THE JOURNAL OF FINANCE • VOL. XXXIX, NO. 2 • JUNE 1984

Intertemporal Commodity Futures Hedging and the Production Decision

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ABSTRACT

This paper deals with the producer's optimal use of commodity futures in hedging. The framework for analysis is an intertemporal consumption and investment model. The producer makes his production decisions at the beginning of the period and realizes his return at the end of the time interval. During the period, he faces both price and output uncertainties. In applying stochastic dynamic programming methods, this paper shows the effect of these risks on his consumption behavior. Further, the paper investigates his optimal hedging positions in the futures market over time and his optimal production decisions. Finally, implications of these results on the futures markets are discussed.

THIS PAPER STUDIES a normative model of the farmer's optimal hedging strategy and consumption behavior in a continuous-time framework. Recent papers (for example, Berck [3], Rolfo [18], Stoll [21]) have analyzed the farmer's hedging strategies in a one-period context. These papers have provided interesting insights into the farmer's production decisions and the optimal design of futures contracts and markets. However, in a one-period model some rather restrictive conditions are imposed on the individual's behavior. For example, in these models an individual is assumed to ignore any information arrival process. As a result, the individual is assumed to adjust neither consumption behavior nor hedging positions to the changing state of the world during the production period. These conditions can be relaxed in a continuous-time framework.

In an intertemporal context, the farmer's hedging problem may be viewed as follows. Since the farm income (at harvest) depends on both the spot price and the output (in bushels), the farmer is subject to both price and output uncertainties during the production period.[1] When the farmer can hedge only in the commodity futures market, he cannot eliminate both the price and output uncertainties by any hedging strategy.[2] As the farm income represents a significant portion of his wealth, he must necessarily hold an undiversified portfolio during the production period. Therefore, in essence, the farmer must solve for his investment/consumption behavior and his hedging strategy, facing the non-marketable asset problem (Mayers [14]) in a multiperiod framework.

In this context, one can see intuitively the major difference between a one-

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[1] Discussions on the significance of the output uncertainty can be found in, for example, Rolfo [18] and *Cotton and Wool—Outlook and Situation* [8].

[2] Under some specific situations, the farmer may be able to deal with price and output risks by futures hedging. The paper will discuss these situations.

period model and an intertemporal model. In an intertemporal model, the individual seeks to stabilize the expected *consumption stream.* Stabilization is accomplished by: first, adjusting present consumption to allow for investment for future consumption, hedging against any unanticipated shortfall in income; and second, holding short positions in commodity futures. The former is called *consumption hedging* and the latter *futures hedging.* We show that they both crucially depend on the time to harvest. In contrast, in a one-period model which ignores the intertemporal consumption and hedging strategies, the effect of duration of the production process on the individual's optimal behavior is not properly incorporated into the analysis. For this reason, a multiperiod model can provide additional insight.

In this paper we analyze the farmer's optimal behavior as an extension of the nonmarketable asset problem to a multiperiod problem. We propose a simple model to describe the combined effect of consumption hedging and futures hedging on derived utility. It determines the optimal intertemporal consumption behavior and hedging strategies, showing that both the consumption and futures hedging strategies must necessarily be jointly optimally determined. We also discuss the effects of futures hedging on the farmer's production decisions. This paper extends the previous literature in several ways.

First, the paper deals with the optimal investment/consumption behavior of an individual with an undiversified portfolio. To a certain extent, this problem has previously been addressed; but in these cases, it is analyzed under restrictive assumptions. Anderson and Danthine [2] study a similar farmer's problem limited only to a three-period model. This restriction does not permit the analysis to describe fully the intertemporal consumption and hedging behavior. Mendelson and Amihud [15] and Miller [17] investigate the individual's consumption behavior under labor income uncertainty. However, they have not considered hedging in the financial market and the resolution of uncertainty is specified differently than ours. The framework utilized in the paper is similar to that of Williams [23] and Breeden [5]; however, we differ in focus. They interpret their results in terms of the individual's derived utility; in contrast, we analyze the derived utility, thus allowing us to study the individual's intertemporal consumption behavior and hedging strategies.

Second, this paper solves for the farmer's production decision. It shows how his expected output level depends on his hedging strategy in the commodity futures market and the duration of the production period. The paper therefore provides a framework that relates production level and futures hedging. Further, we argue that the commodity futures (for example, wheat futures) enable the farmers to hedge only the price risk. We show that in the presence of output uncertainty, commodity futures have limited value to the producers. Therefore, we suggest that the trading of futures on *commodity output indices*, which enables the farmers to hedge output risk, should have significant economic value.

Third, the paper studies the optimal intertemporal hedging strategy for the farmers during the production period. We have shown that the ratio of the number of futures contracts he should hold to the expected output level depends on the time to harvest. Specifically, we show that the ratio, in general, should increase in time during the production period. Utilizing the Commodity Futures

Trading Commission data on the level of open interest in short positions for hedging in wheat, we show that the empirical evidence, which cannot be explained by previous hypotheses, is consistent with our theory.

The paper proceeds in the following format. Section I presents the model of the farmer's use of commodity futures, describing his production uncertainties, investment opportunities, and the utility function that he maximizes. The farmer's optimal behavior is then discussed in Section II. The farmer's derived utility and consumption behavior are determined. Also, in this section, we analyze the intertemporal optimal hedge ratio. Section III studies the effect of hedging on the production decisions. Section IV investigates some empirical regularities. Section V contains the summary and conclusions.

I. The Model

In this section, the assumptions of the model are presented and some preliminary results are discussed.

A. Assumptions

A.1 The farmer's optimal behavior is determined in a continuous-time, finite horizon framework. At the beginning of the period (the planting season), the farmer makes his production decision on the quantity of crops, Q bushels, to be produced. At the end of the period (the harvest time), he sells all the crops at the prevailing spot price P (per bushel), and hence, his farm income is given by PQ. The harvest time T is assumed to be known.

A.2 During the production period (time between the planting and harvest seasons) the farm income is subject to two sources of uncertainty: first, the price at which the crop will be sold, P, and second, the quantity of the crop which will be sold, Q. The former is called *price risk*, the latter *output risk*. At each time, t, the farmer forms expectations of the spot price and output at harvest, and then at the next instant, with more information, the farmer revises the expectations. Therefore, the anticipated spot price, P_t, and anticipated output, Q_t, are the relevant state parameters on which the optimal decisions depend. Following other continuous-time models, the state parameters P_t and Q_t are assumed to follow Itô processes. Notice that since the farmer should not expect to revise the decision either upwards or downwards at the next instant, we require that the drift term be zero. Specifically, we assume:

$$dQ_t = \sigma_Q Q_t \, dZ_t \tag{1}$$

$$dP_t = \sigma_P P_t \, dw_t \tag{2}$$

and

$$dZ \, dw = \rho \, dt \tag{3}$$

where σ_Q and σ_P are the constant instantaneous standard deviation of the anticipated output and anticipated spot price, respectively. dZ and dW are standardized Wiener processes, and ρ is the instantaneous correlation coefficient.

A.3 There are frictions in the real sector.

(i) During the production period, the farmer cannot affect the output level by committing further investment (for example, buy more acreage) or disinvestment (land abandonment or selling off portions of the business) to the production process.

(ii) Significant agency costs prohibit the farmer from shifting the uncertainty of farm income to the market through issuance of shares. Moral hazard problems prevent the farmer from participating in other risk sharing schemes, such as insurance.

A.4 The futures market is assumed to be perfect in that participants can trade costlessly and continuously. The futures contracts are perfectly divisible. The contracts have the settlement date at the time of harvest. Each contract calls for a delivery of one bushel. Those holding long positions promise to take delivery of the underlying crop and make payment at the futures price of the contract; those holding short positions promise to make delivery of the underlying commodity and receive payment at maturity at the futures price of the contract. Assume that mark-to-market settlements occur continuously through time, so that the net value of the contract always remains zero. Let F_t denote the equilibrium settlement price on the contract at time t. F_t is stochastic, and it is characterized by

$$F_t = P_t \tag{4}$$

A.5 During the period, at each instant t, the farmer has to determine optimal consumption rate c_t^* and the futures position x_t^*. The farmer has a cash account W_t that may be positive (negative) on which a constant rate of interest r is received (paid). Any net cash flow resulting from either position in futures contracts or from consumption is deposited or withdrawn from this cash account. Therefore, the borrowing rate equals the lending rate. For simplicity, we assume that the farmer does not invest in any risky capital assets.

