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ABSTRACT

This paper develops the relation between the real rate of return on the stock market and changes in the price level using a multiperiod economy with production. The observed relation between real ex post stock returns and inflation is shown to be consistent with equilibrium in an economy with rational investors. The relation between expected real returns and expected inflation is shown to depend on the form of the economy's production function and on the form of investor preferences. When the production function exhibits stochastic constant returns to scale, the model explains the negative relation between expected real returns and expected inflation which has frequently been observed in empirical studies.

CLASSICAL INVESTMENT THEORISTS such as J. B. Williams [18] and Graham, Dodd, and Cottle [9] viewed inflation as a proportionate increase in all prices and incomes. As a consequence, the real rate of return on claims to production, such as common stocks, was thought to be unaffected by the rate of inflation. However, the empirical evidence of Bodie [2], Fama and Schwert [6], Jaffe and Mandelker [12], and Nelson [16] suggests that real stock returns are adversely affected by both the expected and unexpected components of the rate of inflation. This evidence is inconsistent with both the classical view and with the view (sometimes attributed to Irving Fisher) that all assets should be priced to reflect the expected component of the rate of inflation.

The empirical evidence has inspired further theoretical research on the link between stock returns and the rate of inflation. Modigliani and Cohn [15] contend that the low value of shares during periods of high inflation has resulted from the failure of investors to adjust corporate profits for the inflation premium component of interest expense (which, they argue, represents a return of capital rather than an expense), and from the capitalization of corporate profits at the nominal rate (rather than the theoretically correct real rate) of interest. Feldstein [7] concludes that the existing structure of the tax laws makes the share prices of nonfinancial corporations sensitive to changes in the expected rate of inflation. His analysis incorporates the effect of changes in the expected rate of inflation on the firm's net monetary asset position and the real value of the tax shields provided by depreciation, inventory costs, and the corporate deduction for interest expense.

The work of Feldstein and Modigliani-Cohn is characterized by a partial equilibrium framework in which the real rate of interest is assumed to be constant

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(although Feldstein explicitly recognizes that the risk differential between stocks and bonds is a function of the expected rate of inflation). Changes in the expected rate of inflation are a purely monetary phenomenon in each model in the sense that all prices and incomes are implicitly assumed to increase at the rate of inflation.

This paper will develop the relation between real stock returns and inflation in a multiperiod economy with production. Investor demand for fiat money, and hence the potential for inflation, will be introduced into the model through a "cash-in-advance" constraint similar to that discussed in Clower [3] and Stockman [17]. The manipulation of the money supply by a government agency in order to obtain control over the output of the economy gives inflation a direct effect on the asset pricing mechanism in the model. Thus, inflation is much more than a purely monetary phenomenom in this context. Since investors are assumed to value securities in real terms, the results which will be presented do not depend on any form of money illusion or other irrational behavior, such as that hypothesized by Modigliani and Cohn. In contrast to the assumption by Feldstein that the firm maintains a constant retention ratio (which implies that investment is unaffected by changes in the expected rate of inflation), both the level of investment and the retention ratio may vary with the current state of the economy.

A related model has been used by Grossman and Weiss [11] to study the effect of monetary disturbances when investors are subject to "unsystematic" productivity and money demand shocks. In contrast, the objective of this paper is to focus on the effect of nondiversifiable systematic shocks on the variability of real stock returns and inflation. The model abstracts from the effect of inflation on the firm's net monetary asset position by assuming that firms are financed solely by common equity in a world with no corporate income taxes. This allows attention to be focused on the correlation between inflation and the underlying real value of the firm's assets. As will be shown in Section III, the variation of the level of investment is an important determinant of the ex ante relation between stock returns and inflation.

Perhaps the most important implication of the model concerns the fundamental nature of the relation between real stock returns and inflation. While much previous work assumes that the rate of return on stocks is a dependent variable, which responds to variations in the expected and/or unexpected components of inflation, the present model emphasizes the joint dependence of both inflation and real stock returns on exogenous productivity and government policy stocks. The effect of consumption on stock prices in the model is similar in many respects to that described in Grossman and Shiller [10] and Long and Plosser [14]. The simultaneous effects of random shocks to the economy result in an associative type of relation between stock returns and inflation, rather than the causative relation which is implicit in previous work. This is consistent with the empirical evidence in Fama [5].

