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Author(s): Donald Dutkowsky
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The Demand for Borrowed Reserves: A Switching Regression Model

DONALD DUTKOWSKY*

ABSTRACT

A microeconomic model of bank demand for borrowed reserves from the Federal Reserve is developed based upon constrained cost minimization. The derived demand function was found to correspond to behavior appropriate to the unknown switchpoint switching regression problem. When estimated, parameters generally conformed to theoretical expectations. The model was also tested for existence of switching regression behavior against a model similar to Goldfeld and Kane [12]. Significance exceeded 99% in all cases. With the advent of reserve intermediate targeting, it appears especially necessary to reinvestigate the behavior determining this important source of reserves.

WITH THE ADVENT OF reserve intermediate targeting as of October 6, 1979, it appears especially necessary to reinvestigate the behavior determining one source of these reserves, borrowings from the Discount Window. Although the behavior of borrowed reserves has concerned researchers since the 1920s, interest in the subject generally waned after 1970.

The reason has much to do with the studies of Polakoff [21] and Goldfeld and Kane [12]. Polakoff develops a structural model for Discount Window borrowing based upon the individual bank minimizing its costs among alternative borrowing sources, adapting the seminal work of Turner [29]. Goldfeld and Kane extend and estimate Polakoff's original model. Because of their well-developed microeconomic theory, seemingly accurate empirical adjustments and proxy variables, and excellent all-around empirical results, Goldfeld and Kane's study is widely hailed as a leading study in the field of borrowed reserve demand. Evidence exists, through studies such as Keir [18], that the Federal Reserve continues to use this same basic model.

In this study, a borrowed reserve demand function is derived using a constrained minimum cost setup. Included in the cost of Discount Window borrowing is an implicit cost function reflecting bank reluctance to borrow from the Discount Window caused by Federal Reserve surveillance. The derived demand function found in Section I conforms to behavior appropriate to a switching regression model. In this case, switching occurs dependent upon the size of the interest rate spread relative to an unknown switchpoint.

After making minor adjustments in Section II, the model is estimated employing monthly data from 1955–78. These results and the tests for the existence of

* Clarkson College of Technology. Excerpted from the author's Ph.D. thesis, SUNY at Buffalo. I wish to thank my thesis committee—A. Butler, N. S. Revankar, D. Hamberg, and M. Polakoff—for their comments. N. Yoshino, J. Holmes, W. Foote, and an anonymous referee also provided helpful suggestions.

switching regression behavior against the commonly used model similar to Goldfeld and Kane's are presented in Section III. Strong evidence was given in favor of the switching regression model as significance of the existence statistics consistently exceeded 99%. Concluding remarks are given in Section IV.

I. An Alternative Least Cost-Reluctance Model

The Discount Window borrowing decision of the individual bank is now considered. Having previously allocated its asset portfolio among loans, securities, and reserves according to profit maximization or some other optimization scheme, it is assumed that this bank faces an exogenous reserve need, some unexpected reserve outflow causing its present reserves to fall below requirements. Reasons for this reserve need may include seasonal factors, a change in the float, unforeseen deposit withdrawal, or an unexpected increase in loan demand that the commercial bank wishes to accommodate. Assuming that the individual bank does not wish to liquidate any of its other assets to meet this reserve need, it is forced to borrow reserves from some source. Denote this reserve need variable R^N, not necessarily constant with respect to time.

For simplicity, assume two alternative sources of obtaining reserves to fill this need. One source is the Federal Reserve with costs, neglecting those due to transactions, arising from two different origins. One source of cost is the discount rate, denoted by i_D, assumed unchanging with respect to the level of borrowings.[1] The other cost considered is implicit, based upon the individual bank's reluctance to borrow at the Discount Window reinforced by Federal Reserve surveillance. This behavior can be modeled using a harassment cost function, $c(R^B)$, assumed continuous with positive first and second derivatives, reflecting positive and increasing marginal implicit cost of borrowing from the Federal Reserve. Harassment cost-reluctance behavior is well-known in the borrowed reserve literature, having previously been stated in Brunner and Meltzer [7], Shull [26], and Goldfeld and Kane [12].[2] The alternative source of borrowing, assumed to be the closest substitute to the Discount Window, may either involve obtaining reserves directly from a market such as Federal Funds or selling a liquid security and then purchasing it again as the need subsides. Interest and transactions are assumed to constitute the only costs of obtaining reserves from this source.

Proceeding, then, the bank will choose optimal quantities of borrowing from each source so as to minimize the total cost of borrowing subject to filling the reserve need.[3] If it is additionally assumed that transactions costs are constant,

[1] Different discount rates offered to individual banks based upon the level, term, and frequency of borrowings are assumed away for simplicity.

[2] Goldfeld and Kane argue (correctly) that this implicit cost should be a function not only of present but past borrowings as well, indicating that the Federal Reserve is concerned with the member bank's past borrowing record as well as the magnitude of their current request. Although the adjustment is not included in the subsequent analysis, the inclusion would not substantially alter the later results. See footnote 13 for further explanation.

[3] Another constraint, of course, is the existence of a maximum borrowing limit imposed by the Federal Reserve. It is assumed that this point is never reached, implying that the Federal Reserve's

known, and linear with respect to the level of borrowings with zero fixed cost, the appropriate Lagrangian would be given by:

$$\mathscr{L} = i_S R^S + \theta_S R^S + i_D R^B + \theta_D R^B + c(R^B) + \lambda(R^N - R^B - R^S), \qquad (1)$$

where

i_D = discount rate,
i_S = interest rate of alternative source,
R^B = level of borrowings from the Discount Window,
R^S = level of borrowings from the alternative source,
R^N = level of reserve need,
θ_D = transactions cost of borrowing from the Discount Window,
θ_S = transactions cost of borrowing from the alternative source.

