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# Conditions for Myopic Valuation and Serial Independence of the Market Excess Return in Discrete Time Models

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**ABSTRACT**

In a multiperiod pure exchange world with investors displaying HARA-preferences, conditions for period-by-period application of one-period asset pricing models are derived first. The future investment opportunity set may be uncertain, provided that in every period a specific market portfolio variable depending on preferences is known as of the preceding date. This variable need not be completely deterministic.

Second, conditions for a serially independent market excess return are derived. These conditions render serial independence very unlikely. Hence estimation methods assuming serial independence are likely to yield biased results.

WHILE MULTIPERIOD capital asset pricing models generally involve intertemporal interdependencies in the pricing of assets, there exist conditions under which the multiperiod model becomes indistinguishable from a sequence of one-period models. If these conditions are met, then the price of an asset with cash flows at various dates can be derived by applying the one-period pricing model recursively period by period. The valuation process then is called quasi-myopic at every date.

The quasi-myopic valuation problem has been addressed by various authors in both continuous and discrete time contexts (see Bhattacharya [1], Constantinides [5], [6] and the references cited therein). Since this paper is confined to discrete time models, we shall mention explicitly only those previous analyses which are cast in discrete time. Stapleton and Subrahmanyam [19] derived an equilibrium model in which investors have constant absolute risk aversion, risk-free interest rates are completely deterministic, and all asset cash flows are multivariate normally distributed. The authors proved that the market price of risk in any period is completely deterministic so that myopic valuation obtains.

From an investor's point of view, the assumptions of Stapleton and Subrahmanyam are sufficient for the future investment opportunity set to be completely deterministic. Stapleton and Subrahmanyam considered a pure exchange model while Constantinides [5], [6] analyzed a model with endogenous production. He proved that quasi-myopic valuation can obtain even if the future investment opportunity set facing the individual investor is random. He assumed that the capital market is Pareto-efficient and that at every date the joint probability distribution of returns on all assets is exogenously given and independent of preceding events. Despite these strong assumptions, the market return distribu-

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tion does depend on preceding events, as does investment in different productive technologies. Moreover the risk-free interest rate may evolve stochastically over time.

The first purpose of this paper is to derive necessary and sufficient conditions for quasi-myopic valuation in a discrete time, pure exchange model without any assumptions about the distribution of asset returns. In order to keep the analysis tractable, conditions for aggregation are assumed to hold. More precisely, all investors are assumed to share the same beliefs and to have sufficiently similar time-additive or time-multiplicative HARA-preferences so that the market is Pareto-efficient. Thus the investigation of quasi-myopic valuation in this paper represents a considerable generalization of Stapleton and Subrahmanyam's work.

This paper derives the constraint imposed by quasi-myopic valuation on the future investment opportunity set. From an investor's point of view, this set may be summarized by a single variable which depends on preferences and on the market portfolio return. Constraints on returns of individual assets are irrelevant as every investor always buys a fraction of the market portfolio. Quasi-myopic valuation obtains at date $(t-1)$ if the future investment opportunity set-variable of date $t$ is known as of date $(t-1)$. However, this variable need not be completely deterministic, i.e., known as of date 0.

The precise nature of the constraint on the future investment opportunity set depends on preferences. If utility is time-additive, the constraint can be expressed in terms of aggregate wealth and consumption. With constant proportional risk aversion, quasi-myopic valuation at date $(t-1)$ obtains if the ratio of aggregate consumption over aggregate wealth of date $t$ is known as of date $(t-1)$. With constant absolute risk aversion, quasi-myopic valuation obtains if the ex-dividend market value of the market portfolio of date $t$ is known as of date $(t-1)$. No constraint is needed in the case of logarithmic utility because the logarithmic investor is a myopic investor.

The empirical application of a quasi-myopic valuation model is simplified if the distribution of the market excess return can be estimated easily. Serial independence of the market excess return facilitates this estimate considerably and is, in fact, usually assumed in empirical work. Therefore, the paper derives also the conditions for a serially independent market excess return. Rosenberg and Ohlson [13] have shown that a fundamental degeneracy obtains if the returns of all individual assets and of the market portfolio are required to be serially independent and stationary. This paper shows that even serial independence of the market excess return, disregarding other conditions, is very unlikely to be encountered in empirical tests. If, in addition, other conditions are imposed, then, a fortiori, there is hardly any hope of all conditions being fulfilled.

Serial independence of the market excess return obtains

—for constant proportional risk aversion only if the risk-free interest rates are completely deterministic,

—for constant absolute risk aversion only if the ex-dividend values of the market portfolio are completely deterministic,

—otherwise only if the risk-free interest rate is serially *dependent* and if, simultaneously, a certain function of risk-free interest rates is serially *independent.*

Therefore serial independence of the market excess return is very unlikely to exist.

Finally it will be shown that serial independence of the market excess return implies quasi-myopic valuation under constant proportional and constant absolute risk aversion and under logarithmic utility. Otherwise both are mutually exclusive. This points to the danger of making assumptions about both preferences *and* the properties of asset returns since the assumptions may be inconsistent.[1]

The paper is organized as follows. In Section I, the economic setting and some basic implications for equilibrium are presented. Section II discusses the conditions for quasi-myopic valuation, Section III those for serial independence of the market excess return, and Section IV the conditions for both.

## I. The Economic Setting

### A. Assumptions

The economic setting is described by the following assumptions.

(A1) The capital market is perfect and competitive; there are no taxes, no transaction and information costs, securities are infinitely divisible, short selling is not restricted, and investors act as price takers. All assets are traded in the capital market.

(A2) Every investor optimizes his investment consumption decisions at dates $0, 1, 2, \cdots, t, \cdots, T-1$. His utility depends on his real consumption at dates $0, 1, \cdots, T$. His utility function is state-independent. Relative commodity prices do not change over time.[2]

This assumption allows us to define all prices, consumption, dividends, and wealth in real terms. Let $\alpha_t$, $\beta_t$, $\gamma$, and $\rho_t$ denote parameters of an investor's utility function and $c_t$ denote his consumption at date $t$ $(t = 0, 1, \cdots, T)$.

(A3) The investor's utility is either

time-additive $$U = \textstyle\sum_{t=0}^{T} \dfrac{1-\gamma}{\gamma} U_t$$

or

time-multiplicative[3] $$U = \dfrac{1-\gamma}{\gamma} \textstyle\prod_{t=0}^{T} U_t$$

and $U_t$ belongs to the HARA-class $U_t = \rho_t\left(\alpha_t + \dfrac{\beta_t c_t}{1-\gamma}\right)^{\gamma}$. The parameters are specified without loss of generality in footnote 4.

[1] This corresponds to Brennan's finding [3] that in discrete time models under certain distributional assumptions a risk-neutral valuation relationship for a contingent claim can be assumed to exist only if risk aversion is constant proportional.

[2] In general, this is not sufficient to justify a state-independent utility function. See Laux and Schneeweiss [11].

[3] Time-multiplicative utility has been used by Grauer et al. [9], for example.

[4] For time-additive utility with

—finite $\gamma$, $0 \neq \gamma \neq 1 : \rho_t = 1, \beta_t > 0, \beta_0 = 1$;

—$\gamma = 0$ (logarithmic utility): $\rho_t > 0, \rho_0 = \beta_t = 1$;

—$\gamma = -\infty$ (exponential utility): $\rho_t > 0, \rho_0 = \alpha_t = 1, \beta_t > 0$.