The basic model of a production economy with a stock market and a government sector is developed in Section I. The asset pricing relation is derived from the intertemporal consumption-investment decision of a representative investor in Section II. The actual or ex post real rate of return is shown to have an inverse relation to the ex post rate of inflation. In Section III, the model is extended to facilitate an analysis of the ex ante relation between real stock returns and inflation. The correlation between expected real returns and expected inflation is shown to be affected by investor preferences and the form of the production process. An example shows that a negative correlation between expected real returns and expected inflation is consistent with an equilibrium model in which investors have rational expectations.

I. The Model

Consider a single good economy in which output is a function of the stock of capital per capita, K_t, and a random shock to the production process, $\tilde{z}_t$. Per capita output, $\tilde{G}_t$, is expressed as

$$\tilde{G}_t = f(\tilde{z}_t, K_t) \tag{1}$$

with partial derivatives

$$f_k > 0, \qquad f_z > 0, \qquad f_{kk} < 0, \qquad f_{zz} < 0.$$ [1]

The economy's productive capacity is owned by a single firm, which in turn is owned by N identical investors each endowed with one share of stock.[2]

The capital stock at time t is assumed to be completely used up at the instant of production so that a portion of $\tilde{G}_t$ must be invested in order to restore the economy's productive capacity. Thus, output in the following period is generated by the current period's investment, denoted by K_{t+1}. Output is also consumed by investors and demanded by a government agency for transfer payments which have no direct effect upon the real value which investors attach to their shares in the firm.[3]

Cash balances are used to acquire all output. The firm purchases output for investment with earnings retained from the revenues of the preceding period. Investors purchase output for consumption with dividend income from their shares in the firm. The government agency purchases output using newly printed money (i.e., deficit spending).[4] All government debt is immediately monetized so that the relation between per capita money supply in periods $t-1$ and t is

$$M_t = (1 + g_t)M_{t-1}, \tag{2}$$

[1] For the case where $f(\tilde{z}, K) = \tilde{z}K$, the random shock can be interpreted as the productivity per unit of capital.

[2] A stock market consisting of a single firm endowed with the economy's productive capacity is equivalent to a model in which the distribution of capital across firms is given and identical investors hold the market portfolio.

[3] Government transfers of this type can be thought of as either foreign aid or transfers to investors who do not affect prices in the securities market. An alternative formulation which results in identical implications for asset pricing is that the government agency produces a public good whose production has no effect on the marginal utility of direct consumption.

[4] The omission of taxes from the model has no effect upon the results which follow. The effect of either a personal or corporate income tax on real stock returns would be similar to the effect of a government deficit. However, since these taxes would have no effect upon inflation, their inclusion has no bearing on the correlation between real stock returns and inflation.

where

M_t = the per capita money supply at time t,
g_t = the growth rate of the money supply between periods t and $t - 1$.

The government demand for output is random. The realizations of z and g are assumed to be contemporaneously and serially uncorrelated. This assumption implies that government policy is chosen without knowledge of the current shock to the production process. The random nature of policy can be attributed to random shocks to an exogenous policy making process.

The price level in this economy is determined by the Quantity Theory,

$$M_t v = P_t G_t, \tag{3}$$

where P_t is the price per unit of output at time t and v is the velocity of money. For notational convenience, the velocity of money is assumed to equal one.[5] This implies that the firm's revenues are always equal to the money supply.

At time t, the firm uses the revenues of the preceding period to pay a cash dividend of $(1 - r_t)M_{t-1}$ $(0 \le r \le 1)$ to the shareholders. Dividend income is then used by the shareholders to purchase output for private consumption, C_t. Each shareholder's consumption can be expressed as

$$P_t C_t = (1 - r_t)M_{t-1}. \tag{4}$$

The earnings retained by the firm, $r_t M_{t-1}$, are used to purchase output for investment purposes.

By definition, the quantity of output which can be financed by a government deficit of $(M_t - M_{t-1})$ is $\frac{g_t}{1 + g_t} f(z_t, K_t)$. The firm's retained earnings $(r_t M_{t-1})$ can be used to acquire $\frac{r_t}{1 + g_t} f(z_t, K_t)$ units of output. Since investors are identical and there is no trading in shares, consumption is constrained to equal the after tax purchasing power of dividend income. Consequently, private consumption can be expressed as

$$C_t = \left(1 - \frac{g_t}{1 + g_t} - \frac{r_t}{1 + g_t}\right) f(z_t, K_t), \tag{5}$$

which can be simplified to give

$$C_t = (1 - r_t)(1 + g_t)^{-1} f(z_t, K_t). \tag{6}$$

Substituting Equation (6) into (4) gives

$$P_t = \frac{M_{t-1}(1 + g_t)}{f(z_t, K_t)}. \tag{7}$$

The price level, P_t, is driven up if consumer demand for output exceeds output

[5] Although the seigniorage from the creation of money (i.e., a government of deficit) increases (decreases) for a velocity of money less (greater) than one, government deficits can always be thought of in terms of the magnitude necessary to purchase a desired fraction of output, given the price level implied by the velocity of money and a given nominal money supply.

available after government purchases (financed by deficit spending) and corporate retained earnings.