Corner solutions are meaningful in this analysis as R^{B^*} and R^{S^*} could easily equal zero. Therefore, analysis proceeds according to the Kuhn-Tucker conditions for minimization given below:

$$\frac{\partial \mathscr{L}}{\partial R^B} = i_D + \theta_D + c'(R^B) - \lambda \geq 0, \qquad R^B \frac{\partial \mathscr{L}}{\partial R^B} = 0, \qquad (2a)$$

$$\frac{\partial \mathscr{L}}{\partial R^S} = i_S + \theta_S - \lambda \geq 0, \qquad R^S \frac{\partial \mathscr{L}}{\partial R^S} = 0, \qquad (2b)$$

$$\frac{\partial \mathscr{L}}{\partial \lambda} = R^N - R^B - R^S \leq 0, \qquad \lambda \frac{\partial \mathscr{L}}{\partial \lambda} = 0, \qquad (2c)$$

$$R^B \geq 0, \quad R^S \geq 0, \quad \lambda \geq 0.$$

Since there is a positive cost to borrowing from either source at any level, it can be assumed that total borrowing does not exceed the reserve need.[4] This behavior would imply that the first-order condition of Equation (2c) holds with equality. Four cases, then, are left to consider, depending upon the equality or inequality of the conditions given by Equations (2a) and (2b). The case that both conditions hold with inequality can also be deleted as R^B and R^S cannot simultaneously equal zero under a positive reserve need. Some level of positive borrowing from at least one source must take place.

If condition (2a) is nonbinding and (2b) is binding, then $R^{B^*} = 0$ and $R^{S^*} = R^N$; all borrowing takes place from the alternative source. Since (2b) holds with equality, λ^* can be solved for, equalling $i_S + \theta_S$. Substituting the solutions for λ^* and R^{B^*} into (2a), the following necessary condition for zero borrowings at the

primary weapon of surveillance will cause a bank to stop its borrowing before this limit. Evidence that this limit is not a constraint does exist. It is well-known (Whittlesey [30], Goldfeld [11]) that the Federal Reserve rarely outright refuses loans.

[4] Some large money market banks, permanent borrowers of Federal Funds, may indeed borrow more than their own reserve need. One reason for this added borrowing behavior might be involved with satisfying the anticipated needs of a correspondent bank. This behavior is assumed away here for simplicity.

Discount Window is obtained:

$$K \leq c'(0) - (\theta_S - \theta_D), \tag{3}$$

where $K \equiv i_S - i_D$. If the exogenous interest rate spread, regardless of magnitude, does not cover the positive minimum marginal implicit cost of borrowing from the Federal Reserve less the difference in transactions costs, Discount Window borrowings will be zero for the individual bank.

Now consider the case where both conditions hold with equality, indicating a positive level of borrowing from each source. Then λ^* again can be solved for as equalling $i_S + \theta_S$. Substituting the above solution into Equation (2a), the following first-order condition is obtained:

$$c'(R^{B^*}) = K + (\theta_S - \theta_D). \tag{4}$$

In this paradigm, a member bank will borrow from the Discount Window until the marginal cost of doing so, $i_D + \theta_D + c'(R^B)$, equals the constant marginal cost, $i_S + \theta_S$, of borrowing from the alternative source. Since c' is assumed monotonically increasing, the function can be inverted and R^{B^*} can be solved for as:

$$R^{B^*} = c'^{-1}(K + \theta_S - \theta_D). \tag{5}$$

Because of increasing marginal implicit cost, the above borrowings demand function can be easily shown to be increasing with respect to the interest rate spread.

Since this behavior occurs when R^{B^*} is positive, this condition implies that $c'(R^{B^*}) > c'(0)$ as well. Substituting into Equation (4) for $c'(R^{B^*})$, the necessary condition for positive borrowing from the Discount Window is obtained:

$$K > c'(0) - (\theta_S - \theta_D). \tag{6}$$

Note that the point of determination, $c'(0) - (\theta_S - \theta_D)$, is identical to the value dictating zero Discount Window borrowing in Equation (3). These conditions would indicate the existence of switching behavior dependent upon the value of the interest rate spread relative to the above switchpoint. Since the minimum marginal implicit cost is unobservable, this switchpoint is unknown. Also, because $R^{B^*} \leq R^N$, this behavior occurs only when $K \leq c'(R^N) - (\theta_S - \theta_D)$.

Finally, consider the behavior when Equation (2a) is binding and (2b) is nonbinding. Since the latter equation holds with inequality, $R^{S^*} = 0$ and $R^{B^*} = R^N$, all borrowing takes place at the Discount Window. From this solution, λ^* can be solved for in Equation (2a) as equalling $i_D + \theta_D + c'(R^N)$. If the solution for λ^* is substituted into (2b), the following necessary condition for borrowing the total need from the Discount Window is obtained, given by:

$$K \geq c'(R^N) - (\theta_S - \theta_D). \tag{7}$$

This analysis would imply a second switchpoint at the other corner solution.