A.6 Finally, it is assumed that the farmer during the period would choose the consumption rate c_t and the futures position x_t, such that the expected additive utility of consumption is maximized. Specifically, it is assumed,

$$\max_{c,x} E\left[\int_0^T U(c_t, t)\, dt + B(Y_T', T)\right] \tag{5}$$

where $Y_T' = PQ + W_T$, the total wealth at time T being the sum of the cash account value and the crop value; $E[\ ]$ is an expectation operator, and $U(\cdot)$ is an instantaneous utility function for consumption such that $\frac{\partial U}{\partial c} > 0$ and $\frac{\partial^2 U}{\partial c^2} < 0$; and $B(\cdot, T)$ is the terminal utility of wealth and is assumed to be a concave function.

The model represents a direct extension of the one-period farmer's problem to an intertemporal model. A.1 is the standard assumption used in the one-period models, for example, Rolfo [8] and Stoll [21]. A.2 and A.6 are standard assumptions used in intertemporal models, for example, Merton [16]. Other assumptions require further discussion. A.3 (i) allows us to analyze the consumption and hedging behavior isolated from the intertemporal production decisions. It is also consistent with general practice for some crop production. The production level for the season depends crucially on the anticipated selling price at the planting season. Once the planting period passes, the change in anticipated price would not significantly affect the production level.[3] A.4, Equation (4), allows us to focus on the use of futures contracts for hedging.[4] In the study of normative models in hedging, it is convenient to require the futures price (i) to follow a process perfectly correlated with the anticipated spot price P_t, and (ii) to give an expected return such that the individual would only participate in the futures market when $Q_t \neq 0$. Condition (i) allows the individual using the futures to hedge the price risk effectively. Condition (ii) allows us to isolate the use of futures to hedge as distinct from investing in the futures as a risky asset. Specification of F_t by Equation (4) satisfies these two conditions. In fact, A.4 is equivalent to Breeden's assumption of an existence of a "futures contract of instantaneous maturity on state variable S_1." In our case, the state variable is simply the anticipated spot price P_t. A.5 as a simplifying assumption is also adopted in Miller [17], Mendelson and Amihud [15], and Anderson and Danthine [2]. It allows us to abstract from interest rate risk which would result in a complicated mark-to-market effect. Further, we also do not consider the problem of the impact of limited liability and optimal portfolio decision on his hedging and consumption behavior. These issues are not considered to be central to the farmer's hedging problem, so they are not considered in this paper. This model, thus far, has not provided a framework to determine the farmer's optimal production decision at the planting season. It is assumed that the anticipated output level is given at the beginning of the period, and is not endogenously determined. This problem of the farmer's production decision is discussed in Section III.

B. Formulation of the Optimal Decisions

Let x_t denote the number of contracts held by the farmer at time t in a short position. When there is an increase in the settlement price of the futures (dF), the mark-to-market procedure would require the farmer to pay the clearing corporation immediately, and in cash, $x_t\, dF_t$. It follows, therefore, at each time t prior to harvest, the change in the farmers cash account is the sum of three cash flows: the interest earned from the cash account ($rW\, dt$), the consumption ($-c\, dt$), and the mark-to-market settlement of his futures position ($-x_t\, dF_t$). This is summarized by the following budget constraint equation:

$$dW = (rW - c)\, dt - x\, dF \tag{6}$$

The farmer has to decide on the optimal consumption rate c^* and hedge position x^* at each instant of time given the state variables; time to harvest T, anticipated output level Q, futures price F, and cash account balance W. To

[3] For details of the production problem, see [7].

[4] This paper is not concerned with the contango and backwardation controversy found in Keynes [13], Cootner [9], and Telsar [22]. This paper also does not suggest that empirically the futures price is the expected spot price (see for example Jurt and Rausser [12]). The model is framed to analyze the optimal hedging strategy.

derive the optimal decisions, we utilize the Bellman stochastic dynamic programming technique. Define a function J^T given by:

$$J^T(W, F, Q, t) = \max_{\substack{c_t, x_t \\ t<T}} E_t\left[\int_t^T U(c, s)\, ds + B(Y', T)\right]$$

where E_t is the expectation operator, conditional on $W(t) = W$, $F(t) = F$, $Q(t) = Q$, and $c_s > 0$.

Then J^T is the solution to the Bellman equation:

$$\max_{c,x}[dJ^T + U(c, t)\, dt] = 0 \tag{7}$$

satisfying the boundary condition $J^T(W, F, Q, T) = B(FQ + W, T)$. The boundary condition requires that the optimized derived utility at the time of harvest be equal to the terminal utility of wealth. Wealth is given by the sum of the farm income and the cash account.

Taking the total differential of the function J^T, and applying Îto's lemma, we may express Equation (7) as follows:

$$\begin{aligned}-J_t^T &= J_W^T rW + \tfrac{1}{2}J_{FF}^T F^2\sigma_F^2 + J_{FQ}^T FQ\sigma_{FQ} + \tfrac{1}{2}J_{QQ}^T Q^2\sigma_Q^2 \\ &\quad + \max_{c,x}[U(c, t) - J_W^T c + \tfrac{1}{2}J_{WW}^T x^2F^2\sigma_F^2 - J_{FW}^T xF^2\sigma_F^2 - J_{WQ}^T xFQ\sigma_{FQ}]\end{aligned} \tag{8}$$

Equation (8) shows that the decrease in the optimized derived utility in time depends on three components: (1) The current increase in derived utility as a result of receiving interest payment (the first term); (2) the effect of price and output risks on the derived utility (the next three terms); and (3) the current increase in derived utility through the optimal consumption rate and hedging (the next line).

The last component is of particular interest. First, note that it can be expressed as two optimal decision rules.

$$\max_{c}[U(c, t) - J_W^T c] \tag{9}$$

$$\max_{x}[\tfrac{1}{2}J_{WW}^T x^2F^2\sigma_F^2 - J_{FW}^T xF^2\sigma_F^2 - J_{WQ}^T xFQ\sigma_{FQ}] \tag{10}$$

This shows that when the derived utility is known, the two optimal decisions (consumption rate and hedging positions) are determined separately. Indeed, expression (9) shows that an increase in consumption would lead to an increase in the instantaneous utility but, on the other hand, a decrease in wealth and, hence, a decrease in the derived utility. The optimal consumption rate is determined such that the marginal utility of consumption equates the marginal derived utility in wealth.

$$u_c(c^*, t) = J_W^T(W, F, Q, t) \tag{11}$$

Hence, c^* is determined independently of the hedging decisions.

Expression (10) shows that the optimal hedge position is determined as a result of optimally neutralizing the price risk (second term) and output risks (third term) by the risk of mark-to-market settlement (first term). The optimal hedge position is given by:

$$x^* = \frac{J_{FW}^T F \sigma_F^2 + J_{WQ}^T Q \sigma_{FQ}}{J_{WW}^T F \sigma_F^2} \tag{12}$$

Finally, substituting Equations (11) and (12) into Equation (8), we have

$$-J_t^T = J_w^T(rW - c^*) + U(c^*, t) + \frac{1}{2} J_{FF}^T F^2 \sigma_F^2 + J_{FQ}^T F Q \sigma_{FQ} + \frac{1}{2} J_{QQ}^T Q^2 \sigma_Q^2 - \frac{(J_{FW}^T F \sigma_F^2 + J_{WQ}^T Q \sigma_{FQ})^2}{2 J_{WW}^T \sigma_F^2} \tag{13}$$

where c^* is given by Equation (11). All the terms in Equation (13) have been explained earlier with the exception of the last term. This term gives the effect of using futures on the derived utility. It always increases the utility. It has no effect only when the numerator is zero, or when there is no reduction of risks in hedging, as explained earlier.