Footnote 4 continues on following page.

(A4) Investors share the same beliefs and have sufficiently similar preferences so that a representative investor exists. The representative investor's initial wealth and his consumption and wealth in every state at every date equal $(1/I)$ times the corresponding aggregate figure. $I$ denotes the number of investors in the market.

This assumption restricts beliefs and parameters of utility functions across investors. Rubinstein [16] has shown for time-additive HARA-preferences that the capital market is Pareto-efficient if all investors share the same beliefs, the same $\gamma$, and the same $\beta$-structure $\beta_0 : \beta_1 : \cdots : \beta_T$. Then (a) every investor's consumption at any date is a completely deterministic function of aggregate consumption at that date and (b) he buys at date 0 a fraction of the market portfolio and keeps it until date $T$ (in the case of logarithmic utility this requires in addition that all investors share the same time preference $\rho_0 : \rho_1 : \cdots : \rho_T$; see [18, p. 33]). The same can be shown to hold for time-multiplicative utility. Thus a *representative* investor exists who buys at date 0 the fraction $(1/I)$ of the market portfolio (see [4], [15]). This allows us to focus on the market portfolio return without any distributional assumptions such as the assumption of a separating distribution [14].

(A5) In every state at every date aggregate consumption equals the exogenous dividend of the market portfolio, i.e., the net amount of money paid by all firms together to all investors.

This assumption is important for two reasons. First, since the dividend of the market portfolio (= aggregate dividend) is assumed to be exogenous, a pure exchange economy is considered. Second, (A5) rules out lending and borrowing on the aggregate level.[5] Thus, the representative investor consumes the fraction $(1/I)$ of the aggregate dividend. Any other investor, however, may consume the dividend which he receives and, in addition, part of his wealth.

By (A5), the essential exogenous input of the model is the stochastic process governing aggregate dividends. No assumption is made with regard to this process beyond some regularity assumptions. This raises the question of whether an arbitrary process is consistent with investors' intertemporal investment consumption optimization. The answer is yes. The investor's optimal investment consumption strategy depends on the stochastic processes which govern the market return and the risk-free interest rate. Both processes are determined by the equilibrium conditions and by the exogenous dividend process, hence both

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Footnote 4 continued from previous page.

For time-multiplicative utility with

—$\infty > \gamma > 1$ or $(T+1)^{-1} > \gamma > -\infty$, $\gamma \neq 0 : \rho_t = \beta_t = 1$;[†]
—$\gamma = -\infty$ (exponential utility): $\rho_t = \alpha_t = 1$, $c_t = 0$, $\beta_t = \beta_T$ $(t = 0, \cdots, T-1)$, $\beta_T > 0$.[‡]

[†] The restriction $\gamma \notin [(T+1)^{-1}, 1]$ is required by risk aversion. Multiplicative logarithmic utility ($\gamma = 0$) is excluded since it violates Proposition 1.

[‡] This utility function induces the same investment strategy as $U = -\prod_{t=0}^{T} \exp(-\beta_t c_t) = -\exp(-\sum_{t=0}^{T} \beta_t c_t)$ if $\beta_{t-1}/\beta_t$ equals the gross risk-free interest rate in period $t$. This condition is necessary to rule out profitable lending or borrowing of unlimited amounts. As the risk-free interest rate is not completely deterministic, $\beta_{t-1}/\beta_t$ cannot generally satisfy the condition. Therefore, I assume terminal wealth maximization $U = -\exp(-\beta_T c_T)$.

[5] The economy considered is either the world economy or an economy with a closed capital market.

processes are endogenous. If, e.g., investors desire to consume more than the aggregate dividend in some state, then market returns and risk-free interest rates adjust until the balance is achieved. This adjustment mechanism does not require any intertemporal optimality of exogenous aggregate dividends; any stochastic process of aggregate dividends is feasible. This type of model has been analyzed previously, especially by Rubinstein [16], [17].

### *B. The Representative Investor's Optimization*

Every investor faces a multiperiod investment consumption optimization problem. Given an initial endowment of wealth at date 0, he has to decide how much to consume at date 0, how much to invest in the market portfolio, and how much to invest in risk-free pure discount bonds of varying maturity. At every subsequent date $t$ ($t = 1, \cdots, T-1$) he faces the same decision problem, which depends on the state of nature which has obtained. This multiperiod decision problem has been solved by standard techniques of dynamic programming (see [8]).

For notational clarity, small letters denote individual variables and capital letters denote aggregate variables.

$w_t$ ≡ investor's (random) wealth before consumption at date $t$, $t = 0, \cdots, T$;
$c_t$ ≡ investor's (random) consumption at date $t$, $t = 0, \cdots, T$;
$v_t$ ≡ (random) amount invested at date $t$ in the market portfolio by the investor, $t = 0, \cdots, T-1$;
$I$ ≡ the number of investors;
$R_t$ ≡ 1 + risk-free interest rate between dates $(t-1)$ and $t$ (= period $t$), $t = 1, \cdots, T$;
$R_{Mt}$ ≡ 1 + random return of the market portfolio in period $t$;
$r_{Mt}$ ≡ $R_{Mt} - 1$;
$R_{Et}$ ≡ random market excess return in period $t$, $R_{Et} \equiv R_{Mt} - R_t$;
$C_t$ ≡ aggregate consumption at date $t$, $t = 0, \cdots, T$;
$D_t$ ≡ dividend of the market portfolio at date $t$, $D_t = C_t$, $t = 0, \cdots, T$;
$W_t$ ≡ cum-dividend market value of the market portfolio at date $t$, $t = 0, \cdots, T$, without loss of generality $W_T \equiv C_T$;
$V_t$ ≡ ex-dividend market value of the market portfolio at date $t$, $t = 0, 1, \cdots, T-1$, $V_t = W_t - D_t = W_t - C_t$;
$\emptyset_t$ ≡ information available at date $t$; it reveals the state of nature which has obtained at date $t$, $t = 0, \cdots, T$.

The representative investor's optimization problem can be stated as follows. Choose the investment consumption strategy which maximizes $E[U(c_0, c_1, \cdots, c_T) \mid \emptyset_0]$, subject to $c_T \equiv w_T$, given the initial endowment $w_0^0$, $c_t^0 = 0$, $t = 0, \cdots, T$.

The following proposition is not new. Based on the pioneering work of Hakansson [10], Rubinstein [16] has derived the results for time-additive preferences. As Rubinstein's paper is not published and his results simplify considerably for a representative investor, the proof of the proposition is given in Appendix 1. The proposition can be stated more easily with the following symbols.

—In the case of non-exponential utility ($\gamma > -\infty$):

$b_t \equiv$ date $t$-market value of a risk-free bond which pays

$\alpha_\tau(1-\gamma)/\beta_\tau$ at date $\tau$, $\tau = t, t+1, \cdots, T$; $t = 0, 1, \cdots T$;

$B_t \equiv Ib_t$; $\alpha_t^* \equiv I\,\alpha_t$; $\beta_t^* \equiv \beta_t$.