In the discussion which follows, the price relative of output in periods t and $t - 1$ will be referred to as the rate of inflation. From (7) and the fact that $M_t = (1 + g_t)M_{t-1}$, the rate of inflation at time t, I_t, is

$$I_t = \frac{P_t}{P_{t-1}} = \frac{f(z_{t-1}, K_{t-1})(1 + g_t)}{f(z_t, K_t)}. \tag{8}$$

The expected rate of inflation will refer to the expectation of the price relative of output in periods t and $t + 1$.

$$E(\tilde{I}_{t+1}) = f(z_t, K_t)E\left(\frac{1 + \tilde{g}_{t+1}}{f(\tilde{z}_{t+1}, K_{t+1})}\right). \tag{9}$$

In Section II, an asset valuation model is developed in which real stock returns are negatively related to the measure of inflation that has been developed.

II. Real Stock Returns and Unanticipated Inflation

The equilibrium real value of the firm, and thus the real rate of return, depends upon the firm's investment (dividend) decision. Since investors have been assumed to be identical, this decision is implicitly assumed to satisfy the first-order conditions for the intertemporal utility maximization problem of a representative investor.[6] Assume that individual preferences have the form

$$U = U(C_0) + \textstyle\sum_{t=1}^{L} \rho^t E(U(\tilde{C}_t))$$

with

$$U_c > 0, \qquad U_{cc} < 0$$

where

ρ = the subjective rate of time preference,
C_0 = current consumption,
$\tilde{C}_t$ = random consumption at time t,
L = a finite investment horizon.

Then upon observing z_t and g_t, the level of investment is the K_{t+1} which solves

$$J(K_t, z_t, g_t) = \underset{K_{t+1}}{\text{Max}}\ U(C) + \rho E(J(K_{t+1}, z_{t+1}, g_{t+1})) \tag{10}$$

subject to

$$C_t = \left[\frac{f(z_t, K_t)}{(1 + g_t)} - K_{t+1}\right], \tag{10a}$$

where J represents the indirect utility of wealth. Differentiating (10) with respect

[6] This approach is similar to that found in Brock [1] and Leland [13].

to K_{t+1} and rearranging gives[7]

$$1 = \frac{\rho E(J_K)}{U_c}. \tag{11}$$

Equation (11) is simply the marginal rate of substitution between current consumption and investment. This marginal rate of substitution determines the real present value of a unit of output which is reinvested in the firm. At the optimal level of investment (which implies the optimal value for r_t), this price is simply one unit of current consumption. Therefore, the real value of one share in the firm at time t can be expressed as

$$V_t = K_{t+1}. \tag{12}$$

The definition of the real value of the firm's current dividend and Equation (12) imply that the ex post real rate of return on a share in the firm can be expressed as

$$R_t = \frac{V_t}{V_{t-1}} + \frac{C_t}{V_{t-1}} = \frac{K_{t+1} + \left(\dfrac{f(z_t, K_t)}{(1 + g_t)} - K_{t+1}\right)}{K_t}. \tag{13}$$

Since $K_t = \dfrac{r_{t-1}}{1 + g_{t-1}} f(z_{t-1}, K_{t-1})$, Equation (8) can be used to rewrite (13) as

$$R_t = \frac{(1 + g_{t-1})}{r_{t-1}} \frac{1}{I_t}. \tag{14}$$

Equation (14) expresses the ex post real rate of return as a decreasing function of the current rate of inflation. Note that Equation (14) in no way implies that inflation is the causal factor in determining the ex post real return. Instead, both inflation and real returns are simultaneously determined by the current resolution of economic uncertainty concerning z and g. Theorem 1 proves that, as has been observed empirically, the current rate of inflation and the ex post real returns on securities are negatively correlated.

