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Seasonality Estimation in Thin Markets

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ABSTRACT

The greater availability of daily data in the U.S. has led to a number of studies of the seasonality of daily stock (index) returns. While the studies recognized the potential impacts of nontrading and price-adjustment delays in general, no formal analyses of such impacts were presented; in this paper analytic results are presented for the articulation between these phenomena. The implications of the analysis are discussed and shown to be consistent with a sample of U.K. index data. A modified form of the negative weekend effect is found to be present in the U.K. data analyzed.

TESTS FOR THE PRESENCE of seasonality in equity returns have been carried out by a number of authors at both the monthly and daily return levels. At the monthly level, Rozeff and Kinney [18] discovered evidence of seasonality in the U.S., while Officer [15] detected the presence of seasonality in Australian data. Fama [7] carried out some early work on U.S. daily data, and more recent contributions have been made by Cross [5], French [9], and Gibbons and Hess [10]. While these studies recognized the presence of thin trading and its potential impacts upon the experimental results, no formal analyses were presented. In this paper, we derive analytic results for the articulation between seasonality and nontrading effects[1] (Section I). The implications of this analysis are tested using a sample (described in Section II) of U.K. daily index data[2] in Section III. Section IV discusses the implications of the results for capital market efficiency. Finally,

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[1] Nontrading is, of course, not the only cause of "observed" prices differing from "true" prices. More generally, this phenomenon arises due to delays in the price-adjustment process (see Cohen et al. [2, 3]). The causes of these delays are many, but include delayed portfolio rebalancing (for an extended discussion see Schwartz and Whitcomb [20]), dealer inventory adjustments (see Amihud and Mendelson [1]), and the sequential dissemination of information (see Copeland [4] and Jennings et al. [11]). The results of the analytic section of this paper can be interpreted both within the more general price-adjustment delays and more restricted thin trading frameworks (see Section I for further discussion). However, the experimental design is developed in terms of trading delays (since these are observable) and it is for this reason that the thin trading terminology is used more often in this paper.

[2] The empirical work was carried out using index data as a broadly based data base containing individual security daily prices suitable for academic research not available in the U.K. (The London Business School Data Base contains only monthly security price data). Our results correspond to and corroborate those of Cross [5] and French [9], who use U.S. index data only, and the index results reported in Gibbons and Hess [10]. The latter authors looked at day-of-the-week effects for the constituent stocks of the Dow-Jones Index and found the day-of-the-week effect to be robust across these stocks.

the conclusions of this paper are presented in Section V.

The problem of thin trading/price-adjustment delays has been extensively analyzed in the literature. Fisher [8] was the first to recognize the existence of serial correlations in stock market indexes and to identify the associated problem of nontrading. The nontrading effect will lead to a bias in the traditional ordinary least squares estimator of beta (see, for example, Scholes and Williams [19], Dimson [6], and Cohen et al. [3]) which will decline as the interval over which beta is estimated increases (see Cohen et al. [2] and Theobald [21]). Index autocorrelations impact upon the degree of bias in beta, and they are also used as one means of detecting seasonality in a time series. As will be demonstrated in Section I, thin trading will also impact upon the measurement of the means and variances of index returns, which, when classified across different seasonals (e.g., months or days of the week), are used to detect seasonality. There are, then, strong interrelationships between thin trading biases in beta, index autocorrelations, and measured seasonality. Roll [17] used index data to demonstrate that the "size effect" may be partially caused by the downwards bias in the ordinary least squares beta estimator for small firms (due to their less regular trading profiles). Subsequent studies by Reinganum [16] and Keim [12], for example, have shown that a small firm effect does still exist after adjusting for these biases. The latter study has also discovered some seasonality in the size effect—almost 50% of the small firm premium arises due to abnormal returns in January. Thus, seasonality and potential thin trading effects arise again in a currently important area of the empirical finance literature. Index seasonality will also impact upon the results of event studies which use daily data, where shifts in the mean returns across days of the week are not controlled.[3]

The presence of thin trading on the London Stock Exchange has been demonstrated in a number of papers (see, for example, Dimson [6]) at the monthly level; the impact at the daily level would naturally be anticipated to be greater. The two indexes used in this study, the Financial Times (FT) Ordinary and FT-All Share Indexes, as well as being easily the most widely cited indexes in the U.K., differ in their compositions; although value weighted, the FT-All Share Index would be predicted to be the more affected by nontrading due to its broader composition. This property is used to evaluate the impact of thin trading upon the measured seasonality across days of the week in the U.K. The empirical results are consistent with the analytics developed in this paper, in that measured seasonality in the mean is found to be weaker for the less regularly traded index. A negative weekend effect was detected (as in U.S. studies) but was of a modified form; persistently negative returns were present only in the second Monday of the Stock Exchange Account. Significant departures from the normal distribution were detected in the daily return data in a number of cases.

[3] See Gibbons and Hess [10], who incorporate seasonal dummy variables on the slope coefficient of the market model to reflect shifts in the mean across days of the week. Thin trading/price-adjustment delays in the individual stocks will induce further noise into the generated residuals series (see Theobald [21]).

I. The Analytics of Seasonality and Thin Trading

The detection of seasonality in stock return series generally involves investigating the moments of the return distributions, classified across different seasonals (i.e., days, months, or quarters).[4] Since nontrading will impact upon the measured means, variances, and autocovariances of the return distributions (see, for example, Scholes and Williams[5] [19]), the need for analyzing the articulation between nontrading and seasonality effects is readily apparent. As we have already noted (see footnote 1), the moments of the return distribution will be affected more generally by delays in the price-adjustment process. We argue, below, that the analytics can be interpreted in both price-adjustment and trading delays terms, and throughout this section both terms are used interchangeably. The thin trading interpretation, however, relates more easily to the empirical section (since price-adjustment delays are not directly observable[6]).