—In the case of exponential utility ($\gamma = -\infty$):

$$b_t \equiv \frac{1-\gamma}{\beta_t} \quad \text{so that} \quad b_t \frac{\beta_t}{1-\gamma} = 1;$$

$$B_t \equiv \frac{1-\gamma}{\beta_t^*} \quad \text{so that} \quad B_t \frac{\beta_t^*}{1-\gamma} = 1;$$

$$\alpha_t \equiv \alpha_t^* \equiv 1; \quad \beta_t^* \equiv \beta_t/I.$$

The intertemporal nature of the investment consumption problem is revealed by the Future Investment Opportunity Set (FIOS)-variable $\xi_t$. The solution assumes an equilibrium in the capital market so that the representative investor neither lends nor borrows money from other investors.

*Proposition* 1: (a) In equilibrium the indirect utility function of the representative investor may be written in the case of time-additive utility as:

$$U_t^{in} = \frac{1-\gamma}{\gamma}\,\rho_{t-1}\left(\alpha_{t-1} + \frac{\beta_{t-1}c_{t-1}}{1-\gamma}\right)^\gamma + \frac{1-\gamma}{\gamma}\rho_t\left(b_t\,\frac{\beta_t}{1-\gamma} + \frac{\beta_t}{1-\gamma}w_t\right)^\gamma \xi_t,$$

$$t = 1, \cdots, T. \quad (1a)$$

This equation holds also for time-multiplicative, exponential utility with $c_{t-1} \equiv 0$ for $t = 1, \cdots, T$. In the case of time-multiplicative, non-exponential utility ($\beta_t \equiv 1$; $\bar{t} \equiv T - t + 1$)

$$U_t^{in} = \frac{1-\gamma}{\gamma}\left(\alpha_{t-1} + \frac{c_{t-1}}{1-\gamma}\right)^\gamma \left(\frac{b_t}{1-\gamma} + \frac{w_t}{1-\gamma}\right)^{\bar{t}\gamma} \xi_t, \qquad t = 1, \cdots, T. \quad (1b)$$

(b) For all utility functions, optimal investment in the market portfolio requires

$$E\left[\left(B_t\,\frac{\beta_t^*}{1-\gamma} + \frac{\beta_t^*}{1-\gamma}\,R_{Mt}V_{t-1}\right)^{\theta\gamma-1} R_{Et}\xi_t \,|\, \emptyset_{t-1}\right] = 0, \qquad t = 1, \cdots, T, \quad (2)$$

with $\theta \equiv \bar{t}$ in the case of time-multiplicative, non-exponential utility and $\theta \equiv 1$ otherwise.

(c) In the case of time-additive utility, for $t = 1, \cdots, T-1$,

$$\xi_t \text{ is a constant if } \gamma = 0 \text{ (logarithmic utility)}, \quad (3a)$$

and

$$\xi_t \equiv \left(\alpha_t^* + \frac{\beta_t^* C_t}{1-\gamma}\right)^{\gamma-1}\left(B_t\,\frac{\beta_t^*}{1-\gamma} + \frac{\beta_t^*}{1-\gamma}\,W_t\right)^{1-\gamma} \quad \text{if } \gamma \neq 0. \quad (3b)$$

In the case of time-multiplicative utility, for $t = 1, \cdots, T-1$,

$$\xi_t \equiv E[\exp(-\beta_T^* r_{M,t+1} W_t)\xi_{t+1} \mid \emptyset_t] \quad \text{if } \gamma = -\infty, \tag{3c}$$

$$\xi_t \equiv E\left[\left(\frac{B_{t+1} + R_{M,t+1} V_t}{B_t - (1-\gamma)\alpha_t^* + V_t}\right)^{(\bar{t}-1)\gamma} \xi_{t+1} \mid \emptyset_t\right] \quad \text{if } \gamma > -\infty. \tag{3d}$$

In all cases $\xi_T \equiv 1$.

This proposition is proved in Appendix 1. The investor's indirect utility function and hence his decisions in period $t$ depend on future events only by the FIOS-variable $\xi_t$. This dependence does not exist in the case of logarithmic utility, as is well known; the logarithmic investor is myopic.

$\xi_t$ is a dimensionless variable which summarizes the FIOSs of all successive periods. More precisely, $\xi_t$ determines together with other factors the marginal utility of wealth at date $t$, given *optimal* decisions in the following periods. Therefore $\xi_t$ summarizes the *optimally exploited* FIOSs. As the representative investor holds a fraction of the market portfolio, $\xi_t$ depends on the market portfolio, but not on individual assets included in the market portfolio. The dependence of $\xi_t$ on the utility function is easily seen from Equations (3a)–(3d).

## II. Conditions for Myopic Valuation

This section derives, first, a necessary and sufficient condition for quasi-myopic valuation and, second, interprets this condition with respect to the admissible uncertainty in FIOSs. By definition, quasi-myopic valuation exists if a one-period pricing model can be applied recursively period by period to derive the price of an asset at date 0. A one-period pricing model is defined as a model which explains an asset's price at date $(t-1)$ by its random price (including dividends, etc.) at date $t$ and the random marginal social utility of wealth at date $t$ with the latter being a function of aggregate wealth, but not of the FIOSs. Thus, the one-period pricing model prices assets disregarding any information on the FIOSs.[6]

Given the assumptions of this paper, the equilibrium excess return, $R_{Et}$, and hence the equilibrium market value of the market portfolio, $V_{t-1}$, are determined by Equation (2) which may be rewritten as

$$E\left[\left(B_t \frac{\beta_t^*}{1-\gamma} + \frac{\beta_t^*}{1-\gamma} W_t\right)^{\theta\gamma-1} R_{Et}\xi_t \mid \emptyset_{t-1}\right] = 0, \qquad t = 1, \cdots, T. \tag{2$^+$}$$

This equation determines equally the equilibrium excess return $R_{it}$ of any asset $i$, if $R_{Et}$ is replaced by $R_{it}$. Since Equation ($2^+$) uses information on the FIOS-variable, a one-period pricing model is obtained from this equation if and only if $\xi_t$ is known as of date $(t-1)$. Then $\xi_t$ is irrelevant for the pricing of assets. As $\xi_t$ is a constant for logarithmic utility, this proves

*Proposition* 2: Quasi-myopic valuation exists if and only if the FIOS-variable

[6] A broader definition of one-period pricing models is that an asset's price at date $(t-1)$ is explained by its random price at date $t$ and the random prices of other assets at date $t$. Under this definition the FIOSs may affect the pricing of assets provided there exists a one-to-one correspondence between the FIOS-variable and the asset prices at date $t$.

$\xi_t$ is known as of date $(t-1)$, $\forall t = 1, \cdots, T-1$. It always exists with logarithmic utility.

The important point is that $\xi_t$ need not be completely deterministic, i.e., known as of date 0. Constantinides [5], [6] arrives at the same conclusion in his model which is based on quite different assumptions. It suffices that $\xi_t$ be known as of date $(t-1)$ so that $\xi$ may evolve stochastically over time.

Before discussing the implications of this finding, the relation between quasi-myopic valuation and portfolio choice will be shown. Suppose an investor derives his optimal investment consumption strategy by dynamic programming as stated in Proposition 1. Then his decisions are called myopic if $\xi_t$ is a constant (because of logarithmic utility) or if $\xi_t$ is completely deterministic (because of a special, preference-dependent stochastic process governing rates of return). The investor's decisions are called here quasi-myopic if $\xi_t$ is known as of the preceding date. Then $\xi_t$ acts as a multiplicative constant in the indirect utility function of date $(t-1)$ such that this constant may depend on the available information $\emptyset_{t-1}$. Given these definitions, myopic decisions are quasi-myopic, but not vice versa. Proposition 2 then states that quasi-myopic valuation exists if and only if the representative investor's decisions are quasi-myopic.