THEOREM 1. *The current rate of inflation and the ex post real rate of return are negatively correlated.*

Proof: In order to prove that I_t and R_t are negatively correlated, it is sufficient to show that their covariance is less than zero.

$$\begin{aligned} \mathrm{Cov}(\tilde{I}_t, \tilde{R}_t) &= E(\tilde{I}_t\tilde{R}_t) - E(\tilde{I}_t)E(\tilde{R}_t) \\ &= \frac{(1 + g_{t-1})}{r_{t-1}} \left[1 - E(\tilde{I}_t)E\left(\frac{1}{\tilde{I}_t}\right)\right] \end{aligned}$$

which is less than zero by Jensen's inequality. Q.E.D.

[7] The optimal level of investment must satisfy the first-order conditions for (10)

$$U_c \frac{\partial C_t}{\partial K_{t+1}} + \rho E(J_K) = 0,$$

where $\partial C_t/\partial K_{t+1}$ is derived from constraint (10a).

The intuition behind Theorem 1 is as follows. The greater the magnitude of the shock to the production process, the greater is the economy's total output. Since a portion of any increment in total output must accrue to the shareholders of the firm, either through real dividends or reinvestment (which results in a greater real value of shares in the firm), the real rate of return on shares will be positively correlated with shocks to the production process. The negative correlation of inflation and real stock returns arises because shocks to the production process have the opposite effect on the rate of inflation, which decreases as z_t increases, since the supply of output increases relative to the supply of money. Similar reasoning applies to the effects of deficit financing.

The literature on real stock returns and inflation often makes the distinction between expected and unexpected inflation. Equation (14) can be interpreted as a description of the relation between real stock returns and unexpected inflation. The implications of this description are consistent with the hypothesis tested empirically by Fama, which suggests that the negative relation of real returns and unexpected inflation is the result of a "proxy effect" for the more fundamental determinants of real equity values (e.g., shocks to the production process, or government deficits). The model which has been developed suggests a functional form for the relation between real stock returns and these fundamental economic variables. This functional form is of particular interest because it describes ex post real returns in terms of economic variables which are observable. This is in contrast to the work of Fama, in which real stock returns are explained, in part, by proxies for anticipated future economic activity.

Note that Equation (14) describes the real rate of return for an unlevered market portfolio. The addition of nominal contracting, in the form of debt and the tax shields from depreciation, to the model might imply that "unexpected inflation" would affect firms with net monetary asset positions differently than firms having net monetary liabilities.[8] However, this effect may well be obscured by the relation between inflation and the real value of the firm's assets. Thus, empirical tests which attempt to examine the cross-sectional effects of inflation on real stock returns should first adjust for the effect of current economic conditions (e.g., z and g) on the real value of the firm's assets. Equation (13), coupled with an appropriate specification of the economy's production process, can provide a tractable means by which to make this adjustment.

III. Expected Real Returns

The analysis of Section II was devoted to the relation between the rate of inflation and real stock returns which should be observed ex post. This relation can be thought of as the relation between "unanticipated" inflation and "unanticipated" real stock returns (e.g., Bodie or Fama and Schwert). One of the more interesting aspects of recent empirical evidence is the apparently negative association between expected inflation and expected real stock returns. This section uses the model developed earlier to show that the observed relation between expected real returns and inflation can be consistent with a securities market equilibrium in which investors behave rationally.

[8] This hypothesis has recently been tested by French et al. [8].

The relation between expected real returns and expected inflation will be analyzed by examining the simultaneous response of expected inflation and real returns to the arrival of unexpected information about the distribution of future economic events. This unexpected arrival of information will be modeled as a shift in the distribution of the random shock to the production process.[9] More precisely, assume that the distribution of z in the next period undergoes an unanticipated multiplicative shift of the form

$$\tilde{x} = (1 + h)\tilde{z}. \tag{15}$$

where $\tilde{x}$ denotes a random shock from the new distribution. In general, a shift in the distribution of $\tilde{z}$ will alter both the expected rate of inflation and the expected real rate of return. If unanticipated shifts in the distribution of $\tilde{z}$ cause the expected real rate of return to increase (decrease) and the expected rate of inflation to decrease (increase), then it can be said that there is a negative association between expected real returns and expected inflation. Using (13) and (15), the expected real rate of return when $h = 0$ (i.e., prior to the unanticipated shift in z) can be expressed as

$$E(\tilde{R}_{t+1}) = E\left(\frac{f(\tilde{x}_{t+1}, K_{t+1})}{(1 + \tilde{g}_{t+1})K_{t+1}}\right). \tag{16}$$

The response of expected real stock returns to new information is determined by differentiating (16) with respect to h and evaluating at $h = 0$.