Note that the expected farm income is $Y = FQ \exp[\sigma_{FQ}(T - t)]$. Let G be some function of three parameters and for the time being, we assume that the following equation holds

$$J^T(W, F, Q, t) = G(W, Y, t). \tag{14}$$

By substituting (14) into Equation (13), the model is reduced to

$$G_t + G_W(rW - c^*) + G_{YY} Y^2\left(\frac{1}{2}\sigma_F^2 + \sigma_{FQ} + \frac{1}{2}\sigma_Q^2\right) + U(c^*, t) - \sigma_F^2 \frac{G_{WY}^2 Y^2}{2G_{WW}}\left(1 + \frac{\sigma_{FQ}}{\sigma_F^2}\right)^2 = 0 \tag{15}$$

$$U_c(c^*, t) = G_W(W, Y, t) \tag{16}$$

$$\frac{x^*}{Q} = \frac{G_{YW}}{G_{WW}}\left(1 + \frac{\sigma_{FQ}}{\sigma_F^2}\right) e^{\sigma_{FQ}(T-t)} \tag{17}$$

$$G(W, Y, T) = B(W + Y, T)$$

The above Equations (15), (16), and (17) show that the derived utility and the hedge ratio depend on F and Q only because they depend on Y, the expected farm income. This result can be explained intuitively by comparing this model with Merton's consumption-investment model [16]. Merton's model has shown that the optimal behavior depends on the farmer's current wealth position. In this model, his wealth would include the present value of his future farm income. Therefore, his optimal behavior depends partly on a measure of his uncertain income, and hence on Y. The resulting hedge ratio can also be explained intuitively. Other one-period models on hedging have shown that the hedge ratio is independent of the price and output levels. But this model differs from these other models by considering the hedgers' consumption behavior (and their utility of terminal wealth) in a multiperiod context. Hence, the hedge ratio considered here would take the hedgers' consumption behavior into account. Since it has been argued that the consumption rate c^* depends on Y one expects the hedge ratio to depend also on Y.

So far, the analysis indicates that the farmer's optimal behavior depends on the production risks. However, such risks may be perfectly hedged by his use of futures (i.e., hedging may eliminate all the risks). Indeed a necessary and sufficient condition for the production risks, under which futures contracting would render the farmer's consumption behavior independent of such risks, is that the residual risk $\epsilon = \sigma_Q(1 - \rho^2)^{1/2}$ has zero value (see Appendix A).

This result shows that futures contracts can be used to hedge the price risk, in the sense that the farmer can consume as if there were no production risk in the case where there is only price uncertainty (without output uncertainty) or where output is perfectly correlated with the price. Furthermore, this result also shows that by continuously readjusting the hedge position, the farmer can achieve a perfect hedge when price and output are perfectly correlated.

II. Optimal Behavior of the Hedger with a Constant Absolute Risk-Averse Instantaneous Utility Function

This section further investigates the optimal behavior of the farmer by deriving the explicit solutions of the model described in previous sections. This is accomplished by further assuming that his utility function takes on a specific form. The simplifying assumptions used here enable the analysis to focus on the essential features of the model. The instantaneous utility of consumption $U(c, t)$ is assumed to be

$$U(c, t) = -a \exp(-rt - Ac) \tag{18}$$

and the terminal utility of wealth function $B(\cdot, T)$ is assumed to be

$$B(x, T) = -a \exp(-Ax - rT) \tag{19}$$

where a and A are some positive constants. Notice that the function $B(\cdot, T)$ is assumed to be similar to the instantaneous utility function $U(\cdot, t)$ because this avoids complicated analysis of the trade-off between the utility of intertemporal consumption and the utility of the terminal wealth. We will not address this issue here. Further, exponential utility is used because it is often used in continuous-time investment and consumption studies. Other commonly used utility functions (log and power functions) are not considered here because they do not admit negative values. In these cases, the individual would not borrow to consume, even when his cash wealth has little value while his expected farm income is significantly large. This is so because, if his cash wealth were negative anytime during the production period, he would confront a positive probability of realizing negative wealth at harvest time. Since such an outcome is inadmissible, he would therefore never borrow against his farm income for intertemporal consumption. This kind of behavior, to a large extent, would separate his consumption decision from his "nonmarketable" wealth level, and is inappropriate for the purpose of this paper. For this reason, we shall only consider exponential utility functions.

For this special form of the utility function, Equations (4)–(17) can be rearranged. Substituting Equations (18) and (19) into Equation (15), we find the derived utility function, G, is the solution to:

$$G_t + G_W\left(rW + \frac{\ln\left(\frac{G_W}{aA}\right) + rt}{A}\right) - \frac{G_W}{A} + MG_{YY}Y^2 - N\frac{G_{WY}^2 Y^2}{G_{WW}} = 0 \quad (20)$$

where

$$M = \frac{1}{2}\sigma_F^2 + \sigma_{FQ} + \frac{1}{2}\sigma_Q^2$$

$$N = \frac{1}{2}\left(1 + \frac{\sigma_{FQ}}{\sigma_F^2}\right)^2 \sigma_F^2$$

Furthermore, the optimal consumption rate can be derived by substituting Equations (18) and (19) into Equation (16). We can derive

$$c^* = \frac{-rt - \ln\frac{G_W}{aA}}{A} \quad (21)$$

The optimal hedge ratio remains as given by Equation (17)

$$\frac{x^*}{Q} = \frac{G_{WY}}{G_{WW}}\left(1 + \frac{\sigma_{FQ}}{\sigma_F^2}\right)e^{\sigma_{FQ}(T-t)} \quad (22)$$

THEOREM A. *When there are production risks, the derived utility depends on the residual risk, ϵ, risk aversion, A, and the expected farm income, Y. In addition, it depends on the time to harvest, T.*[5]

Specifically, the derived utility is

$$G = G_0(1 + ½(\epsilon YA)^2\theta(T)) + O^4(\sigma_Q, \sigma_F; W, Y, T) \quad (23)$$

where

$$G_0 = -aR\exp\left(\frac{-A(W + e^{-rT}Y)}{R}\right) \quad \text{for} \quad R = \frac{1 - e^{-rT}}{r} + e^{-rT},$$

$$\theta(T) = \frac{e^{-rT}\ln R}{(1 - r)R} > 0,$$

$O^4(\sigma_Q, \sigma_F; W, Y, T)$ are functions with homogeneous order four or higher in σ_Q and σ_F.

Proof: Let $G(W, Y, t)$ denote the solution to Equation (20) and let $G(W, Y, t)$ be expressed as a power series in the risk parameters σ_F and σ_Q in the parameter

[5] See Blaquiere [4] for a discussion of the approximation method utilized in the proof of this theorem.

space (W, Y, t). That is,

$$G(W, Y, t) = A_0 + A_{11}\sigma_Q + A_{12}\sigma_F + A_{21}\sigma_Q^2 + A_{22}\sigma_Q\sigma_F + A_{23}\sigma_F^2 \cdots \quad (24)$$

Note that ρ, the correlation coefficient, is not assumed to be small. But σ_F and σ_Q are assumed to be sufficiently small, so that the terms of homogeneous degree of order four and higher can be neglected. Let

$$G_0(t) = -aR(t)\exp\left(\frac{-A(W + e^{-r(T-t)}Y)}{R(t)} - rt\right)$$

where

$$R(t) = \frac{1 - e^{-r(T-t)}}{r} + e^{-r(T-t)}$$

Let

$$G = G_0(t) + \frac{\epsilon^2}{2} aA^2 f(W, Y, t) \quad (25)$$

for some function f. Substituting Equation (22) into Equation (20) we see that the function f has to satisfy the following equation.

$$f_t R(t) + f_W e^{-r(T-t)}(W(r-1) - Y) = Y^2 \exp(-\xi) e^{-2r(T-t)} \quad (26)$$

where

$$\xi = \frac{A(W + e^{-r(T-t)}Y)}{R(t)} - rt \quad \text{and} \quad f(W, Y, T) = 0$$

Then a direct substitution of

$$f(W, Y, t) = \frac{Y^2 e^{-r(T-t)} \ln R(t)}{(1 - r)R(t)a} G_0(t) \quad (27)$$

into Equation (23) shows that it is the solution to Equation (23). The third-order terms can be shown to be zero. Equations (25) and (27) give the desired result. Q.E.D.