Proposition 2 will be discussed now. First, consider time-additive utility. As quasi-myopic valuation does not require $\xi_t$ to be completely deterministic, aggregate consumption is not required to be a completely deterministic function of aggregate wealth (Equation (3b)). It is not even generally true that the function relating aggregate consumption and aggregate wealth at date $t$ must be known as of date $(t-1)$. Suppose $\xi_t$ is known as of date $(t-1)$ so that $\xi_t \mid \emptyset_{t-1} = \bar{\xi}_t$. Then, by Equation (3b),

$$B_t + W_t = \bar{\xi}_t^{1/(1-\gamma)}[\alpha_t^*(1-\gamma)/\beta_t^* + C_t] \quad \text{for } \gamma > -\infty. \tag{4}$$

If social risk aversion is neither constant absolute nor constant proportional, then $B_t$ is the market value of a bond which depends on future risk-free interest rates. Thus, $B_t$ is not known as of date $(t-1)$ if risk-free interest rates are stochastic. Consequently, given the information $\emptyset_{t-1}$, aggregate consumption $C_t$ is a random function of aggregate wealth $W_t$. The random element disappears if social risk aversion is constant proportional or constant absolute.

A testable implication of Equation (4) is that the inhomogeneous rate of consumption $[\alpha_t^*(1-\gamma)/\beta_t^* + C_t]/[B_t + W_t]$ is independent of the market return in period $t$. If social risk aversion is constant proportional, then the consumption rate $C_t/W_t = D_t/W_t = D_t/(V_t + D_t)$ must be known as of date $(t-1)$. Hence, any information on events at dates $t, \cdots, T$ which becomes available in period $t$ and affects the ex-dividend value of the market portfolio at date $t$, must be accompanied by a corresponding change in aggregate dividends at date $t$. This requirement is empirically questionable. If, for instance, large oil reserves are discovered at the end of period $t$, then $V_t$ will increase although the immediate dividend $D_t$ is unlikely to increase.[7] Possibly these discrepancies between $D_t$ and $V_t$ tend to offset each other if the length of the period increases together with the number

[7] If $D_t$ remains the same, the risk-free interest rate will increase so that every investor still achieves optimal intertemporal consumption.

of pieces of information which partly increase and partly reduce $D_t$ relative to $V_t$.

With constant absolute risk aversion, $\xi_t = \exp(\beta_t^* V_t)$ is known as of date $(t - 1)$ if and only if the ex-dividend value of the market portfolio at date $t$ is. This condition is highly counterintuitive. Therefore empirical evidence rules out quasi-myopic valuation under time-additive, exponential utility.

This result seems to contradict the findings of Stapleton and Subrahmanyam [19]. The difference is explained, however, by different assumptions. Stapleton and Subrahmanyam do not require that aggregate consumption equal aggregate dividend; in their model, money can be borrowed or lent from outside. Hence $V_t$ need not be known as of the preceding date in their model.

Second, consider time-multiplicative utility. The recursive form of Equations (3c) and (3d) suggests an autocorrelation test to find out if $\xi_t$ is known as of date $(t - 1)$. Define $q_{t+1} \equiv \exp(-\beta_T^* r_{M,t+1} W_t)$ if utility is exponential and $q_{t+1} \equiv [B_{t+1} + R_{M,t+1} V_t]^{(\bar{t}-1)\gamma}[B_t - (1 - \gamma)\alpha_t^* + V_t]^{(1-\bar{t})\gamma}$ if utility is non-exponential. Hence, $q_{t+1} \equiv R_{M,t+1}^{(\bar{t}-1)\gamma}$ if risk aversion is constant proportional.

From the definition of $\xi_t$, it follows that $\xi_t = E[q_{t+1}\xi_{t+1} \mid \emptyset_t] = E[q_{t+1}q_{t+2} \cdots q_T \mid \emptyset_t]$. If $q$ has positive autocorrelation, then $\xi_t$ cannot be known as of date $(t - 1)$ and thus rules out quasi-myopic valuation. In order to see this, suppose that $q_t$ turns out to be high, i.e., $q_t \mid \emptyset_t$ is high. Hence, $E[q_{t+1}q_{t+2} \cdots q_T \mid \emptyset_t] = \xi_t$ is high because of positive autocorrelation. Conversely, $\xi_t$ is low if $q_t \mid \emptyset_t$ is low. As $q_t$ is not known as of date $(t - 1)$, $\xi_t$ is not known as of that date, thus ruling out quasi-myopic valuation. Hence, quasi-myopic valuation is inconsistent with empirical evidence of positive autocorrelation in $q$.

## III. Conditions for a Serially Independent Market Excess Return

This section derives necessary conditions for serial independence of the market excess return (SIMR). The market excess return $R_{Et}$ is defined as the difference between the return of the market portfolio $R_{Mt}$ and the risk-free interest rate $R_t$. SIMR exists if and only if the market excess return $R_{Et}$ is distributed independently of the information $\emptyset_{t-1}$, available at the beginning of period $t - 1$, $\forall t = 2, \cdots, T$. The issue of serial independence should not be confused with the issue of information efficiency (see also Lucas [12, p. 1444]). The assumptions of this paper are consistent with information efficiency, although the market excess return is serially dependent in general.

SIMR derives its importance from tests and applications of one-period pricing models. Although these models do not require SIMR, tests and empirical applications are greatly simplified by SIMR. Most empirical work, in fact, assumes SIMR. Therefore, this section analyzes the conditions for SIMR and finds that they are very unlikely to be satisfied.

*Proposition* 3: (a) In the case of constant proportional risk aversion ($\alpha_t^* = 0$, $\gamma > -\infty$), SIMR exists if and only if the growth rate of aggregate consumption is serially independent. A necessary condition is that the risk-free interest rates and the FIOS-variables be completely deterministic, i.e., known as of date 0.

(b) In the case of constant absolute risk aversion, a necessary condition for

SIMR is that the ex-dividend values of the market portfolio be completely deterministic.

(c) In the other cases ($\alpha_t^* \neq 0$, $\gamma > -\infty$), necessary conditions for SIMR are that the risk-free interest rate be random and serially *dependent* and that, simultaneously, aggregate wealth, normalized to zero mean and unit variance, depend on the stochastic process of risk-free interest rates in subsequent periods only, such that normalized aggregate wealth is serially *independent.* The FIOS-variable $\xi_t$ must not be known as of date $(t - 1)$ unless utility is logarithmic.

Proposition 3 is proved in Appendix 2. Proposition 3 reveals that SIMR is very unlikely to exist. It is well known that in the case of constant proportional risk aversion, serial independence of the growth rate of aggregate consumption implies completely deterministic risk-free interest rates. Proposition 3(a) is discomforting, however, since it shows that completely deterministic risk-free interest rates are a necessary condition for SIMR. Empirical evidence clearly refutes the hypothesis that risk-free interest rates are completely deterministic. This evidence suggests that SIMR and constant proportional risk aversion are conflicting hypotheses.

