.0008)		(0.0009)		(0.0019)	
Coupon	−0.0102	7.880	−0.0071	7.520	0.0314	8.370
	(0.0151)		(0.0256)		(0.0233)	
Price	−0.0008	81.680	0.0006	84.510	−0.0069	77.810
	(0.0016)		(0.0026)		(0.0026)	
Senior/Junior (Dummy)	−0.0032	0.240	0.0305	0.102	−0.0084	0.434
(Junior = 1, Senior = 0)	(0.0103)		(0.0157)		(0.0143)	
Bond Rating	0.0235	4.101	0.0075	2.830	0.0271	5.830
(1–9 1= AAA, 9 = C)	(0.0045)		(0.0079)		(0.0172)	
Current Yield	2.8015	0.098	2.4934	0.090	−0.2471	0.109
	(1.1678)		(1.9982)		(1.7832)	
Financial Leverage	−0.0901	0.372	−0.0621	0.244	−0.1747	0.547
	(0.0286)		(0.0363)		(0.0521)	
1978 (Dummy)	−0.0541	0.193	−0.0654	0.198	−0.0363	0.186
	(0.0129)		(0.0134)		(0.0232)	
1979 (Dummy)	−0.1053	0.203	−0.1012	0.187	−0.1075	0.224
	(0.0134)		(0.0149)		(0.0234)	
1980 (Dummy)	−0.1001	0.206	−0.0973	0.198	−0.0965	0.217
	(0.0166)		(0.0198)		(0.0287)	
1981 (Dummy)	−0.1664	0.200	−0.1471	0.210	−0.1763	0.186
	(0.0197)		(0.0244)		(0.0372)	
R^2	0.539		0.453		0.502	

Standard errors are in parenthesis below coefficient estimates.

could be related to an intertemporal bias in variance estimation or interest rate uncertainty. Variance, or business risk, is significant for the entire sample, as well as for the investment and non-investment grade bond subsamples. In all cases variance enters with a negative coefficient which suggests that a high estimated variance could be associated with an overestimate of the variance and therefore an underestimate of the bond price. Maturity is also significant and enters with a negative coefficient. This is undoubtedly related to the variance effect being amplified for long term bonds.

Current yield is significant for the entire sample. This could be due to the fact that the IRS allows investors to amortize a secondary market premium against interest income while allowing realized gains due to secondary market discounts to be taxed at capital gains rates. However, current yield is not significant in the non-investment grade subsample. This could be due to another dimension of the tax effect which has to do with risk.

Consider low quality par bonds versus high quality par bonds. The expected capital loss on the low quality bonds is larger in absolute terms than the expected capital loss on the high quality bonds. Hence the low quality bonds will have a higher coupon rate than the high quality bonds. Since the higher taxes on the low quality bond are ignored, any tax effect will cause low quality to be overpriced relative to high quality bonds. In particular, since government par bonds are perfectly safe, any tax effect will cause corporate par bonds to be overpriced in general. Similar considerations say that any tax effect will cause junior par bonds and long maturity par bonds to be overpriced relative to senior par bonds and short maturity par bonds respectively.

5. *Conclusions*

The objective of this paper is to test the predictive power of a CCA model of typical capital structures. A distinction is made between the model's performance vis à vis investment grade and non-investment grade bonds. The CCA model is not an improvement over a naive (riskless) model for investment grade bonds. However, the CCA model does appear to have incremental explanatory power over the naive model for non-investment grade bonds. There is evidence that introducing stochastic interest rates, as well as taxes, would improve the model's performance. Variance estimation errors, as with all option based models, appear to be important.

Establishing the empirical validity of the CCA model is an important goal. Numerous problems remain including better bond prices, more efficient numerical techniques and adjustments to the standard set of assumptions. We view the contributions of JMR (1983) and this paper as (1) identifying and resolving a number of analytical issues in the formulation of the CCA valuation problem for typical capital structures and (2) demonstrating empirical results which are crucial in establishing research priorities in what will be a large and complex task.