The complete demand for borrowed reserves from the Discount Window under this revised Least Cost motive can now be summarized as:

$$R^{B^*} = \begin{cases} 0, & K \leq K^* \\ c'^{-1}(K + \theta_S - \theta_D), & K^* \leq K \leq K^{*\prime} \\ R^N, & K^{*\prime} \leq K, \end{cases} \tag{8}$$

where

$$K^* \equiv c'(0) - (\theta_S - \theta_D),$$
$$K^{*\prime} \equiv c'(R^N) - (\theta_S - \theta_D).$$

Two unobservable switchpoints are obtained, each dependent upon relative transactions costs and the bank's reluctance to borrow from the Federal Reserve. If transactions costs from each source are equal, the switchpoints are clearly positive, indicating that the spread has to be favorable enough to overcome minimum marginal implicit cost for the bank to undertake Discount Window borrowing. The switchpoints may be negative, though, if θ_S is large relative to the other terms. Since zero borrowing from the Discount Window is a common occurrence for most banks, it appears that the lower regime of nonborrowing at unfavorable spreads should be considered in the estimation of this demand function.[5] Estimation, including the need constrained regime, presents a more difficult task as the switchpoint is dependent upon the changing reserve need variable.

For estimation purposes, consider a functional approximation to the implicit cost function given by:

$$c(R^B) = \alpha e^{\beta R^B}, \qquad \alpha, \beta > 0. \tag{9}$$

Clearly, c' and c'' are positive. In addition, α and β have intuitive interpretations as reluctance parameters. The parameter $\alpha = c(0)$ corresponds to a "fixed reluctance," an innate reluctance for the bank to be in debt and even approach the Discount Window to borrow. On the other hand, β can be interpreted as a "suasion" parameter, reflecting a type of acquired reluctance due to central bank actions as well as the "tradition against borrowing." Clearly, $\partial c/\partial\alpha$ and $\partial c/\partial\beta$ can be shown to be positive, with a higher amount of either reluctance corresponding to greater implicit cost. Also, consider the switchpoints under this function, given by:

$$K^* = \alpha\beta - (\theta_S - \theta_D),$$
$$K^{*\prime} = \alpha\beta e^{\beta R^N} - (\theta_S - \theta_D). \tag{10}$$

It is easily shown that the switchpoints are increasing functions of α and β.[6] This behavior appears to correspond well with the evidence concerning the use of the Discount Window by continuous borrowers. These borrowers would appear to have relatively low switchpoints caused by little reluctance to borrow from the Federal Reserve.

Substituting the implicit cost function of Equation (9) into Equation (5), the

[5] Goldfeld and Kane [12, p. 501] find the existence of a corner solution in their model as well, but dismiss this behavior in the subsequent estimation.

[6] It can be questioned whether α and β can be assumed to be even approximately stable with respect to time or Federal Reserve policy. However, studies throughout, such as Whittlesey [30], Roosa [25], and Axilrod [3], have argued for the consistency of Discount Window policy. Axilrod [3], in particular, states that the primary function of the Discount Window remains as a buffer to the individual bank against unforeseen forces affecting reserves. Polakoff and Silber [22] did find different reactions of borrowed reserves to the interest rate spread in predefined periods of "tight" and "easy" money. This hypothesis will be tested.

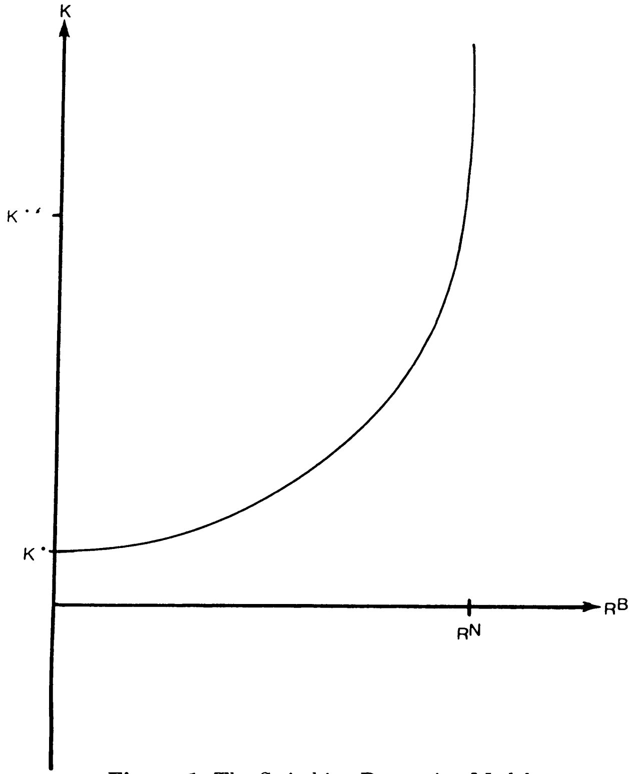

Figure 1. The Switching Regression Model

following borrowed reserve demand function is obtained:

$$R^{B^*} = -\frac{1}{\beta}\log\alpha\beta + \frac{1}{\beta}\log(K + \theta_S - \theta_D), \qquad K^* \leq K \leq K^{*\prime}. \tag{11}$$

Note that, in this regime, borrowing is a decreasing function of either type of reluctance. The derived model also possesses a decreasing marginal propensity to borrow. Following Polakoff's [21] similar result, this behavior would be attributed to the increased Federal Reserve surveillance arising from the higher borrowing request. The linear model does not cover this behavior.

The above switching regression model is drawn in Figure 1 for fixed values of α, β, θ_S, θ_D, and R^N.[7] If K was placed on the horizontal axis, R^B on the vertical axis, and the curve redrawn, this function becomes strikingly similar to the behavior found in the scatter diagrams of Polakoff [21] or Tinsley et al. [28]. Three regions of this borrowings function can be defined corresponding to the well-known conflicting motives for Discount Window borrowing. When $K < K^*$, borrowing is zero or, at best, minimal, corresponding to the borrowing for "needs" behavior put forth by Riefler [24]. Borrowed reserves do not depend on the interest rate spread in this region. At values of K above the switchpoint, implicit costs are low relative to the spread, and the profit behavior described by Turner

[7] A similar curve for borrowed reserve demand can be found in Holmes [17].