We will demonstrate the impact of nontrading effects upon the measured means, variances, and autocovariances of the returns on an index[7] within a framework which has features in common with those of Dimson [6] and Cohen et al. [3]. This framework, has the advantage over that of Scholes and Williams [19] in that a minimum consecutive trades period does not have to be assumed or used. The presence of trading or price-adjustment delays will mean that the measured (or observed) returns series will differ from the true series (i.e., those returns which would arise in the absence of frictions in the trading process). The measured return on an index in period t, I_t^m, is related to the true (and unobservable) return on the index in period $t-j$, I_{t-j}, by an adjustment factor, π_j, as[8]

$$I_t^m = \textstyle\sum_{j=0}^{n} \Pi_j I_{t-j} + u_t \qquad (1)$$

where u_t is a white noise disturbance term. The adjustment factor, Π_j, is assumed to be a random variable, and it may be interpreted as the proportion of the index last traded in period $t - j$ (i.e., a thin trading interpretation in light of the empirical data to follow). This interpretation corresponds to that of Dimson [6]; however, the adjustment factor is effectively deterministic within Dimson's framework when determining moments of the return distribution. It may also be interpreted (in sequence) as a delay distribution in an aggregate sense within the

[4] Rozeff and Kinney [18] suggest that, at the monthly level, seasonal variations are picked up in the mean, rather than in the autocorrelations corresponding to the seasonal lags. French [9] and Gibbons and Hess [10] found a similar mean effect for daily returns data.

[5] Scholes and Williams [19] demonstrate that for a true mean stationary process, the measured mean will be unbiased (with i.i.d. nontrading periods). We demonstrate, in this section, that a bias is introduced when the true process is mean seasonal.

[6] The problem of the observability of price-adjustment delays may be circumvented by assuming that trading delays serve as a proxy for the price-adjustment delays (i.e., since trading delays are one cause of the price-adjustment delays it would appear, *a priori*, reasonable that the two variables will be positively associated; it does not seem compelling that price-adjustment delays, other than trading delays, would be negatively associated with trading delays).

[7] While the results are developed here within an index framework, the relationships will apply at the individual security level when suitable adjustments are made to the variable definitions in Equation (1).

[8] A finite maximum trading delay, n, is assumed.

Cohen et al. [3] framework, which is developed within a more general price-adjustment delay scenario. The adjustment factor used here differs from that in Cohen et al. [3] in that only one subscript is necessary, since the measured and true returns are related relative to measured returns in time[9] (and a security indicator is unnecessary). In order to determine the impact of thin trading/price-adjustment delays upon measured seasonality, it is necessary to relate the moments of the measured return distribution to those of the adjustment factor and the true return distributions. The relationships are complicated since the measured return is represented as a sum of products of random variables in Equation (1). The expectation, variance, and autocovariances of the measured series are derived in the Appendix for the case of a nonseasonal process. The results indicate that the measured moments will be a function of contemporaneous and past moments of the true return process (linked by the adjustment factor, Π, moments) and that the measured autocovariance will be positive even when the true series is serially uncorrelated. The covariance will decline with increasing lag. Seasonality can then be introduced into these expressions by appending an appropriate seasonal indicator to those moments of the true return distributions which display some seasonality.

The presence of mean seasonality necessitates relaxing the assumption that the true return distribution is independently and identically distributed through time. Assuming that the adjustment factor is identically distributed through time and is independent of the true return distribution, the expected return on the measured index in season 0 will be, using result (A4) in the Appendix, given by the expression

$$E(I_t^{m,0}) = \textstyle\sum_{j=0}^{n} E(\Pi_j)E(I_{t-j}^{i(j)}) \tag{2}$$

where $i(j)$ is a seasonal indicator and is defined by $i(j) = j - ms$, $\forall j$, and $m = 0$ when $j < s$, $m = \rho$ when $\rho s \leq j \leq (\rho + 1)$, for ρ an integer. s denotes the number of seasonals present in the true returns series. The intuition behind Equation (2) can be demonstrated by considering the simplest scenario possible, where there is a one-period lagged adjustment and two seasons (i.e., $n = 1$ and $s = 2$). The expected return on the measured index in season 0 will be

$$E(I_t^{m,0}) = E(\Pi_0)E(I_t^0) + E(\Pi_1)E(I_{t-1}^1) \tag{3}$$

The measured mean return will differ from the true mean return in season 0 as a result of the impact of true mean returns from season 1, i.e., a diffusion of seasonality across other seasonals will occur. Thus, if 0 is Monday and season 1 represents the other days of the week, and if the true mean Monday (non-Monday) return is negative (positive), then the extent of the negative Monday mean return in the measured series will be less than that in the true return series.

When seasonality in trading proportions exists, the relationship developed in

[9] Interpreting Π_j as the proportion of the measured return in period t relating to the true return in period $t - j$ will ensure that the sum of the realizations of the adjustment factors will be one. In an equally weighted index this property will mean that true returns are fully reflected in measured returns, on average, while in a value weighted index this "full reflection" property will hold approximately.

Equation (2) is modified somewhat. The trading proportion, Π, now needs a seasonal indicator,[10] but, additionally, some dependence between Π and I could arise. Thus, Equation (2) becomes

$$E(I_t^{m,0}) = \sum_{j=0}^{n} E(\Pi_j^0)E(I_{t-j}^{i(j)}) + \sum_{j=0}^{n} \operatorname{cov}(\Pi_j^0, I_{t-j}^{i(i)})$$

The extent to which mean seasonality is diffused over other seasonals will now be additionally affected by the $\operatorname{cov}(\Pi I)$ terms. Where a positive association exists between these variables, the negative Monday effect will be further reduced in the measured series relative to the true series.