Note that G_0 is the derived utility of the individual under certainty (i.e., $\sigma_Q = \sigma_F = 0$). Also note that, when we confine the discussion of our result to $((W, Y, T)$: $0 < W < W_0$, $0 < Y < Y_0$, $0 < T < T_0)$, where W_0, Y_0, T_0 are some arbitrary numbers, then (σ_Q, σ_F) may be sufficiently small such that the second term of Equation (23) is negligible. Therefore, the formulation in Theorem A allows us to focus our analysis on the first term of Equation (23) to understand the impact of risks on the derived utility (and hence optimal behavior) of the farmer.[6]

The simple expression of Equation (23) offers an intuitive explanation of the

[6] In utilizing the approximating method, we do not require the consumption rate to be nonnegative. Under the model assumptions, the individual would consume at a negative rate only when his expected farm income is less than his debt obligation. When the risks are assumed to be sufficiently small, as imposed by the approximating method, such an outcome is not likely to happen and, therefore, the condition $c \geq 0$ may be ignored. Intuitively, one may argue that the condition $c \geq 0$ only affects the discussions of the model results slightly. Specifically, when the consumption rate is nonnegative, the individual would be more adverse to significant negative terminal wealth, and therefore he would increase his propensity to save. Other results described in the paper would not be affected.

result. It shows that, where there are production risks, the percentage change in the derived utility of consumption depends on four factors:

1. *The farmer's risk aversion.* An increase of the farmer's absolute risk aversion as measured by the constant, A, would result in a percentage decrease in the derived utility.

2. *The residual risk.* The presence of price and output uncertainties affects the farmer's optimal consumption and investment decisions. The use of futures allows him to reduce part of the risk. The remaining level of uncertainty is measured by the residual risk, $\epsilon = \sigma_Q(1 - \rho^2)^{1/2}$, which is determined by the standard deviation of the output risk and by the correlation coefficient of the production risks. When the price and output risks are highly correlated either positively or negatively, the use of futures leads to a significant reduction of the total risk facing the farmer. The reason is that when the two risks are highly correlated, the output risk may be viewed as equivalent to the price risk. Since the farmer can hedge the price risk in the futures market he would be able to hedge much of his production risks. In fact, $\sigma_Q(1 - \rho^2)^{1/2}$ gives the portion of the production risk that cannot be hedged by using the futures market.

3. *Level of uncertain farm income.* The farmer's optimal consumption and investment decisions depend on his wealth position. When his wealth position is uncertain, we shall show that he would consume less (invest more) at the present time. Consequently, his derived utility of consumption is reduced. Further, the higher the level of uncertain wealth, the larger the reduction of his utility.

4. *Time to harvest.* The percentage change of the derived utility of consumption, for given levels of production risks, does not change uniformly with the time to harvest. In fact, the relationship is given by a function, the *time factor* $\theta(T)$.

The behavior of the time factor requires more explanation. This time factor determines how the marginal change of the derived utility depends on the rate of interest, r, and time to harvest, T. First, note that $R(0, r) = 1$ and hence it follows immediately that $\theta(0, r) = 0$. Further, following a direct calculation, we have

$$\frac{d\theta}{dT} = \frac{e^{-rT}}{(1 - r)R}\left[-r \ln R + \frac{(1 - \ln R)}{R}\frac{dR}{dT}\right]$$

For small values of T, R is approximately equal to unity and therefore $\dfrac{\partial\theta}{\partial R}$ is positive. Since R is an increasing function of T, $\dfrac{d\theta}{dT}$ is also positive. Similarly, for large values of T, R is approximately equal to $\dfrac{1}{r}$. In this case $\ln(R)$ is negative and hence $\dfrac{d\theta}{dT}$ is negative. Hence, we have shown that θ increases (decreases) with T for small (large) values of T. That is, risks always result in a percentage decrease in utility. But this change is negligible when it is close to the harvest time. As the time to harvest extends further, the percentage change in the derived utility rises. However, when the harvest time is sufficiently far away, risk has a small effect on this percentage decrease in the derived utility. This qualitative behavior of the farmer's derived utility is the same for any constant interest rate, r.

COROLLARY. *Let c_0 denote the farmer's optimal consumption behavior when there is no uncertainty. Let c denote his consumption rate when there are risks specified in the model. Then the impact of risk on the consumption behavior is given by*

$$c_0 - c = \tfrac{1}{2}(\epsilon Y)^2 A\theta$$

where the notation is as given in Theorem A, and higher order terms of (σ_Q, σ_F) are ignored.

Proof: From Equation (21), we have

$$c = \frac{-\ln\left[\dfrac{G_W}{aA}\right] - rt}{A} \tag{28}$$

Theorem A shows that

$$G = G_0\left(1 + \frac{\epsilon^2}{2}\, Y^2 A^2 \theta\right) \tag{29}$$

Substituting (29) into (28), and letting $t = 0$, one can show that

$$c_0 - c = \tfrac{1}{2}(\epsilon Y)^2 A\theta$$

where

$$c_0 = \frac{W + e^{-rT} Y}{R} \qquad \text{Q.E.D.}$$

This corollary shows that an increase in the residual risk, *ceteris paribus*, leads to a decrease in the present consumption rate. That is, in the presence of risks the farmer would invest more for future consumption to minimize the probability of an unanticipated fall in his consumption rate at a future rate. However, the impact of residual risk on his consumption rate depends on the time to harvest. When the harvest is in the immediate future, the uncertainty of his farm income is small and, therefore, his consumption level would be similar to that in the absence of risk. As the time to harvest lengthens, the uncertainty of farm income grows, and the fall in his consumption level is significant. This impact reaches a maximum for some value of the time to harvest such that any greater time interval diminishes it.

Intuitively, this result can be explained by analyzing the consumption time path. In the absence of output risk, the farmer would seek a constant consumption rate to maximize his derived utility. In the presence of residual risk, he would accept a lower consumption rate. Therefore, in expectation, his consumption rate would increase over time because the higher present investment rate yields a higher level of wealth in the future, and therefore a higher rate of consumption. Hence, the optimal present consumption rate represents a trade-off between two effects: first, the effect of a higher future wealth level which is a hedge against an unexpected decline in his farm income; and second, the effect of a rise in his expected consumption rate over time. The former effect leads to a higher expected derived utility while the latter leads to a lower expected derived utility. Next, as the time to harvest lengthens, if the impact of the residual risk on the farmer's consumption does not decrease, he would in expectation accumulate more wealth (as the farmer seeks to invest more). Our analysis shows that the second effect then would dominate the first effect, leading to a reduction of this risk impact. Further, it points out that the farmer would consume as if there were insignificant production uncertainty when the time to harvest is sufficiently far away. The impact of risk on the percentage decrease in the derived utility is negligible when the harvest is in the distant future. This is because in expectation the farmer consumes as if there was little uncertainty for part of the time.

It is interesting to compare his derived utility when he is optimally hedging in futures with the case when he is not hedging at all. It can be shown that the farmer's derived utility,[7] with hedging, is given by

$$H = G_0(1 + \tfrac{1}{2}(\epsilon_0 YA)^2\theta(T)) \tag{30}$$

where

$$\epsilon_0 = (\sigma_Q^2 + 2\sigma_{FQ} + \sigma_F^2)^{1/2}$$

Comparing Equation (23) and Equation (30), we observe that the only difference between the derived utilities with and without hedging is the difference in the specification of the impact of risk. When optimal hedging is undertaken, the risk impact is given by $\epsilon^2 = \sigma_Q^2(1 - \rho^2)$, whereas without hedging, it is given by

$$\epsilon_0^2 = \sigma_Q^2 + 2\sigma_{FQ} + \sigma_F^2$$

The difference of the impact is given by

$$\epsilon_0^2 - \epsilon^2 = \left[1 + \rho\left(\frac{\sigma_Q}{\sigma_F}\right)\right]^2 \sigma_F^2 > 0$$

Hence, this reduction of uncertainty would be large when there is substantial price uncertainty.

So far, we have considered the optimal consumption behavior and the expected derived utility of the farmer when he pursues continuous optimal hedging in the futures market. This entails holding a position in the commodity futures at each instant of time for the given estimates of the future spot price and output of the commodity. The following theorem describes his continuous time optimal hedge ratio, the ratio of the number of futures contracts he should hold to the expected output level.