Proposition 3(b) is even more discouraging. SIMR obtains under exponential utility only if all future ex-dividend values of the market portfolio are completely deterministic. This condition is completely unrealistic.[8,9]

Proposition 3(c) states necessary conditions for SIMR if risk aversion is neither constant proportional nor constant absolute. SIMR rules out serial independence of the risk-free interest rate. A fortiori, SIMR rules out completely deterministic risk-free interest rates. Uncertainty in all periods is preserved only if risk-free interest rates are random as can be seen from Appendix 2. At the same time the normalized wealth variable must be serially independent, although it depends only on the stochastic process of risk-free interest rates in subsequent periods. Hence, the possibility of preserving uncertainty in all periods and SIMR appears to be very limited. In short, there is very little chance of observing SIMR.

## IV. Myopic Valuation and Serial Independence of the Market Excess Return

Since SIMR derives its importance from tests and empirical applications of the one-period pricing models, it is natural to ask whether the conditions for quasi-myopic valuation are consistent with those for SIMR. From Propositions 2 and 3, Proposition 4 follows immediately.

[8] The following example illustrates this condition for the last period. Consider a single competitive technology with constant returns to scale, the return being serially independent. The market return is thus exogenous and serially independent. Equation (2) yields $E[\exp(-\beta_T^* R_{MT} V_{T-1}) R_{MT} \mid \emptyset_{T-1}] = R_T E[\exp(-\beta_T^* R_{MT} V_{T-1}) \mid \emptyset_{T-1}]$. If $V_{T-1} \rightarrow 0$, $R_T \rightarrow E[R_{MT} \mid \emptyset_{T-1}]$. The higher $V_{T-1}$, the riskier is terminal wealth and hence the smaller $R_T$. Thus, a change in the investment $V_{T-1}$ changes the risk-free interest rate and consequently the market excess return. SIMR, therefore, obtains if and only if $V_{T-1}$ is completely deterministic.

[9] In the case of time-multiplicative, exponential utility a completely deterministic $V_{T-1}$ rules out all uncertainty in periods $1, \cdots, T - 1$ since consumption is zero at these dates. Hence the model essentially collapses to a one-period model.

*Proposition* 4: (a) SIMR implies quasi-myopic valuation if utility is logarithmic or if social risk aversion is constant proportional or constant absolute.

(b) Otherwise, SIMR and quasi-myopic valuation are mutually exclusive.

Proposition 4 shows that SIMR is not always sufficient for quasi-myopic valuation. If it is not, then SIMR and quasi-myopic valuation cannot exist simultaneously.

In the following, Proposition 4 will be briefly related to the findings of Constantinides [5], [6] and Cornell [7]. Constantinides analyzes a model with competitive production in which the rates of return are exogenous and follow a state-independent separating distribution. In addition, he assumes that a representative investor [5] exists or that the capital market is complete [6]; hence, the capital market is Pareto-efficient. Constantinides shows that the SLM-model holds at every date, although the market return and the risk-free interest rate may be state-dependent. This dependence follows from state-dependent investments in different technologies (assets). As a consequence, the market price of risk and the covariance of an asset's cash flow with the market return are state-dependent in general. Therefore, it becomes very difficult to apply the SLM-model recursively period by period. This state-dependence disappears if investment in different technologies is state-independent. But, as Constantinides [5, p. 84] notes, state-independent investments occur only under special conditions such as constant proportional risk aversion.

Proposition 4 derives a similar conclusion. Based on weaker assumptions, it shows that the empirical application of one-period pricing models can be simplified by SIMR only under special preferences.

One route to escape this negative result is to replace the wealth based one-period pricing model (CAPM) by the consumption based CAPM as suggested by Breeden [2]. In a world where social marginal utility depends on aggregate consumption only, implicit prices for state-contingent claims depend on aggregate consumption only. With time-multiplicative, non-exponential utility, however, implicit prices for claims, due at date $t$, depend on $C_t$, $C_{t+1}$, $\cdots$, $C_T$. This rules out, in general, the consumption based CAPM with implicit prices depending on $C_t$ only. With time-additive preferences and assumptions as stated in this paper, the consumption based CAPM always holds. Thus, without additional assumptions, it can be applied recursively period by period.

As Cornell [7] points out correctly, however, the parameters of the consumption based CAPM are state-dependent in general, so that empirical estimates are also hard to obtain. This is also true for the model presented in this paper. Therefore, additional conditions are needed if the parameters of the consumption based CAPM are to be estimated by simple methods.

## V. Conclusion

This paper has analyzed the conditions for quasi-myopic valuation and serial independence of the market excess return in a Pareto-efficient capital market with HARA-preferences. Quasi-myopic valuation requires that the FIOS-variable be known as of the preceding date. This constraint is counterintuitive in the case of exponential utility.

Serial independence of the market excess return imposes very strong conditions on the ex-dividend value of the market portfolio or on risk-free interest rates. These conditions are very unlikely to be satisfied by observable data. Quasi-myopic valuation and serial independence cannot obtain simultaneously unless utility is logarithmic or risk aversion is constant proportional or constant absolute.

These findings suggest two avenues for further research. (1) It may prove fruitful to enlarge the class of one-period pricing models such that all cases are included where a one-to-one correspondence exists between the asset prices and the FIOS-variable of the same date. (2) Empirical work should start from weaker assumptions than SIMR. Econometrics permits us to deal with certain types of intertemporal interdependencies.

## Appendix 1

### Proof of Proposition 1

Equations in the Appendix will be denoted by "*" to distinguish them from those in the text.

The representative investor's sequential, state-contingent optimization problem is given by Max $E[U(c_0, c_1, \cdots, c_T) \mid \emptyset_0]$ subject to $c_T = w_T$, given the initial endowment $w_0^0 = W_0^0/I$, $c_t^0 = 0$, $t = 0, \cdots, T$. Wealth dynamics are given by

$$w_t = (w_{t-1} - c_{t-1} - v_{t-1})R_t + R_{Mt}v_{t-1}. \tag{1*}$$

This problem is solved by recursive dynamic programming.

1) First, consider non-exponential utility. Then $U_t(c_t) = \rho_t(\alpha_t + \beta_t c_t/(1-\gamma))^\gamma$ with $\gamma > -\infty$. The solution is considerably simplified if this optimization problem is transformed into another one which yields the same allocation of consumption across investors so that the original and the transformed economy are supported by the same asset prices. Define $\check{c}_t$ by

$$\alpha_t + \frac{\beta_t c_t}{1-\gamma} = \frac{\beta_t}{1-\gamma}\left[\frac{\alpha_t}{\beta_t}(1-\gamma) + c_t\right] \equiv \frac{\beta_t}{1-\gamma}\check{c}_t, \qquad t = 0, \cdots, T. \tag{2*}$$

Thus, $\check{c}_t$ is defined as consumption $c_t$ plus the constant amount $\alpha_t(1-\gamma)/\beta_t$. As wealth $w_t$ is the market value of $(c_t, c_{t+1}, \cdots, c_T)$, define $\check{w}_t \equiv b_t + w_t$ with $b_t$ being the date $t$-market value of a risk-free bond which pays $\alpha_\tau(1-\gamma)/\beta_\tau$ at date $\tau$, $\tau = t, t+1, \cdots, T$. As $\alpha_t \equiv \alpha_t^*/I$ with $\alpha_t^*$ being the sum of $\alpha_t$ over all investors and $\beta_t$ and $\gamma$ are the same for all investors, aggregation yields with $b_t I \equiv B_t$

$$\check{C}_t = \frac{\alpha_t^*(1-\gamma)}{\beta_t} + C_t, \tag{3*}$$

$$\check{W}_t = B_t + W_t, \tag{4*}$$

$$\check{V}_t \equiv \check{W}_t - \check{C}_t = B_t - \alpha_t^*(1-\gamma)/\beta_t + V_t, \tag{5*}$$

since $W_t - C_t = V_t$.