Inspection of Equation (A8) indicates that mean seasonality will impact upon the measured variance, even when the true series has no seasonal variance shifts, via terms containing $E(I_{t-j}^{i(j)})$. In addition, the variance of the measured series may now incorporate some autocovariance at the seasonal lag (unless, of course, all constituent stocks have been traded since the last corresponding season). The measured variance is then given by[11]

$$\begin{aligned}\operatorname{var}(I_t^{m,0}) = {} & \sum_{j=0}^{n} [E(I_{t-j}^{i(j)})^2\operatorname{var}(\Pi_j) + E(\Pi_j)^2\operatorname{var}(I_{t-j}) \\ & + \operatorname{var}(\Pi_j)\operatorname{var}(I_{t-j})] + \operatorname{var}(u_t) \\ & + 2\sum_{j=0}^{n-1}\sum_{k=1}^{n} E(I_{t-j}^{i(j)})E(I_{t-k}^{i(k)})\operatorname{cov}(\Pi_j\Pi_k) \\ & + 2\sum_{j=0}^{n-s} [E(\Pi_j)E(\Pi_{j+s}) + \operatorname{cov}(\Pi_j\Pi_{j+s})]\operatorname{cov}(I_{t-j}^{i(j)}\, I_{t-j-s}^{i(j)}) \qquad (4)\end{aligned}$$

Some intuitive results can be gleaned from Equation (4). When there are no thin trading/price-adjustment delays, $\Pi_0 = 1$ and will be nonstochastic (i.e., $\Pi_j = 0$, $\forall\ j \neq 0$, and $\operatorname{cov}(\Pi_j\Pi_k) = 0$, $\forall\ j, k$); Equation (4) reduces in this case to $\operatorname{var}(I_t^{m,0}) = \operatorname{var}(I_t)$. However, when $E(\Pi_0) \neq 1$, the seasonal mean terms will impact upon the measured variance, and the degree of impact will depend upon both the variabilities of the adjustment factor and the true index returns. The larger the variability of Π the greater the impact of the seasonal means, *ceteris paribus.* Since the variability of Π would be anticipated to be greater for the more thinly traded index, the measured seasonality would be anticipated to be larger for the less regularly traded index.[12] However, if the variability of the true index returns is very much larger than the variability of the adjustment factors, the impact of seasonality upon measured variances will be minimal. A comparison of the relative magnitudes of the coefficients of variability of the FT-indexes and the total equity bargains (a crude proxy for Π) revealed that the former was always the larger by several orders of magnitude. The impact of mean seasonality

[10] Π_j^0 will now be a function of the proportion of the index traded in j and an adjustment for the changes in trading proportions in moving from season $s - 1$ to 0.

[11] Π_j was assumed to be nonseasonal to simplify the notation and relationships developed. Nothing of a substantive nature was lost by these simplifications. The true process generating returns is assumed to be covariance stationary. Index covariance at the seasonal lag is included for completeness of the analysis; the seasonal lag covariance term has been calculated, on the assumption that $n < 2S$, to simplify the notation.

[12] For a very frequently traded index $\Pi_0 \simeq 1$ with very little variability. As the trading frequency declines, the variability will increase. The impact of seasonal means will also be greater, the greater the heteroskedasticity in Π's (i.e., if $\operatorname{var}(\Pi_j) \simeq \operatorname{var}(\Pi_k)$, $\forall\ j, k$, very little seasonality will be detected). Again, heteroskedasticity in Π's would be anticipated to be greater, the thinner the index.

on the measured variance is predicted to be insignificant for the seasonality type investigated in this paper.

Introducing variance seasonality into the variance formula (A8) necessitates appending a seasonal superscript to variance terms as

$$\begin{aligned}\operatorname{var}(I_t^{m,0}) = {} & \textstyle\sum_{j=0}^{n} [E(\Pi_j)^2 \operatorname{var}(I_{t-j}^{i(j)}) + \operatorname{var}(\Pi_j)E(I_{t-j})^2 + \operatorname{var}(\Pi_j)\operatorname{var}(I_{t-j}^{i(j)})] \\ & + 2 \textstyle\sum_{\substack{j=0 \\ j<k}}^{n-1} \sum_{k=1}^{n} E(I_{t-j})E(I_{t-k})\operatorname{cov}(\Pi_j \Pi_k) + \operatorname{var}(u_t) \\ & + 2 \textstyle\sum_{j=0}^{n-s} [E(\Pi_j)E(\Pi_{j+s}) + \operatorname{cov}(\Pi_j \Pi_{j+s})]\operatorname{cov}(I_t^{i(j)}\ I_{t+s}^{i(j)}) \end{aligned} \tag{5}$$

The expression for the measured variance in Equation (5) indicates that the closer $E(\Pi_0)$ is to one, *ceteris paribus*, the less will be the impact of the variance of other seasonals in the true process. However, diffusion of the true variance seasonality over other seasonals in the measured series will not necessarily always increase with thinness of trading. For the simple case already analyzed, where $n = 1$ and $s = 2$, the measured variance seasonality will correspond more closely to the true variance seasonality, the larger $\operatorname{var}(\Pi_0)$ is relative to $\operatorname{var}(\Pi_1)$. The extent of the diffusion of true variance seasonality across other seasonals in the measured variances will then depend upon the relative magnitudes of $E(\Pi_0)$, $E(\Pi_1)$, $\operatorname{var}(\Pi_0)$, and $\operatorname{var}(\Pi_1)$ in this simple scenario. The coefficients of variability for equity bargains were found to be considerably less than one, indicating that the diffusion of any true variance seasonality over other seasonals will plausibly increase with thinness in the index for the seasonality type empirically analyzed in this paper.