THEOREM B. *The hedge ratio, in general, is less than unity. Furthermore, it falls with the increase in the time to harvest. The hedge ratio,* $\dfrac{x^*}{Q}$, *is given by*

$$\frac{x^*}{Q} = e^{-rT}\left(1 + \frac{\sigma_{FQ}}{\sigma_F^2}\right)\left(1 - \epsilon^2 AY\left(\frac{\ln R}{1 - r}\right)\right)e^{\sigma_{FQ}T} \tag{31}$$

[7] The function H can be analogously derived using Equation (20) with $N = 0$.

Proof: From Equation (17) the optimal hedge position is given by

$$\frac{G_{WY}}{G_{WW}}\left(1 - \frac{\sigma_{FQ}}{\sigma_F^2}\right)e^{\sigma_{FQ}(T-\ t)} \tag{32}$$

Substitute the expression G from Theorem A into Equation (32). By a direct calculation, one can derive the result given in the proposition. See Appendix B for details. Q.E.D.

The proposition integrates four different effects on the optimal hedge ratio. The first pertains to the mark-to-market settlement which has been discussed elsewhere (Anderson and Danthine [2]). When there are upward movements in the expected spot price and thus in the expected return, the farmer with a short position in the futures contract has to pay the additional return in the present time. Since the present value of the change in expected return is less than the settlement payment, the farmer would only hold a hedge position which is a portion of the expected output. This amount is given by the first factor in the expression (31), i.e., e^{-rT}.

The second effect concerns the output uncertainty which has been discussed in the literature (Stoll [21], Rolfo [18], Anderson and Danthine [2]). When the farmer faces both price and output uncertainties, the optimal hedge ratio depends on the correlation of the two uncertainties. If the correlation is negative, the farmer's income is less uncertain, and therefore the farmer would not hedge his entire expected output in the futures market. The portion that he would hedge is given by the second factor in the expression (31), i.e., $\left(1 + \frac{\sigma_{FQ}}{\sigma_F^2}\right)$. This result is consistent with the one-period model discussed in the literature.

The third is the intertemporal investment and consumption effect, and it deserves more attention. We have argued that the consumption rate is lower in the presence of residual risk. As the time to harvest increases, the change in the consumption level is less sensitive to any change in the anticipated farm income. In the framework of continuous time investment/consumption behavior, commodity futures are utilized to stabilize his consumption stream. Since the impact of the change in the state of the world on his consumption stream declines with a greater time to harvest, the farmer's hedge ratio would go down in step with the time to harvest. This effect is expressed by the third factor, i.e.,

$$\left(1 - A\sigma_Q^2(1 - \rho^2)\,\frac{Y \ln R}{1 - r}\right)$$

The fourth is the downward bias of expected farm income. Since the output risk and the price risk, in general, are negatively correlated, the expected farm income would be less than the product of the expected output and the expected spot price. The bias increases with the time to harvest and, hence, the hedger would lower the hedge ratio when he is further away in realizing his farm income. This effect is expressed by the fourth factor, i.e., $e^{\sigma_{FQ}T}$.

III. Effects of Hedging on Production Decisions

In this section we investigate the effects of hedging in the futures market on the farmer's production decisions. We have assumed that the farmer makes only one production decision which is at the beginning of the period (see Section I). We assume that given information on the expected spot price at which the crops may be sold and the production function, the farmer decides on the expected output level of the crops. The production function relates the initial investment outlay I to the expected output level. We assume that additional investment yields higher expected output, but also a drop in marginal expected output. For simplicity we assume that this relationship is given by the Cobb-Douglas function:

$$Q(I) = \alpha I^{\beta} \tag{33}$$

where α, β are some constants such that $\alpha > 0$ and $0 < \beta < 1$. I is the investment outlay and Q is the expected output at the time of harvest.

The farmer has to pay for the investment outlay at the beginning of the period, and can finance it either through a wealth account, W, and/or by borrowing at the risk-free rate, r. Under these assumptions, the farmer would choose the expected output level which maximizes derived utility, i.e.,

$$\operatorname*{Max}_{I}\ G(W - I,\ FQe^{\sigma_{FQ}T},\ 0) \tag{34}$$

where the derived utility G is given by Equation (23). Theorem C gives the farmer's optimal expected output, Q, for a price level F. It therefore provides the individual's supply curve, and demonstrates the effect of hedging on the production decisions.

THEOREM C. *The supply curve is upward sloping. At a fixed price level, an increase in the residual risk leads to a decrease in the level of output. Furthermore, for a given residual risk, the output is further reduced by risk as the price level rises.*

Proof: By Equation (34), together with Equations (33) and (23), we have the following optimizing rule:

$$\operatorname*{Max}_{I} - aR\left(\exp\left[\frac{-A(W - I + e^{(\sigma_{FQ}-r)T}FQ)}{R}\right]\right)\left(1 + \frac{\epsilon^2F^2Q^2A^2\theta e^{2\sigma}FQ^{T}}{2}\right)$$

This, however, is equivalent to the following expression:

$$\operatorname*{Max}_{I} \frac{-(W - I + e^{(\sigma_{FQ}-r)T}QF)A}{R} + \ln(1 + bF^2Q^2) \tag{35}$$

where $b = \frac{1}{2}\epsilon^2\theta A$, and θ is defined in Theorem A, since the log function is a monotonic function. It follows that at the optimal investment level, by differentiating Equation (35) with respect to I and equating it to zero, we find (ignoring higher order ϵ terms),

$$1 - e^{(\sigma_{FQ}-r)T}\left(\frac{\partial Q}{\partial I}\right)F + 2bRF^2Q\left(\frac{\partial Q}{\partial I}\right) = 0$$

Since $Q = \alpha I^{\beta}$ and therefore $\frac{\partial Q}{\partial I} = \beta\alpha I^{\beta-1}$, it follows that

$$1 - e^{(\sigma_{FQ}-r)T}\beta\alpha^{1/\beta}Q^{1-(1/\beta)}F + 2bRF^2Q^{2-(1/\beta)}\alpha^{1/\beta} = 0 \tag{36}$$

Equation (36) is the farmer's supply curve. An analysis of the supply curve gives the results of the proposition. By a straightforward calculation, one can show, from Equation (36), that $\frac{\partial F}{\partial Q} > 0$. Q.E.D.

The above theorem allows us to derive the elasticity of the supply curve.

COROLLARY. *The elasticity of the supply curve is given by*

$$\eta = \frac{dQ}{dF}\frac{F}{Q} = \frac{\beta}{(1-\beta)}\left(1 - \frac{\epsilon^2 AY}{(1-r)(1-\beta)}\ln R\right)$$

Therefore, the supply curve is more elastic with an increase in the residual risk. Further, the impact of the residual risk on the elasticity depends on the length of the harvest season. Its importance grows as the time to harvest lengthens.

The elasticity of the supply curve when the farmer does not hedge in the futures market is given by

$$\eta_0 = \frac{dQ}{dF}\frac{F}{Q} = \frac{\beta}{(1-\beta)}\left(1 - \frac{\epsilon_0^2 AY}{(1-r)(1-\beta)}\ln R\right)$$

where ϵ_0 is given in Theorem A.

Proof: From Equation (36), we have a function relating the price F and the output Q. By a direct calculation, we derive the expression for η (details are given in Appendix C). The expression for η_0 follows directly from Theorem A. Q.E.D.

These results are summarized in Figure 1.

When the production level is primarily determined at the beginning of the planting season, these results show that the impact of the residual risk and the effect of hedging on the expected production level vary according to the characteristics of the commodity. If the commodity has a very short growing season, then risk is relatively insignificant. Both risk and hedging effects increase, but the marginal changes decline with the time to harvest.

IV. Wheat Futures Hedging

In this section, we argue that the intertemporal hedging behavior can explain some of the empirical regularity in the monthly variations of wheat futures open interest, specifically in the short positions in hedging. Table I presents the short hedging positions in all wheat futures (Panel A) and in wheat futures of the same crop year (Panel B), respectively, in each month from 1977–1980. They show that in each calendar year, the amount of open interest increases monotonically beginning in March and reaching a peak in September. Furthermore, when we compare the peak hedging level with the wheat harvested during that year, referring to Table IA (row 13), we find that they are highly correlated. That is, the production level is positively related to the short hedging level.

This observation cannot be explained by the traditional theory,[8] where output risk is ignored. According to this theory, farmers would hedge their full position

[8] See, for example, Working [24].