[29] dominates. In this region, borrowings are elastic with respect to the interest rate spread. However, as K continues to increase, reluctance/surveillance begins to dominate and borrowings become less and less responsive until becoming totally inelastic at the given reserve need.

Finally, note that the function proposed to approximate implicit cost is but one example of functions satisfying positive first and second derivatives that can be postulated. Consider the implicit cost function, $c_1(R^B)$, given by:

$$c_1(R^B) = (\beta R^B + \sqrt{\alpha})^2, \qquad \alpha, \beta > 0. \tag{12}$$

Using the first-order condition of Equation (4), it can be easily shown that a switching regression model linear in the profit regime can be obtained.[8] However, this model does not predict dominating reluctance at higher spreads. It would seem, then, that the linear switching regression model, although theoretically viable, may be less preferred to the model loglinear in K in the profit regime.

II. The Final Switching Regression Model

The switching regression model of Equation (11) with additional adjustments was estimated employing monthly data from February 1955 through December 1978, 287 observations in all. The starting point of the sample is designed to correspond with the "Revision of Regulation A," a major document concerning Discount Window borrowing. As discussed in Anderson [2] or Goldfeld [11], it can be argued that this revision represented an attempt by the Federal Reserve to reinforce the "tradition against borrowing" and to provide additional surveillance toward the Discount Window.

The data used for Discount Window borrowings are designed to be as disaggregated as possible. As explained in Goldfeld and Kane [12], obvious differences in banks' situations (portfolio size, location, deposit mix, competitive pressures) may be so considerable as to make it likely that individual bank responses show important differences in magnitude. Following Goldfeld and Kane [12], the data used are divided according to Federal Reserve classes, given by large New York City banks, large Chicago banks, Reserve City banks, and Country banks. The model was also estimated with the data of all bank borrowing. Despite the usage of the above data, aggregation problems associated with estimation of the switching regression model remain. Aggregation issues will be addressed in the following section.

The Federal Funds rate was used as the short-term rate interest rate in the estimation. It can be argued that even banks, usually considered net Federal Funds suppliers such as Country banks, borrow largely from correspondents at approximately the above rate.[9] Since the rate on repurchase agreements closely follows the Federal Funds rate, this estimation would be valid for a Least Cost theory involving this instrument as well. The parameters θ_S and θ_D are assumed

[8] It can also be shown that there also exists an implicit cost function leading to the borrowed reserve demand model quadratic in K used by Goldfeld and Kane [12] and Polakoff and Silber [22]. This implicit cost function is quite nonintuitive and cumbersome, so it will not be given here.

[9] Unreported regressions were also performed using the 90-day Treasury Bill rate, traditionally used in previous studies such as Goldfeld and Kane [12].

to be zero. Under the Discount Window or Federal Funds, transactions costs of borrowing consisting of telephone calls, paper work, etc., can easily be argued as incidental.

In Sections *A–C*, the final estimated model is formed by adding additional variables to Equation (11). These variables do not change the theoretical results obtained from Least Cost-Reluctance but are instead included in order to test for factors affecting Discount Window borrowing other than what the purely static microeconomic theory would predict. Taken from previous studies on Discount Window borrowing, it will also be interesting to observe if these variables retain their significance in the switching regression model. The reserve need variable is also discussed.

A. *The Reserve Need and Aggregation*

While playing an essential role under the derived Least Cost-Reluctance model, the reserve need variable presents particularly difficult problems in the estimation. Although observable to the individual bank, it is unobservable in the aggregate and therefore must be proxied. A more thorny estimation problem occurs because of the dependence of the upper switchpoint upon the above variable, changing this switchpoint over time.

If it is assumed that some individual banks operate on the need constrained regime, aggregation across the upper switchpoint may produce a smoothing behavior conducive to approximating the three regime model by eliminating the need constrained regime and adding the reserve need variable linearly in the profit regime. A positive sign would be expected on the corresponding parameter. A two regime switching regression model results, the upper regime consisting of spread affected plus need constraining behavior with the lower regime based upon zero or minimal borrowing. As pointed out in Goldfeld and Quandt [13], considerable problems emerge in attempting to estimate a micro switching model using aggregate data. The authors, however, did not address aggregation problems occurring under variable switchpoints, a problem deserving further investigation.

As a proxy variable for reserve need, Goldfeld and Kane [12] used changes in unborrowed reserves, stemming from dividing the true need into permanent and transitory components and assuming the former to be a weighted sum of unborrowed reserve flows. Although it seems doubtful that such Friedmanian type behavior actually occurs, changes in unborrowed reserves would appear to be an acceptable proxy for the reserve need. This proxy would seem to capture the approximate size and reserve flow cause of the reserve need. A problem arises because this variable does not capture changes in the reserve need due to changes in required reserves or composition of deposits. This proxy is also biased due to the inclusion of Federal Funds borrowing, highly difficult data to obtain, in its measure. Still, because changes in unborrowed reserves do not appear to create simultaneity problems in estimation and are also used in the Least Cost models of Goldfeld and Kane [12] and Polakoff and Silber [22], this variable will be employed here, added to both regimes.[10] Even in the lower regime of nonborrow-

[10] Simultaneity of the reserve need variable in the estimation of the demand for borrowed reserves has been previously addressed. A common solution has been to assume borrowings homogeneous of

ing, it would not be surprising for the reserve need to constitute a significant cause. An unexpected inflow of reserves, suddenly decreasing the reserve need, may cause the member banks to possibly repay borrowings sooner than what normally would be the case. Also, aggregation across switchpoints is occurring, possibly causing some "upper regime" type behavior to take place in the lower regime.