The lag x autocovariance in the presence of both mean and variance seasonality for the case where $x < s$ and Π is nonseasonal is obtained from Equation (A11) as

$$\begin{aligned}\operatorname{cov}(I_t^m, I_{t-x}^m) = {} & \textstyle\sum_{j=0}^{n} \sum_{k=x}^{n+x} E(I_{t-j}^{i(j)})E(I_{t-k}^{i(k)})\operatorname{cov}(\Pi_j \Pi_{k-x}) \\ & + \textstyle\sum_{j=x}^{n} [E(\Pi_j)E(\Pi_{j-x}) + \operatorname{cov}(\Pi_j \Pi_{j-x})]\operatorname{var}(I_{t-j}^{i(j)}) \\ & + \textstyle\sum_{j=0}^{x} [E(\Pi_j)E(\Pi_{j+s-x}) + \operatorname{cov}(\Pi_j \Pi_{j+s-x})]\operatorname{cov}(I_{t-j}^{i(j)}\ I_{t-j-s}^{i(j)}) \end{aligned} \tag{6}$$

Autocovariances will be induced into the measured series due to nontrading via the first two terms in Equation (6) even when the true return series is serially independent; the impact will be greater in the thinner index. These autocovariances will mask and diffuse autocorrelations at the seasonal lag. Thus, seasonal lag autocovariances will be stronger and more easily discerned for the more frequently traded index.

II. Sample Details

The levels of the Financial Times Ordinary (FTO) and Financial Times Actuaries All Share (FTAS) Indexes were collected on a daily basis for the period starting June 1, 1975 and ending May 31, 1981 (a total of 1,458 observations on each index, excluding holidays). The FTO is a geometrically averaged index of the leading 30 U.K. equity shares and, as such, is subject to very little in the way of nontrading impacts. The FTAS is a value weighted index of 750, or so, U.K.

equities. It is a broader based index and will contain less frequently traded stocks. However, the value weighting scheme will mitigate against this to an extent (assuming total market value is an instrumental variable for thinness of trading).[13] Nevertheless, there will be differentials between the nontrading levels of the two indexes given their different constituents, with the FTAS being the less regularly traded index of the two.[14,15]

The daily levels of equity bargains (i.e., the number of equity transactions) and equity turnover (in £ millions) were also collected for the whole of the sample period to provide some evidence as to whether there was any daily seasonality present in daily trading "volumes."

III. Seasonality Tests

A. Financial Times Ordinary (FTO) and Actuaries All Share (FTAS) Indexes

The results of the seasonality tests for the two indexes are contained in Tables I and II. In Table I, the means and variances of the return distributions for the FTO and FTAS are presented, together with the studentized range,[16] for the whole data set (June 1, 1975 to May 31, 1981) and four subperiods.

The results in Table I provide considerable evidence of nonnormality in the underlying return distributions. For example, in the case of the whole data set, kurtosis[17] and studentized range results are significantly nonnormal at the 5% critical level for all days of the week; skewness was not significantly different from zero in two cases only (FTO on Thursday and FTAS on Friday). Although nonnormal behavior is less pervasive in the subsamples, significant degrees of nonnormality are still exhibited. Small sample tests assuming normality are thus suspect and nonparametric tests have been carried out to supplement t-tests. The negative Monday effect on the mean, documented in U.S. studies, is very apparent in Table I, where all the Monday mean returns are negative for the

[13] Empirical support for this assumption in the U.K. is provided in Dimson [6], Table 8(a), where the market values of companies which fall in the highest and lowest frequency of trading deciles are analyzed.

[14] An inspection of the constituents of the FTAS revealed that there were 33 stocks with total market capitalizations greater than £1,000,000,000 which had average nontrading profiles of between 0.1 and 1 day nontrading, and two with average nontrading periods between 1 and 7 days. Only four of the constituents of the FTO had nontrading periods in the range 0.1–1 day (source: *Risk Measurement Service*, January–March 1980; published by LBS Financial Services). Empirical support is provided in Section III for the differences in levels of thin trading for the two indexes.

[15] The two indexes will, of course, differ in their response due to the different weighting schemes. A geometric index will always decline more rapidly and rise more slowly than a corresponding arithmetic index. While this could make the negative Monday impact greater for the FTO, it will also reduce the positive Tuesday recovery.

[16] We report statistically significant nonnormal skewness and kurtosis also. Skewness was measured by $\frac{1}{N}\sum_{t=1}^{N}[(I_t-\bar{I})/s]^3$ and kurtosis by $\frac{1}{N}\sum_{t=1}^{N}[(I_t-\bar{I})/s]^4-3$, where $s=\frac{1}{N-1}\sum_{t=1}^{N}(I_t-\bar{I})^2$.

[17] Scholes and Williams [19] demonstrate that kurtosis will be increased above zero in the presence of thin trading. Measured kurtosis was larger for the FTAS (58.019) than the FTO (49.819) when estimated over the whole data set, which is consistent with the prior greater nontrading impacts in FTAS.