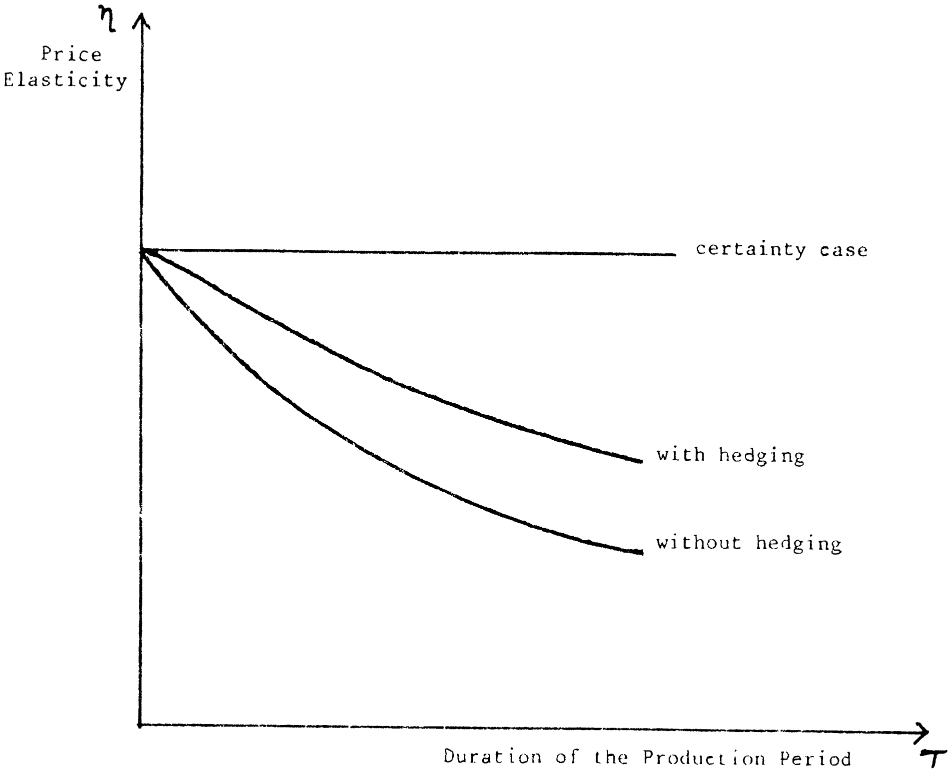

Figure 1. The Price Elasticity and the Duration of the Production Period

Table I

Bushels of Wheat Hedged in Short Positions

	Panel A Short Hedging in the Same Crop Year (in thousands of bushels)				Panel B Short Hedging in all Futures Contracts (in thousands of bushels)			
	1977	1978	1979	1980	1977	1978	1979	1980
January	97450	66500	40900	81790	141615	114835	60540	116095
February	83180	41175	38900	68725	153610	99395	68125	115230
March	36780	37655	11605	26665	116775	104605	52540	74870
April	7980	13445	7070	13465	104445	92060	84310	77420
May	105675	100805	70635	84315	105675	100805	70635	84315
June	111885	67140	119985	113485	112475	67140	120865	115285
July	136070	77125	125125	183855	137000	77125	126045	188575
August	128720	103425	164575	180215	130795	104780	166360	190435
September	145150	98375	165270	182555	150070	101670	170285	197070
October	146485	90870	102345	213280	155350	100490	118795	237075
November	114750	78940	90790	146520	129930	89960	118040	177890
December	101010	40055	93300	110950	135655	55225	145315	148240
Production	2045527	1775524	2134060	2369666				

in the futures market throughout the production period. Now, in the U.S., the planting season for winter wheat (about ⅔ of the total wheat production) is during September and October, and harvest time is during June and July. For spring and Durum wheat, the planting season is during April and May, and harvest time is in August. Therefore, if the traditional theory were to hold, one would expect that the open interest would significantly increase in the months September–October and moderately so in April–May, and would decrease in June–August. Even if the farmers do not sell their crops immediately after harvest, so that they would liquidate their futures positions some time later, this theory cannot explain the observed *increase* in open interest several months prior to the harvest. When output risk is incorporated, Rolfo's one-period model also fails to explain these monthly variations in open interest. This is because his model does not suggest any intertemporal adjustments of the farmer's hedging positions during the production period.

This paper offers an explanation. We argue that in the presence of output uncertainty, futures contracts cannot be viewed as a perfect hedging instrument for the farmers. However, the futures contracts can be used to partially hedge his intertemporal consumption risk over the production period. And since the uncertainty is resolved continuously over the production period, Theorem B shows that each farmer's hedge ratio, *ceteris paribus*, would increase as harvest time approaches. The results of this paper allow us to derive an empirical model of the short hedging open interest in wheat futures.

A. The Model

We first propose an empirical model of an individual farmer's hedging position. We then derive a model of aggregate futures open interest, which we later use to explain the observed open interest positions presented in Table I, A and B.

Let $H_{\tau,T}$ denote the harvest level in the τth month of year T, and let β be the portion of the production that the farmers sell short in the futures market. Consider a representative farmer holding such a short position (the ith farmer, where $1 \leq i \leq N$). Let his harvest be denoted by $H^i_{\tau,T}$. After harvest, we assume that the farmer would store the grains for K months. Therefore his futures (short) positions for months $(\tau, \tau + 1, \cdots, \tau + K)$ would be $H^i_{\tau,T}$. For j months prior to the harvest, he would hedge a portion of his expected harvest in his futures market. Specifically, his hedge position in month $(\tau - j)$ is given by

$$x^i_{\tau-j} = \alpha_j E_{\tau-j}(\tilde{H}^i_{\tau,T}) \tag{37}$$

where

$$\alpha_1 > \alpha_2 > \cdots > \alpha_M \quad \text{for some fixed integer } M \tag{37a}$$

We make two simplifying assumptions. First, we assume that the farmer starts hedging 4 months before the harvest time.[9] This is reasonable in the case of

[9] This assumption may be viewed as consistent with the findings in Stein [20]. The paper shows that the futures prices have little predictive value for the future spot price any time up to 3 months prior to the harvest. This may be the result of the highly uncertain nature of the harvest level during that period.

wheat production, because the output uncertainty is sufficiently large before springtime; the farmer would hold an insignificant level of short positions before then. Second, for lack of additional information, we utilize the simplest way to describe (37a) by assuming linearity, i.e., we assume that

$$\begin{aligned} \alpha_j &= 1, && \text{for } -K \le j \le 0 \\ &= \left(\frac{M-j}{M}\right), && \text{for } 0 \le j \le M \\ &= 0, && \text{for } M < j \end{aligned} \tag{38}$$

Since there is a lack of published information on the anticipated production level prior to the harvest, we shall assume that the expectation of the farmer's anticipated harvest is the realized harvest, i.e.,

$$E_{\tau-j}(\tilde{H}^i_{\tau,T}) = H^i_{\tau,T} + \tilde{\epsilon}^i_{\tau,T} \tag{39}$$

where $\tilde{\epsilon}^i_{\tau,T}$ is the error of the farmer's prediction, such that $E(\epsilon^i_{\tau,T}) = 0$, then, Equations (37) and (38) give,

$$x_{\tau-j} = \textstyle\sum_{i=1}^N x^i_{\tau-j} = \alpha_j \sum_{i=1}^N H^i_{\tau,T} + \alpha_j \sum_{i=1}^N \tilde{\epsilon}^i_{\tau,T} \tag{40}$$

where $x_{\tau-j}$ is the aggregate $(\tau - j)$th month short positions held by the farmers, whose crop harvest time is in the τth month. But, we have argued that

$$\textstyle\sum_{i=1}^N H^i_{\tau,T} = \beta H_{\tau,T} \tag{41}$$

Hence, substituting (41) into (40), we have

$$x_{\tau-j,T} = \alpha_j \beta H_{\tau,T} + \alpha_j \textstyle\sum_i^N \tilde{\epsilon}^i_{\tau,T} \tag{42}$$

Since there are 3 harvest months, June, July, and August, let them be denoted by the suffixes τ_1, τ_2, τ_3, respectively. Then the total short positions in hedging in month $(\tau_1 - j)$ is given by

$$\begin{aligned} X_{\tau-j,T} &= x_{\tau_1-j} + x_{\tau_2-j-1} + x_{\tau_3-j-2} \\ &= \beta(\alpha_j H_{\tau_1,T} + \alpha_{j+1} H_{\tau_2,T} + \alpha_{j+2} H_{\tau_3,T}) + U_{j,T} \end{aligned} \tag{43}$$

where the error term is

$$U_{j,T} = \alpha_j \textstyle\sum_{i=1}^N \epsilon^i_{\tau_1,T} + \alpha_{j+1} \sum_{i=1}^N \epsilon^i_{\tau_2,T} + \alpha_{j+2} \sum_{i=1}^N \epsilon^i_{\tau_3,T}$$

The production level of each state in the U.S. in each year during 1977–1980 is reported in *Agriculture Statistics* [1], and since the harvest time for each type of wheat in each state is reported in *U.S. Wheat Industry* [11], we can estimate $H_{\tau_1,T}$, $H_{\tau_2,T}$, and $H_{\tau_3,T}$.