The transformed optimization problem of the representative investor is defined by Max $E[U(\check{c}_0, \check{c}_1, \cdots, \check{c}_T) \mid \emptyset_0]$ subject to $\check{c}_T \equiv \check{w}_T$, given the initial endowment $\check{w}_0^0 \equiv \check{W}_0^0/I$, $\check{c}_t^0 = 0$, $t = 0, \cdots, T$. $U_t(\check{c}_t) = [\beta_t \check{c}_t/(1-\gamma)]^\gamma$. Wealth dynamics are given by

$$\check{w}_t = (\check{w}_{t-1} - \check{c}_{t-1} - \check{v}_{t-1})R_t + \check{R}_{Mt}\check{v}_{t-1} \tag{6*}$$

with $\check{v}_t$ being the investor's investment in the transformed market portfolio.

$$\check{R}_{Mt} \equiv \check{W}_t/\check{V}_{t-1} \tag{7*}$$

is the transformed market portfolio return.

1.1) In the case of time-additive, non-exponential utility

$$U = \textstyle\sum_{t=0}^T \frac{1-\gamma}{\gamma}\rho_t\left(\frac{\beta_t}{1-\gamma}\check{c}_t\right)^\gamma.$$

The existence of an indirect utility function for period $t$, as defined in (8*), has to be shown.

$$U_t^{in} = \frac{1-\gamma}{\gamma}\rho_{t-1}\left(\frac{\beta_{t-1}\check{c}_{t-1}}{1-\gamma}\right)^\gamma + \frac{1-\gamma}{\gamma}\rho_t\left(\frac{\beta_t\check{w}_t}{1-\gamma}\right)^\gamma\xi_t, \qquad t = 1, \cdots, T. \tag{8*}$$

a) $\check{w}_T \equiv \check{c}_T$ and $\xi_T \equiv 1$. Hence, the indirect utility function for period $T$ is identical with the last two terms of the original utility function. Therefore, it remains to be shown that the existence of $U_t^{in}$ implies the existence of $U_{t-1}^{in}$ for every $t = 2, \cdots, T$.

b) First, the optimal investment $\check{v}_{t-1}$ in the transformed market portfolio will be derived. Insert (6*) in (8*), take expectations, and obtain

$$E[U_t^{in} \mid \emptyset_{t-1}] = \frac{1-\gamma}{\gamma}\rho_{t-1}\left(\frac{\beta_{t-1}}{1-\gamma}\check{c}_{t-1}\right)^\gamma$$
$$+ \frac{1-\gamma}{\gamma}\rho_t\left(\frac{\beta_t}{1-\gamma}\right)^\gamma E[([\check{w}_{t-1} - \check{c}_{t-1} - \check{v}_{t-1}]R_t + \check{R}_{Mt}\check{v}_{t-1})^\gamma\xi_t \mid \emptyset_{t-1}].$$

Differentiate with respect to $\check{v}_{t-1}$ and obtain the optimum condition

$$E[([\check{w}_{t-1} - \check{c}_{t-1} - \check{v}_{t-1}]R_t + \check{R}_{Mt}\check{v}_{t-1})^{\gamma-1}\check{R}_{Et}\xi_t \mid \emptyset_{t-1}] = 0 \tag{9*}$$

with $\check{R}_{Et} \equiv \check{R}_{Mt} - R_t$. The condition for optimal consumption is

$$\rho_{t-1}\frac{\beta_{t-1}}{1-\gamma}\left(\frac{\beta_{t-1}}{1-\gamma}\check{c}_{t-1}\right)^{\gamma-1}$$
$$= \rho_t\left(\frac{\beta_t}{1-\gamma}\right)^\gamma R_t E[([\check{w}_{t-1} - \check{c}_{t-1} - \check{v}_{t-1}]R_t + \check{R}_{Mt}\check{v}_{t-1})^{\gamma-1}\xi_t \mid \emptyset_{t-1}]. \tag{10*}$$

In equilibrium the representative investor neither borrows nor lends at the risk-free rate so that $\check{w}_{t-1} - \check{c}_{t-1} - \check{v}_{t-1} = 0$. Therefore, (9*) yields

$$E[(\check{R}_{Mt}\check{v}_{t-1})^\gamma\xi_t \mid \emptyset_{t-1}] = R_t\check{v}_{t-1}E[(\check{R}_{Mt}\check{v}_{t-1})^{\gamma-1}\xi_t \mid \emptyset_{t-1}]. \tag{11*}$$

Insert (11*) in (10*) and obtain for $\gamma \neq 0$

$$\rho_{t-1}\left(\frac{\beta_{t-1}}{1-\gamma}\,\check{c}_{t-1}\right)^{\gamma} = \rho_t\,\frac{\check{c}_{t-1}}{\check{v}_{t-1}}\left(\frac{\beta_t}{1-\gamma}\right)^{\gamma} E[(\check{R}_{Mt}\check{v}_{t-1})^{\gamma}\xi_t \mid \emptyset_{t-1}]. \tag{12*}$$

Insert (12*) in $E[U_t^{in} \mid \emptyset_{t-1}]$ and obtain for $\gamma \neq 0$

$$E[U_t^{in} \mid \emptyset_{t-1}] = \frac{1-\gamma}{\gamma}\,\rho_{t-1}\left(\frac{\beta_{t-1}}{1-\gamma}\,\check{w}_{t-1}\right)^{\gamma}\xi_{t-1}$$

with

$$\xi_{t-1} \equiv \left(\frac{\check{c}_{t-1}}{\check{w}_{t-1}}\right)^{\gamma-1} = \left(\frac{\check{C}_{t-1}}{\check{W}_{t-1}}\right)^{\gamma-1} = \frac{\left(\alpha_t^* + \dfrac{\beta_t}{1-\gamma}\,C_t\right)^{\gamma-1}}{\left(B_t\,\dfrac{\beta_t}{1-\gamma} + \dfrac{\beta_t}{1-\gamma}\,W_t\right)^{\gamma-1}} \tag{13*}$$

as in equilibrium $\check{c}_{t-1} = \check{C}_{t-1}/I$ and $\check{w}_{t-1} = \check{W}_{t-1}/I$. Hence, an indirect utility function exists for $\gamma \neq 0$. This is true also for the logarithmic function ($\gamma = 0$) but with $\xi_{t-1}$ being a constant (Rubinstein [18]).