Table I

Stock Return Seasonality

	6/1/75–5/31/81			6/1/75–12/31/76			1/1/77–5/31/78			6/1/78–12/31/79			1/1/80–5/31/81		
	Mean	Std. Dev.	Stud. Range	Mean	Std. Dev.	Stud. Range	Mean	Std. Dev.	Stud. Range	Mean	Std. Dev.	Stud. Range	Mean	Std. Dev.	Stud. Range
MONDAY															
FTAS	−0.002	0.012^{b}	9.42^{deh}	−0.004	0.017	6.59^{deh}	-0.001^{f}	0.012	4.42^{de}	−0.000	0.009	4.89	−0.001	0.010^{b}	5.20^{e}
FTO	−0.003	0.020^{b}	12.75^{deh}	−0.005	0.021	6.29^{deh}	-0.004^{f}	0.014	5.21^{de}	−0.001	0.010	4.70^{c}	−0.004	0.029^{b}	8.39^{deh}
TUESDAY															
FTAS	0.001^{agi}	0.016^{c}	14.44^{deh}	0.002^{afgi}	0.013^{b}	5.69^{e}	0.002^{agi}	0.011	4.00^{c}	-0.001^{fg}	0.025^{bc}	8.12^{deh}	0.001^{a}	0.010	5.50^{de}
FTO	0.003^{agi}	0.015^{c}	8.93^{deh}	0.006^{afgi}	0.021^{b}	6.38^{deh}	0.003^{agi}	0.014	4.36^{c}	0.003^{afgi}	0.011^{b}	4.82	0.001^{ai}	0.013^{c}	3.77^{e}
WEDNESDAY															
FTAS	0.002^{agi}	0.016^{c}	14.75^{deh}	0.001^{a}	0.012^{c}	5.58^{c}	0.002^{ag}	0.011	5.56^{de}	0.003^{f}	0.026^{be}	8.61^{deh}	0.003^{agi}	0.008^{b}	5.50^{e}
FTO	0.001^{agi}	0.019	13.68^{deh}	-0.002^{a}	0.013^{c}	4.39^{c}	0.004^{agi}	0.015	6.20^{deh}	-0.002^{f}	0.012^{b}	4.50	0.005^{ag}	0.029^{b}	8.87^{deh}
THURSDAY															
FTAS	0.000^{ai}	0.011	7.36^{deh}	-0.001^{a}	0.013	4.92^{c}	0.002^{agi}	0.012	6.17^{deh}	-0.002^{af}	0.009	7.67^{deh}	0.001^{a}	0.009^{b}	4.89^{e}
FTO	-0.001^{ai}	0.014^{c}	7.71^{ch}	-0.002^{i}	0.017	6.36^{eh}	0.000^{ai}	0.013	4.54	-0.004^{afi}	0.009	5.22	0.001^{a}	0.013^{bc}	4.85^{de}
FRIDAY															
FTAS	0.001^{ai}	0.010	7.50^{ch}	0.002^{ag}	0.013	5.46^{e}	-0.001^{i}	0.009	4.89	0.001^{i}	0.010	6.90^{deh}	0.001^{a}	0.008	5.88^{e}
FTO	0.002^{agi}	0.012^{c}	6.83^{deh}	0.002^{agi}	0.016	5.13^{de}	0.000^{agi}	0.011	5.09	0.002^{ai}	0.011	5.36^{de}	0.001^{a}	0.010^{c}	4.90

[a] Statistically significant at 5% level from corresponding Monday level using conventional t-tests.
[b] Statistically significant at 5% level from the other index for that day using conventional F-tests.
[c] Statistically significant at 5% level from corresponding Monday level using conventional F-tests.
[d] Statistically significant at 5% level from corresponding normal distributional properties (skewness).
[e] Statistically significant at 5% level from corresponding normal distributional properties (kurtosis).
[f] Statistically significant at 5% level from the other index for that day using conventional t-tests.
[g] Statistically significant at 5% level from corresponding Monday level using Mann-Whitney test.
[h] Statistically significant at 5% level from corresponding normal properties (studentized range).
[i] Statistically significant at 5% level from corresponding Monday level using Median test.

Table II

Kruskal-Wallis Chi-Square Statistics

Data Set / Index	FTAS	FTO
6/1/75–5/31/81	19.97*	31.24*
6/1/75–12/31/76	7.72	15.94**
1/1/77–5/31/78	11.67**	12.40**
6/1/78–12/31/78	11.12**	17.78*
1/1/80–5/31/81	8.10	4.18

* Significant at 1% level.
** Significant at 5% level.

whole data set and four subperiods. On a pairwise basis (i.e., Monday with Tuesday, Monday with Wednesday, etc.), considerable evidence of a significant difference in Monday's mean returns was found. The impact is strongest when measured over the whole data set, where significant differences (at the 5% level) were found in all cases using t-tests and Median tests, while the Mann-Whitney test indicated differences in five out of the eight cases. Significant differences are still present when returns are measured over subperiods, although the impact is greater in the first two subperiods[18] and is more pervasive in the case of Monday with Tuesday.

The measured seasonality in the mean was generally stronger in the case of FTO which is consistent with the impact of nontrading[19] discussed in Section I. Test statistics generally had higher values[20] for FTO. Not only was the mean Monday return for FTO more negative in all cases, the mean Tuesday return was generally more strongly positive also (which is consistent with a thin trading interpretation of the differential index results, rather than a weighting scheme interpretation). Further evidence for the differential impact of seasonality in the indexes is provided by the Kruskal-Wallis test statistics reported in Table II. In all but the last subperiod (1/1/80 to 5/31/81) for both indexes, and the first subperiod (6/1/75 to 12/31/76) for the FTAS, the Kruskal-Wallis statistics are significant at least at the 5% level. The statistic is higher for the FTO in four of the five cases, which is consistent with a stronger shift of location across days for the more highly traded index.[21]

The results in Gibbons and Hess [10], Table I, can be interpreted within the price-adjustment/thin trading framework developed in this paper. Since company size may be viewed as an instrumental variable for nontrading (see, Cohen et al. [3] for U.S. evidence), the CRSP equally weighted index would be anticipated to

[18] t-Tests indicated significance in all eight cases in the last subperiod; however, both the Median and Mann-Whitney tests indicated significance in two cases only.