Here, we are only concerned with the open interest level in the pre-harvest months. For the post-harvest months, one may expect that storers and other wheat industry participants would hedge in the futures market, and therefore our model would be inappropriate for those months. For this reason, we only investigate the hedging open interest levels between March and August. Since the post-harvest time hedging open interest is not considered, the length of time farmers choose to store their wheat after harvest would not significantly affect the empirical result. We choose that duration to be two months, i.e., $K = 2$.

B. Empirical Results

When the producers hedge against their production risks, one may expect that they use the futures contracts maturing in that crop year. The open interest in short positions for hedging, $x_{\tau-j,T}$, is published by the Commodity Futures Trading Commission (CFTC). The sample period was from January, 1977 to December, 1980. The OLS test of Equation (43) is given in Table IIb. It shows that the goodness-of-fit of the model with adjusted R^2 is 0.807.

Several interesting insights can be derived from the model results. The regression coefficient, β, is about 7% (highly statistically significant), suggesting a low rate of participation by farmers in the futures market, consistent with the results of a recent CFTC survey [6]. Referring to Figure 2, showing the plot of the estimated and the realized open interest in short hedging, one can further observe the high explanatory power of the model. Between 1977–1980, the highest hedging level varied greatly from a low of 107 thousand contracts in 1978 to a high of 182 thousands contracts in 1980. According to the model, this variation can largely be explained by the variation in the production level.

Since our model depends crucially on the hedger being a producer, and hence only participating in short positions, one might expect that our model would provide less (if any) explanatory power to hedging in long positions. When the OLS test (Equation (43)) is repeated for the long hedging positions in all futures contracts (Table IIc) and the long hedging positions in futures contracts of the crop year (Table IId), one observes a decrease in the explanatory power of the model. Indeed, the model shows insignificant explanatory power for the total hedging in long positions. The variation of the long hedging during a particular crop year can partly be explained by the model. However, a closer examination of the observations of the long hedging in the same crop year shows that there is, in general, an increase in hedging in the September contract, giving rise to the significant explanatory power of the model. But the level of hedging does not relate to the level of production. In contrast, examination of short hedging in the

Table II

OLS Test of the Empirical Model[a]

	Constant	β	$\bar{R}^2$
a. Total short hedging	6.3093×10^4 (5.054)	33.543 (4.078)	0.4046
b. Short hedging in the same crop year	-1.3158×10^4 (−1.189)	72.044 (9.883)	0.8078
c. Total long hedging	6.6905×10^4 (5.350)	9.3026 (1.129)	0.0118
d. Long hedging in the same crop year	-4.3303×10^3 (−0.400)	4.4344 (6.210)	0.6202

[a] t-statistics are reported in parentheses.

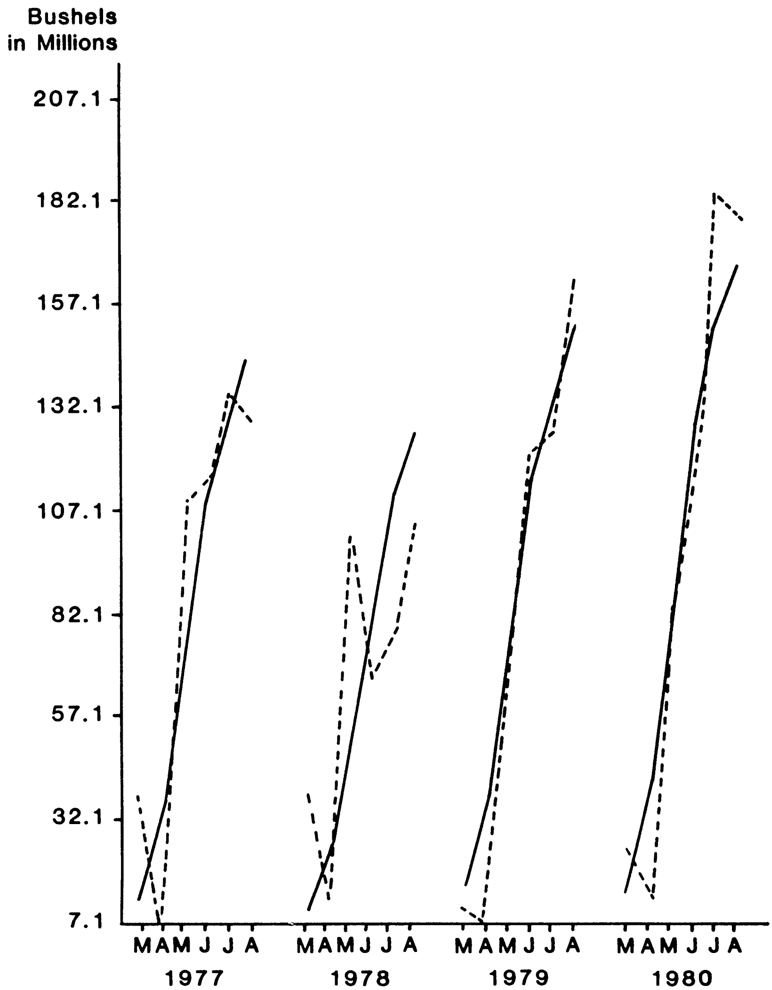

Figure 2. Comparison of the Estimated and the Realized Hedging in Short Positions (solid lines denote the estimated values; dotted lines denote the realized values)

same crop year shows that the level of long hedging is positively related to the production level, indicating that a significant portion of the futures market participants are the producers, whose behavior is what we seek to model. Lastly, we have argued that the model should be more appropriate for short positions in futures of a given crop year. Farmers would not utilize futures contracts of the next crop year to hedge their present production. For this reason, one may expect that the model has less explanatory power for short positions in hedging in all futures contracts. Table IIa presents the OLS test for these short positions and the result is consistent with the hypothesis.

V. Implications and Conclusions

This paper utilizes a continuous-time investment and consumption model to analyze the optimal behavior for the farmer's use of futures in hedging when he must contend with both price and output uncertainties. It investigates his derived utility, his optimal hedging strategy, and his production decisions. The summary of the results of the analysis and its implications are discussed below.

(a) *The value of futures trading to the farmer.* Use of futures could reduce the production risks confronting the farmer. However, it does not eliminate all the risks in general. In fact, the residual risk is proportional to $\sigma_Q(1 - \rho^2)^{1/2}$, where σ_Q^2 is the instantaneous variance of the output uncertainty and ρ is the correlation coefficient of the price and output uncertainties. Consequently, futures trading offers perfect hedging opportunities for the farmer only when there is no output uncertainty or when output and price are perfectly correlated. More interestingly, the impact of hedging on the farmer's utility depends on the time to the end of the period. When harvest time is either close or sufficiently far away, the impact of hedging is negligible. In the presence of transaction costs, a farmer would hedge in futures only when it is sufficiently valuable to him. Therefore, the result indicates that the trading activity in futures for the purpose of hedging should not be uniform during the production period. There should be less hedging when harvest time is near or when harvest time is distant.

(b) *The optimal hedge position.* The hedge ratio, in general, is less than unity and falls with the longer the time to harvest. This is a result of three effects. First, when price and output are negatively correlated (a reasonable assumption in many cases), the income uncertainty is reduced and therefore, the farmer would hedge only a portion of his anticipated output. Second, since the futures position is mark-to-market, the hedge position has to be discounted to the present value. Third, and more important, because the farmer is risk-averse, the higher the level of uncertain income, the lower the portion of the anticipated output that is hedged (the lower the hedge ratio).