REFERENCES

Black, F. and M. Scholes 1973. The pricing of options and corporate liabilities. *Journal of Political Economy* 81: 637–659.

Brennan, M. and E. Schwartz 1977a. Convertible bonds: valuation and optimal strategies for call and conversion. *Journal of Finance* 32: 1699–1715.

——— 1977b. The valuation of American put options. *Journal of Finance* 32: 449–462.

——— 1978. Finite difference methods and jump processes arising in the pricing of contingent claims: A synthesis. *Journal of Financial and Quantitative Analysis*, September: 461–474.

Cox, J., S. Ross and M. Rubinstein 1979. Option pricing: A simplified approach. *Journal of Financial Economics* 7: 229–263.

Fisher, L. 1959. Determinants of risk premium on corporate bonds. *Journal of Political Economy* 4: 269–322.

Ingersoll, J. 1976. A theoretical and empirical investigation of the dual purpose funds. *Journal of Financial Economics* 3: 83–123.

——— 1977a. A contingent claims valuation of convertible securities. *Journal of Financial Economics* 4: 269–322.

——— 1977b. An examination of corporate call policies on convertible securities. *Journal of Finance* 32: 463–478.

Jaffee, D. M. 1975. Cyclical variations in the risk structure of interest rates. *Journal of Monetary Economics* 1: 309–325.

Jones, E. P., S. P. Mason and E. Rosenfeld 1983. Contingent claims valuation of corporate liabilities: Theory and empirical tests. *National Bureau of Economic Research Working Paper* No. 1143.

Mason, S. P. 1978. The numerical analysis of certain free boundary problems arising in financial economics. Harvard Business School, Boston, MA.

——— 1979. The numerical analysis of risky coupon bond contracts. Working Paper No. 79-35. Harvard Business School, Boston, MA.

Merton, R. C. 1974. On the pricing of corporate debt: The risk structure of interest rates. *Journal of Finance* 29: 449–470.

——— 1977. On the pricing of contingent claims and the Modigliani-Miller theorem. *Journal of Financial Economics* 5: 147–175.

Parkinson, M. 1977. Option pricing: the American put. *Journal of Business* 50: 21–36.

Sloan, P. E. 1963. Determinants of bond yield differentials: 1954 to 1959. *Yale Economic Essays, Spring.*

DISCUSSION

LAWRENCE FISHER*: "Contingent Claims Analysis of Corporate Capital Structures: An Empirical Investigation" by E. Philip Jones, Scott P. Mason, and Eric Rosenfeld ("JMR") is an ambitious project that shows substantial use of formal analytical skills. First, JMR extend Merton's [5] contingent claims analysis (CCA) of a firm that has a single, callable debt issue to the case where the firm may have many such issues and where each issue may have sinking-fund requirements and be callable for redemption as a whole or for the sinking fund. Then they attempt to test the implications of their model for the prices of publicly traded debt securities against an alternative "naive" model that assumes there is no risk of default.

It seems to me that there are three crucial assumptions at the heart of Merton's CCA—three assumptions that may be required to make the model tractable but which may not be sufficiently true for the model to provide useful estimates of security prices. These are

1. Perfect liquidity (and irrelevance of firm size—cf. Fisher [3, 4])
2. Irrelevance of taxes
3. Nonstochastic interest rates

I had hoped that JMR would test one or more of these assumptions. If taken at face value, their empirical results would suggest that CCA takes account of risk in only a very rough and unsatisfactory manner. However, both the particular form of CCA model that JMR have used and their empirical methods are ill-suited for testing the three crucial assumptions. Hence, the applicability of Merton's CCA to corporate bond markets remains untested.

Formal Model

One of the so-called "standard" assumptions that the authors use—indeed, they show that it implies strange behavior by the firm because the value of a bond does not increase monotonically with the value of the firm—is needlessly unrealistic.[1] That is assumption A8, "Perfect [sic] anti-dilution protection," which

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[1] The assumption may have been entirely satisfactory in the study from which JMR took it.