Aggregation across switchpoints may also result in spread-affected borrowing behavior taking place. Some banks having low switchpoints relative to the estimate may be undertaking borrowing behavior in the estimated lower regime. It would not be surprising, then, to find $\partial R^{B^*}/\partial K > 0$ in the unfavorable spread regime of the estimated model. Thus, the interest rate spread is added to the lower regime as well.

As pointed out in Goldfeld and Quandt [13], aggregation over a micro switching model may also considerably alter the nature of the estimated model. This would imply that assumptions need to be made concerning the underlying distribution of the switchpoint criterion variable, the interest rate spread. The above issue will be examined further upon formation of the final model.

B. Structural Changes

Dummy variables are added to test for factors affecting Discount Window borrowing due to various structural changes. The variables will be added to both regimes although the regression parameter under nonborrowing would be expected to be zero.

The first change involves the imposition of Lagged Reserve Accounting, beginning in September 1968. Coats [9] claims and provides evidence that the Federal Funds rate became more volatile and that member bank borrowing increased as a result. By adding the dummy variable LRA to both regimes (0 before September 1968, 1 afterwards), it can be tested whether Lagged Reserve Accounting produced increased borrowing through perhaps greater reserve needs caused by its imposition. Even if insignificant, Lagged Reserve Accounting may still have increased Discount Window borrowing but only through increases in the Federal Funds rate.

The second change considered is the Seasonal Borrowing Privilege, instituted by the Federal Reserve in April 1973. Stated in Graham [16], this privilege was involved with making funds available to all member banks demonstrating a substantial seasonal dip in funds available for loans. Despite being available for all eligible member banks regardless of size, the Seasonal Borrowing Privilege was intended to provide funds to small rural banks needing added funds during regularly recurring periods of heavy loan demand, such as the crop planting and growing seasons. In addition, these small banks generally are thought not to have access to the money market as a source for obtaining needed funds. Graham's study involving member banks in the Minneapolis district provides evidence that

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degree one with respect to the need variable, examples including studies such as Meigs [19], Brunner and Meltzer [6], Alperstein [1], Brimmer [4], Burger [8], and Thomson et al. [27]. There is no theoretical basis for such an assumption in this model.

there appears to be problems in the implementation of this privilege. Graham [16, p. 10] claims that banks eligible for the Seasonal Borrowing Privilege have made little use of it. Secondly, according to Graham [16], most banks that have indeed used the privilege are banks having access to the money markets and choosing to use the privilege under a favorable spread. A dummy variable SBP was added to both regimes (SBP = 0 pre-April 1973, 1 otherwise) to test Graham's claims. If his first assertion is true, it would be expected that the coefficient for SBP should be zero, especially with Country banks. Obtaining a positive and significant coefficient only in the upper regime, especially with the city banks, would give supporting evidence to Graham's second claim.

Another question that has been considered is concerned with possibly different Discount Window borrowing behavior depending upon the type of policy the Federal Reserve is instituting based upon the study of Polakoff and Silber [22]. To test for this behavior in the switching regression model, the variable log K was kept in the model and an additional term was added to the upper regime. This term consisted of a dummy variable TE multiplied by log K, with TE defined to be one under a period of "tight money" and zero otherwise. A negative coefficient for TE log K would indicate greater suasion/reluctance in a time when the Federal Reserve was attempting to tighten monetary policy.

The series for tight vs. easy money was obtained from the quantified FOMC series based upon specific Federal Reserve policies first done by Brunner and Meltzer [5] and later extended through 1969 by Meltzer [20].[11] The author further extended this series through 1978. Zero was given when the numbers in the original series were primarily negative, reflecting "easy money" policies. One was given in times where specific policies were deemed restrictive in nature.

Finally, in any empirical study on Discount Window borrowings involving the year 1974, the behavior of the Franklin National Bank must be taken into account. The impending insolvency of this large New York City area bank was a major cause of borrowing from May 1974 until its merger with the European-American bank in October of the same year.[12] The Least Cost theory is not a lender-of-last-resort type model, so this borrowing would be in addition to what the model would predict. It is not known exactly how much was loaned to this bank each month, so the dummy variable FNB (1 from May 1974 to October 1974, 0 otherwise) was created. The time period in question involved large interest rate spreads throughout so this variable was only added to the upper regime. Since this behavior occurred only to a large New York City bank, this dummy variable was only used in the New York City bank class and with borrowing of all banks.

C. *Seasonality and Partial Adjustment*

Seasonality was taken into account by adding monthly seasonal dummy variables. Since borrowing due to seasonality may take place at any interest rate spread, dummy variables were added to both regimes. Greater significance in the upper regime would lend support to the Least Cost hypothesis.

Borrowing may also take place due to partial adjustment. Mentioned in

[11] I am grateful to Professor Meltzer for sending me the latter series.

[12] Source: "Announcements," *Federal Reserve Bulletin*, October 1974, p. 740.