$$\frac{\partial E(\tilde{R}_{t+1})}{\partial h} = E\left(\frac{1}{(1 + \tilde{g}_{t+1})K_{t+1}^2}\left\{K_{t+1}f_z\tilde{z}_{t+1} + K_{t+1}f_k\frac{\partial K}{\partial h} - f\frac{\partial K}{\partial h}\right\}\right). \tag{17}$$

The sign of Equation (17) is determined by the relative strength of the income and substitution effects caused by the change in the next period's investment opportunity. The net effect depends upon both the form of the production function, f, and the form of investor preferences. Thus, the relation of expected real returns and inflation is, in general, indeterminate. For pedagogical purposes, however, assume that the production function has the form

$$f(\tilde{x}, K) = (1 + h)\tilde{z}K. \tag{18}$$

Using (16) and (18), the expected real rate of return can be expressed as

$$E(\tilde{R}_{t+1}) = (1 + h)E\left(\frac{\tilde{z}_{t+1}}{1 + \tilde{g}_{t+1}}\right), \tag{19}$$

which is increasing in h. Thus, the expected real rate of return varies directly with a shift in the distribution of z.

The comparative statics response of the expected rate of inflation to unanticipated shifts in the distribution of z can be determined by substituting Equation (18) into (9) and differentiating with respect to h. Evaluating at $h = 0$ gives

$$\frac{\partial E(\tilde{I}_{t+1})}{\partial h} = \frac{-z_t K_t}{(1 + h)^2 K_{t+1}^2} E\left(\frac{1}{(1 + \tilde{g}_{t+1})}\left\{K_{t+1} + \frac{\partial K_{t+1}}{\partial h}\right\}\right). \tag{20}$$

[9] Similar effects result from a shift in the distribution of the growth rate of the money supply.

It is straightforward to show that

$$K_{t+1} + \frac{\partial K_{t+1}}{\partial h} > 0.^{10} \tag{21}$$

Consequently, the expected rate of inflation will vary inversely with h (i.e., Equation (20) is less than zero). The fact that expected real returns vary directly with h while expected inflation varies inversely implies that, in this example, there would be a negative association between expected real returns and expected inflation.

The negative relation between expected real returns and expected inflation arises, in this special case, because increases in the expected magnitude of the shock to the production process increase the expected real payoff from an investment in the firm. This results in a decline in the expected rate of inflation, since investors adjust their expectation of the supply of goods upward relative to the expected money supply. A similar explanation for the negative relation between real returns and unexpected inflation was given in Section II. In the case of expected real returns and expected inflation, however, the statistical relation depends upon the response of current investment to changing expectations about the course of future economic events.

For general preferences and a general specification of the production function (e.g., decreasing returns to scale), the correlation between changes in expected inflation and expected real returns is unclear. However, the special case examined here is of interest since it proves that the observed negative relation between expected real returns and expected inflation is consistent with at least one production function commonly used in financial models.[11] The special case shows that a negative correlation between changes in expected real returns and expected inflation is plausible in an economy with rational investors. In other words, under certain conditions, a negative relation between real returns and expected inflation is to be expected and is in no way anomalous. This result is consistent with the failure of Fama to completely explain the correlation of real stock returns and expected inflation with proxies for expected real activity.

IV. Conclusion

This paper has analyzed the relation between real stock returns and inflation in a multiperiod economy with production. Inflation was introduced by means of a "cash-in-advance" constraint on transactions and a government agency with the power to alter the economy's supply of fiat money. The model illustrates the fact that a negative correlation between inflation and real stock returns is consistent with equilibrium in a market with rational investors. This relation will hold ex post (i.e., between unanticipated inflation and real returns) and under certain specifications of the economy's production function ex ante (i.e., between expected real returns and inflation).

[10] Given (16) Equation (21) follows from totally differentiating (11) to obtain $\frac{\partial K_{t+1}}{\partial h}$ and substituting.

[11] See, for example, Diamond [4] or Leland [13].

The model has implications which extend beyond the relation between inflation and asset pricing. The important role which the form of the production process plays in the ex ante real return-inflation relation suggests that models which explicitly consider the consumption-investment decision may be useful in the study of the variability of asset prices. The smoothing of the intertemporal consumption pattern via the consumption-investment decision of the firm implies that the variability of the consumption stream may be less than that of total output. The use of a model which incorporates the role of production in determining share prices may well lead to the conclusion that the actual variability of asset prices is consistent with rational expectations and market efficiency.

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