Furthermore, this paper shows that the optimal hedge ratio depends on the state parameter Y, the quasi-expected farm income. Y may change (stochastically) over time and thus the farmer has to continuously adjust his hedge ratio, and so he would benefit from a liquid futures market. Although futures and forward contracting are similar within the extent of this model (Cox et al. [10] show that the two instruments are similar when we have a frictionless capital market and nonstochastic risk-free rate), in the presence of transaction costs in a security market, hedgers may much prefer futures to forward contracting when they have to revise their position frequently, as futures markets reduce and homogenize the transaction cost of forward hedging (see Silber [19]).

(c) *Production decisions.* The output decision is made at the beginning of the production period. Given the expected spot price of the commodity at harvest and the production costs, the farmer decides on the expected output which maximizes the derived utility. The analysis shows that hedging in futures makes the price elasticity of the supply function greater. Moreover, the increment is a function of the duration of the production period. When this period is short, the increase is insignificant. But it gains importance with a longer duration. This implies that futures markets have different effects on the aggregate output of various crops, depending on the duration of their growing seasons. The effect is small when the growing season is short (e.g., tomatoes), but it may be substantial for crops which have long growing seasons (e.g., wheat and cotton).

Furthermore, the analysis shows that producers benefit from a financial instrument to hedge their output risk. Hedging in such financial futures would give rise to their using commodity futures more, thereby increasing their utility and output at any given price. When an individual's production is correlated to the aggregate (national) output of the commodity, the producer can hedge the output risk by trading futures of a *commodity output index.* For example, futures on the aggregate cotton output index can be made available for cotton growers to hedge their output risk. The basic concept of such futures is similar to the futures on stock indices, in that they are futures contracts on nondeliverable goods. However, such a financial instrument, as demonstrated by the analysis, would allow the producer to hedge his own output risk, using the aggregate output as a proxy of his uncertain production level.

If the farmer's output were perfectly correlated with the national output level, the futures contracts on the commodity output index would offer a financial instrument with which he could hedge his output risk perfectly. Employing this instrument with hedging in the commodity futures, the farmer can eliminate all production risks. It follows then that the economic value of the commodity output index for hedging depends on the degree of correlation between the national output level and the individual's output level. However, this degree of correlation will vary among commodities and requires empirical investigation.

Appendix A

Let $G(W, Y, t) = f(X, t)$ where

$$X = Y\exp(-(T-t)r) + W$$

Note that $G(W, Y, T) = B(W + Y, T)$, so that $f(X, T) = B(X, T)$. Then by a direct calculation, we have

$$G_{YY} = f_{XX}[\exp(-(T-t)r)]^2$$

Similarly,

$$G_{WW} = f_{XX}$$

and

$$G_{WY} = f_{XX}\exp[-(T-t)r]$$

Then

$$\frac{G_{WY}^2}{G_{WW}} = f_{XX}[\exp(-(T-t)r)]^2$$

Hence, if we let

$$\begin{aligned}\Phi &= G_{YY}Y^2\left(\frac{1}{2}\sigma_F^2 + \sigma_{FQ} + \frac{1}{2}\sigma_Q^2\right) - \frac{\sigma_F^2}{2}\frac{G_{WY}^2}{G_{WW}}Y^2\left(1 + \frac{\sigma_{FQ}}{\sigma_F^2}\right)^2 \\ &= f_{XX}[\exp(-(T-t)r)]^2Y^2\left\{\frac{1}{2}\sigma_Q^2 - \frac{1}{2}\rho^2\sigma_Q^2\right\} \\ &= \frac{1}{2}f_{XX}[\exp(-(T-t)r)]^2Y^2\sigma_Q^2(1-\rho^2)\end{aligned}$$

But $f_{XX} \neq 0$ if the function f has to satisfy the boundary condition, and hence if $\Phi = 0$, then we require

$$\sigma_Q^2(1 - \rho^2) = 0$$

Appendix B

We wish to derive the hedge ratio given in Theorem B. From Equation (23), it can be shown that, by differentiating wish respect to W and Y,

$$G_W = G_{0W}\left(1 + \frac{\epsilon^2}{2} Y^2A^2\theta\right)$$

$$G_{WW} = G_{0WW}\left(1 + \frac{\epsilon^2}{2} Y^2A^2\theta\right)$$

and also

$$G_{WY} = G_{0WY}\left(1 + \frac{\epsilon^2}{2} Y^2A^2\theta\right) + G_{0W}\epsilon^2 YA^2\theta$$

However, the expression for G_0 is given in Theorem A. By a direct differentiation, we have

$$G_{0WW} = -\frac{aA^2}{R} \exp\left(\frac{-A(W + e^{-rT}Y)}{R}\right)$$

$$G_{0WY} = -\frac{aA^2}{R} e^{-rT} \exp\left(\frac{-A(W + e^{-rT}Y)}{R}\right)$$

and

$$G_{0W} = aA \exp\left(\frac{-A(W + e^{-rT}Y)}{R}\right)$$

Therefore, from the above equations, we can show that

$$\frac{G_{WY}}{G_{WW}} = e^{-rT} - \frac{R(\epsilon^2 YA\theta)}{\left(1 + \frac{\epsilon}{2} Y^2A^2\theta\right)}$$

$$= e^{-rT} - \epsilon^2 AYR\theta$$

where higher orders of ϵ are being ignored. Now since $\theta = \frac{\ln R}{R(1 - r)} e^{-rT}$, it follows that

$$\frac{G_{WY}}{G_{WW}} = e^{-rT}\left(1 - \frac{\epsilon^2 AY \ln R}{(1 - r)}\right)$$

Now, by Equation (22), we have

$$\frac{x^*}{Q} = \frac{G_{WY}}{G_{WW}}\left(1 + \frac{\sigma_{FQ}}{\sigma_F^2}\right)e^{\sigma_{FQ}T}$$
$$= e^{-rT}\left(1 + \frac{\sigma_{FQ}}{\sigma_F^2}\right)\left(1 - \frac{\epsilon^2 AY \ln R}{(1-r)}\right)e^{\sigma_{FQ}T}$$

as required.

Appendix C

We wish to derive the price elasticity

$$\eta = \frac{dQ}{dF}\frac{F}{Q}$$

From Equation (36), the supply function is given by, for $B = bR$,

$$1 - e^{(\rho-r)T}\beta\alpha^{1/\beta}Q^{1-(1/\beta)}F + 2BF^2Q^{2-(1/\beta)}\alpha^{1/\beta}\beta = 0$$

Totally differentiating the equation with respect to Q and F we have

$$-e^{-rT}\left(1 - \frac{1}{\beta}\right)Q^{1/\beta}\, dQF - e^{-rT}Q^{1-(1/\beta)}\, dF$$
$$+ 2BF^2\left(2 - \frac{1}{\beta}\right)Q^{1-(1/\beta)}\, dQ + 4BF\, dFQ^{2-(1/\beta)} = 0$$

Now, we let $Q\eta\, dF = F\, dQ$, and it follows that

$$\eta = \frac{e^{-rT} - 4BFQ}{2BFQ\left(2 - \frac{1}{\beta}\right) - e^{-rT(1-(1/\beta))}}$$

Since $B = R\theta(\frac{1}{2}\epsilon^2 A)$ and let $L = \frac{1}{2}\epsilon^2 A$, we have $\left(\text{note: } \theta = \frac{\ln R}{R(1-r)}\, e^{-rT}\right)$

$$\eta = \left(\frac{\beta}{1-\beta}\right)\left[1 + \frac{2 \ln R}{(1-r)}\, LY\left[\frac{2 - \frac{1}{\beta}}{1 - \frac{1}{\beta}} - 2\right]\right]$$

ignoring higher order terms of L. In simplifying, it can be shown that

$$\frac{2 - \frac{1}{\beta}}{1 - \frac{1}{\beta}} - 2 = \frac{1}{\beta - 1}$$

Then, it follows that

$$\eta = \left(\frac{\beta}{1-\beta}\right)\left[1 - \frac{\epsilon^2 A Y \ln R}{(1-\beta)(1-r)}\right]$$

as required.

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