Goldfeld and Kane [12], commercial banks effect desired changes in their portfolios through a series of partial adjustments, implying that the individual bank may borrow more under partial adjustment than what a purely static theory would predict. Following the stock adjustment mechanism of Deleeuw [10], borrowings lagged one month, R_{t-1}^B, were added to the model to both regimes. A positive regression coefficient would be expected as partial adjustment may take place even under unfavorable spreads.[13]

In sum, the final model for estimation can be expressed as:

$$R_t^B = \begin{cases} \pi_0^L + \pi_1^L K_t + \pi_2^L R_{t-1}^B + \pi_3^L \Delta R_t^{UB} \\ \quad + \pi_4^L \mathrm{LRA}_t + \pi_5^L \mathrm{SBP}_t + \sum_{i=1}^{11} \partial_i^L D_{i_t} + \varepsilon_t^L, & K \le K^* \\ \pi_0^U + \pi_1^U \log K_t + \pi_1'^U \mathrm{TE} \log K_t \\ \quad + \pi_2^U R_{t-1}^B + \pi_3^U \Delta R_t^{UB} + \pi_4^U \mathrm{LRA}_t + \pi_5^U \mathrm{SBP}_t \\ \quad + \sum_{i=1}^{11} \partial_i^U D_{i_t} + \varepsilon_t^U, & K > K^*. \end{cases} \tag{13}$$

where

$$\varepsilon^i \sim N(0, \sigma_i^2 I),\ i = L, U,$$

$$D_j = \begin{cases} 1 & \text{when the observation takes place in the } j^{\text{th}} \text{ month} \\ 0 & \text{otherwise, } j = 1, 2, \cdots, 11. \end{cases}$$

All coefficients are expected to be nonnegative except for the terms involving reserve need, tight and easy money, and possibly seasonality.

In addressing the switching aggregation problem, it is additionally assumed that K_t is distributed uniformly over the range $[\bar{K}_t - S, \bar{K}_t + S]$ with S assumed near zero. Under the above conditions, Goldfeld and Quandt [13] show that the derived aggregate switching model bears close resemblance in functional form to the corresponding micro model. Even if S cannot be assumed near zero, Goldfeld and Quandt [13] conclude that estimation of the micro switching model directly using the aggregate data, while inferior to the results obtained from estimating the derived three regime aggregate model, nonetheless can provide a reasonable approximation to the true results under a uniform underlying distribution. It would be interesting at some point to investigate estimation of Equation (13) using more sophisticated means.

III. Estimation and Testing of the Switching Regression Model

Estimation of the model of Equation (13) was performed with the five sets of borrowings data.[14] The technique employed, originally proposed by Quandt [23],

[13] The value of the coefficient itself does not appear to be meaningful because partial adjustment may not be the only effect R_{t-1}^B has on R_t^B. Returning to the implicit cost function, if it is taken into account that past as well as present borrowings contribute to implicit cost according to: $c(R_t^B, R_{t-1}^B) = \alpha e^{\beta_1 R_t^B + \beta_2 R_{t-1}^B}$, with $\alpha, \beta_1, \beta_2 > 0$, the borrowings function obtained will be negatively related to R_{t-1}^B. The combined effect of past borrowings on present borrowings may be the sum of partial adjustment and reluctance due to past borrowings.

[14] Data for borrowed reserves and total reserves were obtained from the *Federal Reserve Bulletin*,

can also be found in Goldfeld and Quandt [15]. A search procedure was used to estimate the switchpoint using maximum likelihood as the criterion. After this estimate is chosen, the sample is divided according to the estimated switchpoint and the two regimes are estimated by ordinary least squares.[15]

Estimated parameters for the loglinear switching regression model are presented in Table I. Seasonal dummy variable parameters were estimated but not reported. Parameter estimates of a switching regression model with the upper regime linear in the interest rate spread are presented in the Appendix. Also given in the Appendix are estimated parameters of a nonswitching model, obtained by regressing borrowed reserves linearly against the interest rate spread, reserve need, and the remaining variables added to both regimes of the switching model. Comparable to Goldfeld and Kane [12] and Keir [18], this nonswitching model will be employed in order to test for the existence of switching behavior. It should also serve as a ready basis of comparison for the estimated switching regression models.

With regard to the estimated switchpoints, all were found to be at relatively small but positive interest rate spreads, suggesting the existence of small but positive implicit costs involved with Discount Window borrowing. Surprisingly, the estimated switchpoint was smaller for the Country banks. The estimated switchpoint range of 0.195 for the five classes would indicate relatively homogeneous switching behavior.

In the lower regime, the constant term and the coefficient for the interest rate spread were positive and significant in all regressions. The reserve need coefficient was significant only under Reserve City, large New York City, and Chicago banks. It would seem that the city banks borrow from the Discount Window much more readily given an increased reserve need even under unfavorable spreads. The coefficients for Lagged Reserve Accounting and for the Seasonal Borrowing Privilege were not significantly positive for any class in the unfavorable spread regime. This result would indicate that more Discount Window borrowing in general did not necessarily take place as a result of these procedures.

For the upper regime, the coefficient for log K carried the expected positive sign in all cases. It was insignificant, however, with the regressions involving New York City banks, Chicago banks, and Country banks. With respect to the Country Bank case, borrowing behavior of the needs constrained regime might be occurring. For the other two classes, such behavior is difficult to explain.

With respect to the reserve need in the favorable spread regime, the coefficient was highly significant and of expected sign in all cases except for overall

Table: "Reserves and Borrowing Data," various issues. Interest rate data on the Federal Funds rate and the 90-day Treasury Bill rate (market yield) were obtained from *Banking and Monetary Statistics: 1941–70* for the time period 1955–70 and *Federal Reserve Bulletin*, Table: "Interest Rates: Money and Capital Markets" for 1971–78. Discount rates were obtained from the *Federal Reserve Bulletin*, Table: "Federal Reserve Interest Rates," various issues. All data are monthly averages. A complete listing of the data is available upon request.

[15] A unique maximum was obtained in all cases (up to consecutive switchpoint estimates which divided the sample identically). There were, however, numerous points of local maximums on the log likelihood function. This would indicate that, if a researcher chose to estimate this switching regression model using a more elegant technique, consistent initial estimates would be critical.