[19] It should, of course, be noted that price-adjustment delays other than trading delays could impact upon the empirical results reported in this section and reduce the internal validity of the experiment. To the extent that these two delay types are strongly associated, the impact will be minimal.

[20] The mean t-value for the FTO was 4.25 versus 3.20 for the FTAS, for example. Similar results were obtained with the Mann-Whitney and Median tests.

[21] To investigate whether monthly seasonality impacted further on the results reported here, we analyzed day effects by month. The FTO (FTAS) index return on Mondays was negative in eleven (ten) cases but no additional impacts were detected.

be more affected by nontrading than the corresponding value weighted index. Thus, measured mean seasonality, in particular the Monday effect, would be anticipated to be lower in the former. In fact, of the seven cases presented, six had the CRSP equally weighted index with the less negative Monday return.[22] As further potential evidence of a nontrading impact (and diffusion of seasonality over the time series), the mean returns on Tuesdays are lower on the equally weighted index in six of the seven cases presented also. The S&P 500 Index has the most negative Monday effect of all three indexes in all seven cases; this result is also potentially amenable to a thin-trading type rationalization. That is, the S&P 500 Index is less broadly based than CRSP with a potential reduction in the incidence of thinly traded stocks in its composition.

Returning to the results in Table I, F-tests on variance equalities across days indicated less pervasive patterns of variance seasonality. Only twelve cases were found (seven in the FTO and five in the FTAS). The greatest incidence occurred when variances were estimated using the whole data set (five cases).[23] The discussion in Section I indicated that mean seasonality could induce variance seasonality into the measured series. The impact would be expected to be greatest for a thinly traded index with heteroskedastic trading proportions distributions; however, the empirical evidence cited in Section I suggested that the actual impact of mean seasonality on measured seasonality was likely to be slight. The evidence in this section suggests very little difference between the indexes in measured variance seasonality, indicating that some variance seasonality in the true process is present, but that it is not a strongly pervasive phenomenon.

The sample autocorrelation function was inspected for both indexes for all four subperiods.[24] In all four cases, the lag one autocorrelation was more than twice the standard error for the FTAS, while in no case did this occur for the FTO. This result is consistent with the analytics developed in Section I, where it was predicted that the presence of nontrading or price-adjustment delays would induce measured autocovariances into an otherwise independent series. Autocorrelations with twice their standard errors were present for both indexes at lag ten in the first subperiod and lag fifteen and thirty for the second subperiod. These significant higher order autocorrelations are indicative of a potential "Account" effect which is more fully discussed and analyzed in Section *B*. An unambiguous interpretation of the differential impacts of nontrading on seasonal autocorrelations is difficult due to the generally weaker seasonal effects in the correlogram. The Box and Pierce Q-statistic will reflect both seasonal and thin trading impacts with the consequent difficulty of interpretation of seasonality impacts.[25]

[22] If the comparison of the CRSP equally weighted and value weighted index mean returns is viewed as an experiment comprising seven repeated independent Bernoulli trials with equal probability of either index mean exceeding the other, the probability of observing this set of observations is 0.055.

[23] Potentially, this result arose due to heteroskedasticity across subperiods.

[24] We do not report these results in detail in order to keep the paper at a manageable length. As in other studies, seasonality is more strongly evidenced in the means rather than the correlogram.

[25] For example, in the first subperiod, the Q-statistic was greater for FTAS (and significant at the (traditional) 10% level) while in period two, FTO had the larger Q-statistic (although neither were statistically significant at the 10% level).

B. The Ex Div Effect and the Settlement Date/Account Effect

A potential contributing factor to the negative Monday mean return is that stocks typically go ex div on Mondays (or, more specifically, the first Monday of the Account). Ex div dates were collected for the whole sample period and Monday returns were partitioned down into those which correspond to an ex div date and those which did not (generally, the second Monday of the Account).[26] The results which were obtained with this partition are contained in Table III. At first sight, the results appear somewhat perplexing—the non ex div Mondays are more strongly negative over the total data set and each of the four subperiods for both of the indexes. A potential rationalization for these results is to be found in the Account/Settlement Date system employed on the London Stock Exchange.[27]

The trading year is divided into a set of Accounts, which are typically of 2-week duration and commence on a Monday. Settlement, however, is not due until the second Monday after the end of the Account. Thus, the first Monday's price[28] incorporates a 21-day interest grossing-up adjustment. The base period Friday price used to calculate the Monday return only incorporates a 10-day interest grossing-up effect. This "interest effect" will contribute a strong and positive element to the returns on the first Monday of the Account; the positive impact will, of course, be tempered by the fact that the first Monday is generally the ex div date.[29] The generally nonnegative return on ex div Mondays will reflect these two phenomena. The return on the second Monday of the Account will have a 14-day grossing-up effect, versus a 17-day grossing-up effect for the first Friday of the Account. On a Settlement Date analysis, then, the second (or non ex div) Monday returns will be negative. The Settlement Date argument cannot fully explain the observed seasonality on the London Stock Exchange since for days other than Monday, a negative 1-day interest effect would be predicted, which is not apparent in the returns contained in Table I. Nevertheless, it does provide a plausible rationalization for the differing Monday effects and indicates that the weekend effect[30] in the U.K. is of a somewhat more complex nature than that in the U.S. Inspection of the results of Table III also indicates that the seasonality effects, while persistent when measured across the whole data set, are not homogeneous within subperiods (when evaluated in terms of Kruskal-Wallis statistics). The last subperiod provided no evidence of seasonality in an analogous fashion to the results reported in Tables I and II, using all Monday's data. The third subperiod provided evidence of a seasonal ex div date and

[26] On the very rare occasions when Friday was an ex div day, the following Monday's return was excluded.