1.2) Consider time-multiplicative, non-exponential utility. The indirect utility function is

$$U_t^{in} = \frac{1-\gamma}{\gamma}\left(\frac{\check{c}_{t-1}}{1-\gamma}\right)^{\gamma}\left(\frac{\check{w}_t}{1-\gamma}\right)^{\bar{t}\gamma}\xi_t, \qquad t = 1, \cdots, T, \quad \bar{t} \equiv T - t + 1.$$

Insert (6*), take expectations, and obtain again optimum condition (9*) with the exponent $(\gamma - 1)$ being replaced by $(\bar{t}\gamma - 1)$. Optimal consumption is determined by

$$\begin{aligned} &E[((\check{w}_{t-1} - \check{c}_{t-1} - \check{v}_{t-1})R_t + \check{R}_{Mt}\check{v}_{t-1})^{\bar{t}\gamma}\xi_t \mid \emptyset_{t-1}] \\ &\qquad = \check{c}_{t-1}R_t\bar{t}E[((\check{w}_{t-1} - \check{c}_{t-1} - \check{v}_{t-1})R_t + \check{R}_{Mt}\check{v}_{t-1})^{\bar{t}\gamma-1}\xi_t \mid \emptyset_{t-1}]. \end{aligned} \tag{14*}$$

Again in equilibrium $\check{w}_{t-1} - \check{c}_{t-1} - \check{v}_{t-1} = 0$. Insert (11*) with modified exponents in (14*) and obtain

$$\check{v}_{t-1} = \bar{t}\check{c}_{t-1} \quad \text{or} \quad \check{w}_{t-1} = (\bar{t} + 1)\check{c}_{t-1}. \tag{15*}$$

Insert (15*) in $E[U_t^{in} \mid \emptyset_{t-1}]$ and obtain ($\check{w}_t = \check{R}_{Mt}\check{v}_{t-1}$)

$$E[U_t^{in} \mid \emptyset_{t-1}] = \frac{1-\gamma}{\gamma}\left(\frac{\check{w}_{t-1}}{1-\gamma}\right)^{(\bar{t}+1)\gamma}\xi_{t-1}$$

with

$$\xi_{t-1} \equiv \frac{\bar{t}^{\bar{t}\gamma}}{(\bar{t}+1)^{(\bar{t}+1)\gamma}}\,E[\check{R}_{Mt}^{\bar{t}\gamma}\xi_t \mid \emptyset_{t-1}]. \tag{16*}$$

The constant factor in $\xi_{t-1}$ is negligible.

2) Consider exponential utility. Then

$$U_t^{in} = -\rho_{t-1}\exp(-\beta_{t-1}c_{t-1}) - \rho_t\exp(-\beta_t w_t)\xi_t, \qquad t = 1, \cdots, T. \qquad (17^*)$$

Insert (1*) in (17*), take expectations, and obtain for $v_{t-1}$ the optimum condition

$$E[\exp(-\beta_t[(w_{t-1} - c_{t-1} - v_{t-1})R_t + R_{Mt}v_{t-1}])R_{Et}\xi_t \mid \emptyset_{t-1}] = 0$$

which simplifies to

$$E[\exp(-\beta_t R_{Mt}v_{t-1})R_{Et}\xi_t \mid \emptyset_{t-1}] = 0. \qquad (18^*)$$

Hence, the optimal $v_{t-1}$ is independent of $w_{t-1}$. Moreover, $w_{t-1} - c_{t-1} - v_{t-1} = 0$ in equilibrium so that $w_t = v_{t-1}R_{Mt}$. Consequently, the second term in (17*) is independent of $w_{t-1}$ so that it can be neglected in the indirect utility function of wealth $w_{t-1}$. Then obtain

$$E[U_t^{in} \mid \emptyset_{t-1}] = -\rho_{t-1}\exp(-\beta_{t-1}c_{t-1}) = -\rho_{t-1}\exp(-\beta_{t-1}w_{t-1})\xi_{t-1}$$

with

$$\xi_{t-1} \equiv \exp(-\beta_{t-1}c_{t-1})/\exp(-\beta_{t-1}w_{t-1}) = \exp(-\beta_{t-1}^* C_{t-1})/\exp(-\beta_{t-1}^* W_{t-1})$$

with

$$\beta_{t-1}^* \equiv \beta_{t-1}/I.$$

For time-multiplicative utility $c_{t-1} = 0$ and $w_{t-1} = v_{t-1}$, then

$$\begin{aligned} E[U_t^{in} \mid \emptyset_{t-1}] &= -E[\exp(-\beta_T R_{Mt}w_{t-1})\xi_t \mid \emptyset_{t-1}] \\ &= -\exp(-\beta_T w_{t-1})E[\exp(-\beta_T r_{Mt}w_{t-1})\xi_t \mid \emptyset_{t-1}] \\ &= -\exp(-\beta_T w_{t-1})E[\exp(-\beta_T^* r_{Mt} W_{t-1})\xi_t \mid \emptyset_{t-1}] \\ &\equiv -\exp(-\beta_T w_{t-1})\xi_{t-1}. \end{aligned}$$

3) Finally Equation (2) has to be proved. It follows immediately for exponential utility from (18*) with $\beta_t v_{t-1} = \beta_t^* V_{t-1}$. For non-exponential utility (9*) yields in equilibrium

$$E[\check{R}_{Mt}^{\theta\gamma-1}\check{R}_{Et}\check{V}_{t-1}\xi_t \mid \emptyset_{t-1}] = 0. \qquad (19^*)$$

From (4*), (5*), and (7*) follows

$$\begin{aligned} \check{R}_{Et}\check{V}_{t-1} &= B_t + W_t - R_t[B_{t-1} - \alpha_{t-1}^*(1-\gamma)/\beta_{t-1} + V_{t-1}] \\ &= [W_t - R_t V_{t-1}] + [B_t - R_t(B_{t-1} - \alpha_{t-1}^*(1-\gamma)/\beta_{t-1})]. \end{aligned}$$

Hence, $\check{R}_{Et}\check{V}_{t-1}$ is the sum of the random risk premium of the market portfolio and the random risk premium of the risk-free bond which pays $\alpha_\tau^*(1-\gamma)/\beta_\tau$ at date $\tau$, $\tau = t, t+1, \cdots, T$. Since the capital market is Pareto-efficient, $E[\check{R}_{Mt}^{\theta\gamma-1}\pi_t\xi_t \mid \emptyset_{t-1}] = 0$ for each asset with $\pi$ begin its random risk premium. Hence, $E[\check{R}_{Mt}^{\theta\gamma-1}R_{Et}V_{t-1}\xi_t \mid \emptyset_{t-1}] = 0$. Divide by $V_{t-1}$ and multiply by $\check{V}_{t-1}^{\theta\gamma-1}$. As $\check{R}_{Mt}\check{V}_{t-1} = B_t + W_t = B_t + R_{Mt}V_{t-1}$, Equation (2) follows.

## Appendix 2

### Proof of Proposition 3

Rewrite Equation (2)

$$E\left[\left(B_t \frac{\beta_t^*}{1-\gamma} + \frac{\beta_t^*}{1-\gamma}(R_{Et} + R_t)V_{t-1}\right)^{\theta\gamma-1} R_{Et}\xi_t \mid \emptyset_{t-1}\right] = 0, \qquad (20^*)$$

$$t = 1, \cdots, T.$$

1) Proof of Proposition 3(a). Assume constant proportional risk aversion ($B_t = 0$, $\gamma > -\infty$). For $t = T$, $\xi_T \equiv 1$. As SIMR requires $R_{ET}$ to be distributed independently of $\emptyset_{T-1}$, $R_T$ must be completely deterministic (Equation (20*)).