Table I

Estimated Parameters from the Switching Regression Model

	Lower Regime						Upper Regime				
Bank Class	All	New York	Chicago	Reserve City	Country	Bank Class	All	New York	Chicago	Reserve City	Country
Constant	53.790	41.258*	16.232*	32.242*	26.676*	Constant	369.296*	154.236*	49.320*	153.559*	−10.685
	(1.855)	(3.297)	(2.827)	(2.159)	(3.687)		(2.993)	(3.419)	(4.345)	(3.244)	(−0.264)
K	63.604*	26.336*	10.443*	22.840*	22.660*	log K	198.705*	24.960	2.279	85.636*	13.774
	(3.463)	(2.985)	(2.366)	(2.318)	(4.647)		(2.350)	(1.046)	(0.402)	(2.679)	(0.616)
ΔR^{UB}	−0.028	−0.102*	−0.448*	−0.093*	−0.032	ΔR^{UB}	−0.050	−0.302*	−0.246*	−0.197*	−0.210*
	(−1.252)	(−4.041)	(−12.594)	(−3.725)	(−1.517)		(−1.007)	(−4.346)	(−4.942)	(−4.448)	(−4.162)
R^B_{t-1}	0.887*	0.676*	0.839*	0.895*	0.811*	R^B_{t-1}	0.353*	0.395*	0.385*	0.396*	0.790*
	(34.373)	(12.975)	(22.371)	(34.100)	(26.182)		(3.757)	(4.189)	(3.823)	(4.360)	(11.393)
LRA	−28.633	7.752	3.329	−16.598	−4.201	LRA	114.142	−0.197	−10.812	70.504*	19.067
	(−1.300)	(0.694)	(0.568)	(−1.395)	(−0.626)		(1.494)	(−0.007)	(−1.466)	(2.270)	(0.793)
SBP	−11.550	−17.646	−13.015*	−8.970	−5.883	SBP	221.500*	−11.189	−6.167	99.643*	57.414*
	(−0.438)	(−1.359)	(−1.985)	(−0.621)	(−0.783)		(3.135)	(−0.435)	(−0.991)	(3.600)	(2.234)
						TE	63.663	−2.829	11.266	20.839	14.267
							(0.787)	(−0.107)	(1.670)	(0.642)	(0.658)
Estimated Switchpoint	0.295	0.105	0.145	0.295	0.145	FNB	717.424*	649.293*	—	—	—
							(4.424)	(5.998)			
SE/df	89.250/	40.309/	20.676/	48.527/	23.071/	SE/df	236.118/	104.010/	24.750/	94.634/	80.953/
	183	168	172	183	172		68	83	80	69	80
$\bar{R}^2$	0.905	0.604	0.826	0.902	0.852	$\bar{R}^2$	0.866	0.864	0.482	0.822	0.890

Note: Figures in parentheses are t-statistics. Parameters corresponding to seasonal dummy variables were estimated but not reported.
* Significant at the 5% level.

borrowing, where the coefficient was still negative. For all banks, there may be aggregation problems taking place either in the formulation of the reserve need proxy or a smoothing behavior corresponding to aggregation across banks where the reserve need does not present a constraint. In the two regressions involving Country banks, the reserve need coefficient was significant in the upper regime and insignificant in the lower regime, giving support to the Least Cost hypothesis. Even in the case of the Reserve City, New York City, and the Chicago classes, where the coefficients were significant in both regimes, the need reaction appears much stronger under favorable spreads.

There appears to be increased Discount Window borrowing in the upper regime due to the imposed structural changes. With respect to Lagged Reserve Accounting, the regression coefficient was significantly positive in the Reserve City bank class. The regression coefficient for the Seasonal Borrowing Privilege was also positive and significant with the Reserve City banks, providing evidence that large banks indeed appear to use this privilege under favorable spreads. The coefficient was significantly positive as well for the Country banks and for All banks, lending further support to Graham [16]. The imposition of the above procedures appeared to result in increased borrowing at the Discount Window, but only during times when the spread was favorable. Despite these appealing results, it should be noted that inferences based upon regression coefficients of dummy variables should be made with caution as the observed significance may be due to a structural change or variable other than the one hypothesized. Still, the results lend strong support to the derived switching regression Least Cost model.

The coefficient for tight versus easy money policy was insignificant in all reported regressions, arguing for the constancy of Federal Reserve policy toward the Discount Window.[16] Borrowing behavior due to seasonality was generally erratic but seemed to be a more significant cause in the upper regime (17 significant regression coefficients as opposed to 6 in the lower regime). The greatest borrowing due to seasonality occurred in April through August, most likely indicating increased borrowing due to agriculturally related loan demand.

The models fit well for the most part. The estimated variances of the upper regime were found to be consistently greater than the corresponding variances in the unfavorable spread regime. The only classes that fit poorly were that of New York and Chicago banks.

In previous studies, problems have been encountered in the estimation of borrowed reserve demand models with the two above bank classes. Goldfeld and Kane [12] found that the fit for the regression involving New York City banks was considerably poorer than for the other classes and that the interest rate spread did not constitute a significant cause of borrowings in the Chicago bank class. Perhaps the Least Cost model does not present a complete explanation of

[16] In the estimations using the Treasury Bill rate, this coefficient was found to be significant and negative for All banks and the Country bank class. This result provides an interesting explanation to a phenomenon first observed by Polakoff [21], the apparent backward bending behavior at higher interest rate spreads. Under the above situation, two loglinear demand curves in the upper regime result. Then given the imposition of tight money with probable increased spreads, the commercial bank would shift inward to the "tight money curve" decreasing borrowings.