[27] Gibbons and Hess [10] and Lakonishok and Levi [13] had mixed success in explaining seasonality via settlement-type phenomena in the U.S.

[28] As Gibbons and Hess [10] point out, the index levels represent forward prices, where the forward contract has zero value.

[29] In all but one of the subperiods (1/1/80–5/31/81), turnover was significantly larger on ex div dates; total bargains were similarly higher in these corresponding periods, although significantly so in two cases only.

[30] The non ex div Monday effects are, as expected, stronger for the more regularly traded FTO Index. This was manifested via generally larger Kruskal-Wallis and Mann-Whitney Statistics.

Table III

Mean Monday Returns Classified by Whether Monday is Ex Div Date or Not

	6/1/75–5/31/81		6/1/75–12/31/76		1/1/77–5/31/78		6/1/78–12/31/79		1/1/80–5/31/81	
	Ex Div	Non Ex Div	Ex Div	Non Ex Div	Ex Div	Non Ex Div	Ex Div	Non Ex Div	Ex Div	Non Ex Div
FTAS	0.001**	−0.004*	0.002	−0.007	−0.000	−0.002	0.002*	−0.002	−0.001	−0.002
FTO	0.000*	−0.007*	0.000	−0.010*	−0.002	−0.004*	0.002*	−0.003	0.000	−0.008

* Statistically significant Chi-Square at the 1% level for the Kruskal-Wallis test across days of the week.
** Statistically significant Chi-Square at the 5% level for the Kruskal-Wallis test across days of the week.

Table IV

Equity Bargains and Turnover (Means)

	6/1/75–5/31/81	6/1/75–12/31/76	1/1/77–5/31/78	6/1/78–12/31/79	1/1/80–5/31/81
TURNOVER (£ millions)					
Monday	74.4[c]	49.6[c]	66.6[c]	74.7[c]	110.4[c]
Tuesday	82.8[ab]	54.3[b]	74.5[a]	87.0[a]	119.2[a]
Wednesday	85.8[ab]	55.71[ab]	77.4[ab]	88.0[ab]	125.3[a]
Thursday	90.5[ab]	60.1[ab]	82.6[ab]	91.4[ab]	132.8[ab]
Friday	90.0[ab]	58.4[ab]	78.0[ab]	91.4[ab]	135.1[ab]
BARGAINS					
Monday	15181	13743	15563	14923	16793
Tuesday	15499	13796	15999	15625	16874
Wednesday	15146	13559	15804	15203	16236
Thursday	15570	13658	16291[a]	15559	17066
Friday	15628[a]	13933	16239[a]	15488	17148

[a] Statistically significant at 5% level from corresponding Monday level using conventional t-tests.
[b] Statistically significant at 5% level from corresponding Monday level using Mann-Whitney test.
[c] Statistically significant Chi-Square at 5% level for Kruskal-Wallis test.

inspection of the Mann-Whitney statistics indicated that Monday's and Thursday's returns were statistically significant at the 5% level for this subperiod.

C. *Seasonalities in Trading Volumes*

The analytics in Section I were developed, in the main, with the simplifying assumption that seasonalities were not present in the nontrading profiles or price-adjustment delays. We will investigate in this section whether any evidence exists for such seasonalities, given the available data.

Table IV contains the mean equity turnover (in £ millions) and mean equity bargains (a crude instrument for the volume of equity trading) for the whole data set and each of the four subperiods. The data indicate a gradual (if not perfect) increase in both average turnover and bargains as the week progresses. However, the increase in turnover through the week is of a considerably greater order of magnitude than that of bargains. Further, it is only in three cases that the bargains are statistically significant among days at the 5% level using t-tests. No significant differences were obtained with the Mann-Whitney (and Median) tests and the Kruskal-Wallis statistic was not significant. In the case of turnover, more pervasive patterns of seasonality are present as measured by t-tests, Mann-Whitney tests, and Kruskal-Wallis tests; turnover is statistically significantly lower on Mondays. F-tests on the variances of equity turnover and equity bargains could not reject the null hypothesis of variance equality across days of the week. In summary, then, the seasonality appears to arise from a price effect (the turnover results) with very little evidence of seasonality in trading volumes (the bargains results).

IV. Implications for Market Efficiency

The empirical results reported in Section III may be interpreted as being indicative of an inefficiency in the U.K. equity market. Potentially, by buying the FTO Index late on the second Monday of the Account and selling on any other day, a positive incremental return could be achieved. Although transactions costs will be incurred, the marginal costs of changing the timing of trades to any investor who is regularly trading in the market will be zero.

The results reported in this paper are, however, not necessarily inconsistent with the market being efficient. First, part of the negative Monday mean return has been attributed to the prices used being forward, rather than spot, prices. Further, the "prices" used are calculated on the basis of mid-market quotes, rather than transactions prices.[31] Buying on Monday at the top end of the spread and selling at the lower end of the spread will cut into any trading profits based on mid-market quotes. Finally, if indeed there is a market inefficiency, the results in Tables I and III indicate that it has become a less pervasive phenomenon for more recent data.

[31] The mid-market quotes are not revised by jobbers when a stock is not traded; thus quotes will become out-of-date for infrequently traded stocks. All the Financial Times Indexes are compiled using mid-market quotes for the constituent stocks.