In the case of time-additive utility, Equation (10*) yields after aggregation

$$\frac{\rho_{t-1}}{\rho_t}\frac{\beta_{t-1}}{\beta_t}\left(\alpha_{t-1}^* + \frac{\beta_{t-1}C_{t-1}}{1-\gamma}\right)^{\gamma-1} \qquad (21^*)$$

$$= R_t E\left[\left(B_t \frac{\beta_t}{1-\gamma} + \frac{\beta_t}{1-\gamma}(R_{Et} + R_t)V_{t-1}\right)^{\gamma-1} \xi_t \mid \emptyset_{t-1}\right],$$

$$t = 1, \cdots, T.$$

In the case of constant proportional risk aversion, $\alpha_{t-1}^* = B_t = 0$ and SIMR requires $R_T$ to be completely deterministic. As $R_{ET}$ must be distributed independently of $\emptyset_{T-1}$, Equation (21*) implies that $V_{T-1}/C_{T-1} = W_{T-1}/C_{T-1} - 1$ is completely deterministic. Then, by Equation (3b), $\xi_{T-1}$ is completely deterministic. Proceeding backwards, the same arguments show that $R_t$, $\xi_{t-1}$, and $C_{t-1}/W_{t-1}$ must be completely deterministic if SIMR is true, $\forall t = 2, \cdots, T$.

As $R_{Mt} = W_t/V_{t-1} = W_t[W_{t-1} - C_{t-1}]^{-1} = (W_t/C_t)[(W_{t-1}/C_{t-1}) - 1]^{-1}C_t/C_{t-1}$, the foregoing results imply that $R_{Mt} = R_{Et} + R_t$ is distributed independently of $\emptyset_{t-1}$ only if this is true of the growth rate of aggregate consumption, $C_t/C_{t-1}$. Rubinstein [17, p. 414] has shown that this condition is sufficient for SIMR under constant proportional risk aversion and time-additive preferences. The same results can be proved for constant proportional risk aversion and time-multiplicative preferences. This proves Proposition 3(a).

2) Proof of Proposition 3(b). Assume constant absolute risk aversion. Then Equation (20*) yields

$$E[\exp(-\beta_t^* R_{Et} V_{t-1}) R_{Et}\xi_t \mid \emptyset_{t-1}] = 0, \qquad t = 1, \cdots, T. \qquad (22^*)$$

Hence, for $t = T$ with $\xi_T \equiv 1$, $R_{ET}$ is distributed independently of $\emptyset_{T-1}$ only if $V_{T-1}$ is completely deterministic. Therefore, by Equation (3b), $\xi_{T-1}$ is completely deterministic in the case of time-additive utility. Then, by (22*), SIMR requires that $V_{T-2}$ be completely deterministic and so forth.

In the case of time-multiplicative, exponential utility, consumption at dates $t = 0, 1, \cdots, T-1$ is zero so that a completely deterministic $V_{T-1}$ rules out uncertainty at all dates $t = 0, 1, \cdots, T-1$.

3) Proof of Proposition 3(c). Assume $\alpha_t^* \neq 0$ ($t = 0, 1, \cdots, T$) and $\gamma > -\infty$.

Rewrite Equation (20*)

$$E\left[\left(\frac{B_t}{V_{t-1}} + R_t + R_{Et}\right)^{\theta\gamma-1} R_{Et}\xi_t \,|\, \emptyset_{t-1}\right] = 0, \qquad t = 1, \cdots, T. \tag{23*}$$

(i) First we show that SIMR requires $V_{T-1}$, $C_{T-1}$, $W_{T-1}$, and $\xi_{T-1}$ to be functions of $R_T$ only. Define $B_t/V_{t-1} + R_t \equiv \Lambda_t$. For $t = T$, $\xi_T \equiv 1$. Hence, by (23*), $R_{ET}$ is distributed independently of $\emptyset_{T-1}$ only if $\Lambda_T$ is completely deterministic. In the case of time-additive utility, (21*) yields

$$\frac{\rho_{t-1}}{\rho_t}\frac{\beta_{t-1}}{\beta_t}\left(\alpha^*_{t-1} + \frac{\beta_{t-1}}{1-\gamma}C_{t-1}\right)^{\gamma-1}$$
$$= R_t\left(\frac{\beta_t V_{t-1}}{1-\gamma}\right)^{\gamma-1} E[(\Lambda_t + R_{Et})^{\gamma-1}\xi_t \,|\, \emptyset_{t-1}], \qquad t = 1, \cdots, T. \tag{24*}$$

If $\Lambda_T$ is completely deterministic, then, by definition of $\Lambda_T$, $V_{T-1}$ is a function of only $R_T$ so that, by (24*), $C_{T-1}$ is, too. Hence, $W_{T-1}$ is a function of $R_T$ only so that, by Equation (3b), $\xi_{T-1}$ is, too. This can be shown to hold also in the case of time-multiplicative, non-exponential utility.

(ii) Second we show that SIMR requires $V_{T-2}$, $C_{T-2}$, $W_{T-2}$, $R_{T-1}$, and $\xi_{T-2}$ to be functions of $R_T | \emptyset_{T-2}$ only. $R_T | \emptyset_{T-2}$ is the distribution of $R_T$ given the information $\emptyset_{T-2}$. From $R_{E,T-1} = W_{T-1}/V_{T-2} - R_{T-1}$, it follows that $V_{T-2} = \sigma(W_{T-1})/\sigma(R_{E,T-1})$. As, by (i), $W_{T-1} = W_{T-1}(R_T)$ and SIMR requires $\sigma(R_{E,T-1})$ to be completely deterministic, $V_{T-2}$ depends on $R_T | \emptyset_{T-2}$ only. Moreover, $R_{T-1} = E(W_{T-1})/V_{T-2} - E(R_{E,T-1})$ so that SIMR requires $R_{T-1}$ to depend on $R_T | \emptyset_{T-2}$ only. $B_{T-1} = B_{T-1}(R_T)$. Hence, $\Lambda_{T-1}$ depends on $R_T | \emptyset_{T-2}$ and the realization of $R_T$ only so that in the case of time-additive utility $C_{T-2}$ depends on $R_T | \emptyset_{T-2}$ only (Equation (24*)). Then this must be true for $W_{T-2}$ and, by Equation (3b), for $\xi_{T-2}$, too. Proceeding backwards, it can be shown that $V_t$, $C_t$, $W_t$, $R_{t+1}$, and $\xi_t$ are required to be functions of only the stochastic process which governs risk-free interest rates in subsequent periods, $t = 0, \cdots, T-1$. The same can be shown in the case of time-multiplicative, non-exponential utility.

(iii) From (ii) it follows that $W_t$, $C_t$, and $\xi_t$ are completely deterministic if the stochastic process governing risk-free interest rates in subsequent periods is independent of $\emptyset_t$. Hence, uncertainty in periods 1, 2, $\cdots$, $T-2$ is preserved only if the stochastic process depends on $\emptyset_t$. As $R_{t+1}$ depends on this process only, the risk-free interest rate must be serially dependent. Then, given $\emptyset_t$, $\xi_{t+1}$ is not known as of date $t$ unless utility is logarithmic.

From $W_t = (R_{Et} + R_t)V_{t-1}$, it follows for the normalized wealth variable that

$$\frac{W_t - E(W_t)}{\sigma(W_t)} = \frac{R_{Et} - E(R_{Et})}{\sigma(R_{Et})}.$$

SIMR requires the right-hand side to be serially independent, hence the left-hand side, too. As $W_t$ is a function of only the stochastic process governing risk-free interest rates in subsequent periods, the same must be true of the normalized wealth variable. This variable must be serially independent. This ends the proof.

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