Table II

Test Statistics for the Existence of Switching Regression Behavior

Bank Class	Likelihood Ratio[a]	Asymptotic Chow[b]
All	195.07	6.15
New York	123.92	3.65
Chicago	66.18	4.54
Reserve City	140.08	5.08
Country	279.96	13.06

[a] The appropriate degrees of freedom are 20 for the cases including the Franklin National Bank, 19 otherwise. The chi-squared statistic for the 1% level with 20 degrees of freedom is 37.57.

[b] The appropriate degrees of freedom are (18,251) for the cases involving the Franklin National Bank, (17,253) otherwise. An upper bound for the significant F-statistic at the 1% level is given by $F_{(16,200)}\,0.01 = 2.09$.

Discount Window borrowing by large money market banks. Still, it will be seen that, even with these banks, the switching regression model represents a significant improvement over the usual nonswitching linear model.

The statistics employed to test for the existence of switching regression behavior were the Likelihood Ratio statistic proposed by Quandt [23, p. 876] and the Asymptotic Chow statistic derived by Goldfeld and Quandt [14]. Care must be used in the selection of these test statistics due to the significantly greater variances in the upper regime, making the standard Chow test invalid. Test statistics for the linear switching regression model are given in Table II. Exact existence tests could not be performed with the loglinear upper regime model as the nonswitching linear hypothesis is not nested in the latter switching regression model.[17]

Consider the existence of switching regression statistics. Each is highly significant at the 99% level for every bank class. Clearly, behavior appropriate to the switching regression problem appears to be the case. Furthermore, given the previous empirical results, the overall evidence strongly supports the derived Least Cost switching regression model.

IV. Concluding Remarks

If the loglinear switching regression model is valid, borrowings from the Federal Reserve may present difficulties in controlling the money stock. The potential for disruption appears especially applicable when the interest rate spread is

[17] It can be argued that the loglinear upper regime model can be at least approximately tested against the usual linear nonswitching model. The hypotheses are nearly nested and each statistic is dependent upon the upper regime only through its estimated variance. These "pseudo existence test statistics," formed by substituting the estimated variance of the loglinear switching model, were found to be quite similar to the statistics corresponding to the formal test.

Appendix

Estimated Parameters from the Nonswitching Model and the Upper Regime of the Linear Switching Regression Model

	The Nonswitching Model						The Linear Switching Regression Model				
Bank Class	All	New York	Chicago	Reserve City	Country	Bank Class	All	New York	Chicago	Reserve City	Country
Constant	134.842*	93.000*	22.556*	59.206*	16.050	Constant	141.737	124.686*	38.234*	65.236	−31.678
	(3.293)	(5.982)	(4.074)	(3.250)	(1.222)		(1.325)	(2.867)	(3.491)	(1.520)	(−0.900)
K	122.722*	30.691*	6.611*	34.163*	34.297*	K	177.322*	29.556*	7.606*	53.098*	25.598*
	(7.207)	(6.064)	(3.960)	(4.811)	(5.545)		(4.794)	(2.595)	(2.851)	(3.511)	(2.080)
ΔR^{UB}	−0.063*	−0.211*	−0.347*	−0.179*	−0.154*	ΔR^{UB}	−0.058	−0.306*	−0.248*	−0.219*	−0.206*
	(−2.637)	(−7.371)	(−11.400)	(−7.375)	(−5.708)		(−1.193)	(−4.581)	(−4.965)	(−4.950)	(−4.136)
R^B_{t-1}	0.730*	0.424*	0.745*	0.823*	0.804*	R^B_{t-1}	0.386*	0.401*	0.428*	0.492*	0.765*
	(21.089)	(9.322)	(19.137)	(25.986)	(24.536)		(4.480)	(4.633)	(4.305)	(5.900)	(11.353)
LRA	−24.833	−6.967	−7.502	0.094	1.876	LRA	103.083	−8.784	−9.277	68.024*	16.551
	(−0.862)	(−0.589)	(−1.735)	(0.007)	(0.192)		(1.369)	(−0.299)	(−1.246)	(2.134)	(0.700)
SBP	4.865	−16.078	−4.015	−7.791	16.226	SBP	192.924*	−9.696	−8.875	77.406*	56.905*
	(0.153)	(−1.233)	(−0.883)	(−0.557)	(1.419)		(2.866)	(−0.396)	(−1.429)	(2.912)	(2.369)
FNB	224.862*	582.172*	—	—	—	FNB	562.449*	607.338*	—	—	—
	(2.514)	(10.489)					(3.633)	(5.816)			
SE/df	167.721/	69.428/	24.811/	76.547/	57.396/	SE/df	233.097/	101.394/	25.159/	97.577/	79.783/
	269	269	270	270	270		69	84	81	70	81
$\bar{R}^2$	0.907	0.857	0.694	0.882	0.924	$\bar{R}^2$	0.869	0.870	0.464	0.810	0.893

Note: Figures in parentheses are t-statistics. Parameters corresponding to seasonal dummy variables were estimated but not reported.

* Significant at the 5% level.

positive and small. The results also lend support to a policy of pegging the Discount Rate to the Federal Funds rate, particularly at a negative spread. As the lower regime estimated variance is seven times smaller than its upper regime counterpart in the All banks regressions, the results suggest that a penalty rate policy would lead to substantially improved forecasts for borrowings in the reserve intermediate targeting procedure.

In any case, the overall theoretical and empirical results imply that the switching regression model, either linear or loglinear in the interest rate spread, appears quite superior theoretically and empirically to the commonly used nonswitching linear model in explaining the demand for borrowed reserves from the Federal Reserve. It would seem that this model represents a significant step toward a completely developed theory of Discount Window borrowings.

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