V. Conclusions

The presence of seasonality in U.K. daily index returns data was detected, and found to be strongest for the means of the return distributions. However, while finding a negative weekend effect, as in recent U.S. studies, this was found to be only a pervasive phenomenon for non ex div Mondays (typically the second Monday of the Account) and for two of the four subperiods studied. This difference was partially rationalized via the Settlement Date system employed on the London Stock Exchange. Some variance seasonality was detected; in many cases, strong evidence of nonnormality was discovered in the daily return distributions.

The predictions of the analytic section of the paper were detected in the data. The mean seasonality was stronger for the more regularly traded index. Measured variance seasonality was reported (but to a very much lesser degree than measured mean seasonality). While the analytics indicated that mean seasonality in the true process could induce some measured variance seasonality, its incidence was predicted to be greater (from this cause) for the less regularly traded index with heteroskedastic trading proportions. Since the measured variance seasonality was not strongly associated with the less regularly traded index, it is inferred that the small amount of measured seasonality has been caused by variance seasonality in the true process.

Appendix

Details of Derivations of Results

The relationship used in Section I is

$$I_t^m = \sum_{j=0}^n \Pi_j I_{t-j} + u_t \tag{A1}$$

with the assumptions that

(i) Π_j is identically distributed through time, $\forall j$
(ii) I_{t-j} is i.i.d. $\forall j$
(iii) u_t is i.i.d. $\forall t$, with zero expectation
(iv) u_t, Π_j, and I_{t-j} are pairwise independent $\forall t, j$
(v) $E \sum_{j=0}^n \Pi_j = 1$, $\forall t$

where E is the expectation operator.

A. Derivation of $E(I_t^m)$

From Mood et al. [14], Equation (12), p. 180, we have

$$E(XY) = E(X)E(Y) + \text{cov}(X, Y) \tag{A2}$$

where cov is the covariance operator. Applying the expectation operator to (A1)

$$E(I_t^m) = \sum_{j=0}^n E(\Pi_j I_{t-j}) + E(u_t) \tag{A3}$$

since $E(A + B) = E(A) + E(B)$. Using (A2) we then have

$$E(I_t^m) = \sum_{j=0}^{n} E(\Pi_j)E(I_{t-j}) \tag{A4}$$

from assumptions (iii) and (iv).

B. Derivation of var(I_t^m)

When X, Y are independent, then, with var the variance operator,

$$\text{var}(XY) = E(X)^2\text{var}(Y) + E(Y)^2\text{var}(X) + \text{var}(X)\text{var}(Y) \tag{A5}$$

(see Mood et al. [14] Corollary to Theorem 3, p. 180). From (A1),

$$\text{var}(I_t^m) = \sum_{j=0}^{n} \text{var}(\Pi_j I_{t-j}) + \text{var}(u_t) + 2 \sum_{\substack{j=0 \\ j<k}}^{n-1} \sum_{k=1}^{n} \text{cov}(\Pi_j I_{t-j}, \Pi_k I_{t-k}) \tag{A6}$$

from assumption (iv). Substituting for (A5) in (A6) yields

$$\text{var}(I_t^m) = \sum_{j=0}^{n} [E(I_{t-j})^2\text{var}(\Pi_j) + E(\Pi_j)^2\text{var}(I_{t-j}) + \text{var}(\Pi_j)\text{var}(I_{t-j})] + \text{var}(u_t) + 2 \sum_{\substack{j=0 \\ j<k}}^{n-1} \sum_{k=1}^{n} \text{cov}(\Pi_j I_{t-j}, \Pi_k I_{t-k}) \tag{A7}$$

Extending and generalizing (A5) to covariances then yields

$$\text{var}(I_t^m) = \sum_{j=0}^{n} [E(I_{t-j})^2\text{var}(\Pi_j) + E(\Pi_j)^2\text{var}(I_{t-j}) + \text{var}(\Pi_j)\text{var}(I_{t-j})] + \text{var}(u_t) + 2 \sum_{\substack{j=0 \\ j<k}}^{n-1} \sum_{k=1}^{n} E(I_{t-j})E(I_{t-k})\text{cov}(\Pi_j \Pi_k) \tag{A8}$$

as a result of assumptions (ii), (iv), and (v).

C. Derivation of Covariance I_t^m, I_{t-1}^m

Considering one term in the expansion in (A1), we have

$$\text{cov}(\Pi_1 I_{t-1} + u_t, \Pi_0 I_{t-1} + u_{t-1}) = [E(\Pi_0)E(\Pi_1) + \text{cov}(\Pi_0\Pi_1)]\text{var}(I_{t-1}) + E(I_{t-1})^2\text{cov}(\Pi_0\Pi_1) \tag{A9}$$

Generalizing this result to n lag terms yields

$$\text{cov}(I_t^m, I_{t-1}^m) = \sum_{j=0}^{n} \sum_{k=1}^{n+1} E(I_{t-j})E(I_{t-k})\text{cov}(\Pi_j \Pi_{k-1}) + \sum_{j=1}^{n} [E(\Pi_j)E(\Pi_{j-1}) + \text{cov}(\Pi_j \Pi_{j-1})]\text{var}(I_{t-j}) \tag{A10}$$

The result in (A10) is easily extended to an "x" period serial covariance (provided $n > s$) as

$$\text{cov}(I_t^m, I_{t-x}^m) = \sum_{j=0}^{n} \sum_{k=x}^{n+x} E(I_{t-j})E(I_{t-k})\text{cov}(\Pi_j \Pi_{k-x}) + \sum_{j=x}^{n} [E(\Pi_j)E(\Pi_{j-x}) + \text{cov}(\Pi_j \Pi_{j-x})]\text{var}(I_{t-j}) \tag{A11}$$

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