



American Finance Association

Conglomerate Merger, Wealth Redistribution and Debt: A Note

Author(s): Chun H. Lam and Kenneth J. Boudreaux

Source: *The Journal of Finance*, Vol. 39, No. 1 (Mar., 1984), pp. 275-281

Published by: Blackwell Publishing for the American Finance Association

Stable URL: <http://www.jstor.org/stable/2327684>

Accessed: 27/11/2009 07:38

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at <http://www.jstor.org/page/info/about/policies/terms.jsp>. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at <http://www.jstor.org/action/showPublisher?publisherCode=black>.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

JSTOR is a not-for-profit service that helps scholars, researchers, and students discover, use, and build upon a wide range of content in a trusted digital archive. We use information technology and tools to increase productivity and facilitate new forms of scholarship. For more information about JSTOR, please contact support@jstor.org.



Blackwell Publishing and American Finance Association are collaborating with JSTOR to digitize, preserve and extend access to *The Journal of Finance*.

<http://www.jstor.org>

Conglomerate Merger, Wealth Redistribution and Debt: A Note

CHUN H. LAM and KENNETH J. BOUDREAUX*

THAT MERGER CAN REDISTRIBUTE wealth among capital claimants is now well recognized [2], [3], [4], and corporations even seem to take equity wealth-preserving actions from this perspective [5]. We provide herein an analysis of this redistribution effect more completely specified than in models to date, and portray certain aspects of merger not previously developed, including:

- (i) the set of relationships between merging firms' cash flows that determines the size (and direction) of wealth redistribution,
- (ii) the necessary and sufficient conditions for wealth redistribution neutrality in conglomerate merger (which requires more than perfectly correlated cash flows), and
- (iii) the determinants of wealth redistribution among debt claimants in merger, in addition to those between debt and equity.

This analysis of the unanticipated redistribution effect may be useful in developing better-specified models of agency effects upon security prices.

Because redistribution in its most pure form is best seen in the case of conglomerate merger, we limit our discussion to the merger of jointly normal cash flows $\tilde{X}_i$ and $\tilde{X}_j$ where:

$$\begin{aligned} E(\tilde{X}_i) &= \bar{X}_i; & E(\tilde{X}_i - \bar{X}_i)^2 &= \sigma_i^2 \\ E(\tilde{X}_j) &= \bar{X}_j; & E(\tilde{X}_j - \bar{X}_j)^2 &= \sigma_j^2, \end{aligned}$$

where $\tilde{X}_i$, $\tilde{X}_j$, $\tilde{X}_i | X_j$, and $\tilde{X}_j | X_i$ are all normal, as is the sum of $\tilde{X}_i$ and $\tilde{X}_j$:

$$\tilde{X}_{ij} = \tilde{X}_i + \tilde{X}_j,$$

and

$$\sigma_{ij}^2 = \sigma_i^2 + \sigma_j^2 + 2\rho_{ij}\sigma_i\sigma_j,$$

with $\rho_{i,j}$ as the correlation of $\tilde{X}_i$, $\tilde{X}_j$.

The market value of any particular corporate cash flow $\tilde{X}_j$ is

$$V_j = r^{-1}\{\bar{X}_j - \lambda\rho_{j,m}\sigma_j\}$$

with

- $\rho_{j,m}$ = the correlation of $\tilde{X}_j$ with the market portfolio,
- λ = the market risk premium, and
- r = one plus the risk free rate.

* Edwin L. Cox School of Business, Southern Methodist University, and School of Business, Tulane University, respectively.

Using the assumptions of Brennan [1] and the partial moment form of Winkler et al. [7], the firm's debt value can be found by:

$$D_j = r^{-1}\{rV_jF^*(C'_j) - \sigma_j f^*(C'_j) + C_j[1 - F^*(C'_j)]\} \quad (1)$$

where

$C_j = j$'s debt promise,

$C'_j = \frac{C_j - rV_j}{\sigma_j}$ is C_j 's standardization, and

$f^*(\cdot)$ and $F^*(\cdot)$ are standard normal density and distribution function.

Since V_j denotes the market value of firm j , the term rV_j in (1) represents the certainty equivalent of the uncertain cash flow $\bar{X}_j$.

I. Conglomerate Merger

When two firms (i and j) with jointly normal cash flows merge,

$$V_{ij} = r^{-1}\{\bar{X}_{ij} - \lambda\rho_{ij,m}\sigma_{ij}\} \quad (2)$$

but

$$\rho_{ij,m}\sigma_{ij} = \rho_{i,m}\sigma_i + \rho_{j,m}\sigma_j$$

and given a pure conglomerate merger,

$$\bar{X}_{ij} = \bar{X}_i + \bar{X}_j$$

Substituting these two relationships into (2) yields

$$V_{ij} = r^{-1}\{(\bar{X}_i - \lambda\rho_{i,m}\sigma_i) + (\bar{X}_j - \lambda\rho_{j,m}\sigma_j)\},$$

therefore

$$V_{ij} = V_i + V_j$$

and the Value Additivity Principle (VAP) holds at the overall corporate level [6].

Assuming that $C_{ij} = C_i + C_j$ so that

$$C'_{ij} = \frac{C_{ij} - rV_{ij}}{\sigma_{ij}},$$

debt value of the merged firm is

$$D_{ij} = r^{-1}\{r(V_i + V_j)F^*(C'_{ij}) - (\sigma_i^2 + \sigma_j^2 + 2\rho_{i,j}\sigma_i\sigma_j)^{1/2}f^*(C'_{ij}) + (C_i + C_j)[1 - F^*(C'_{ij})]\} \quad (3)$$

Assuming exchanges for merged-firm shares and bonds by premerger claimants are aliquot,¹ the merger will leave debt and equity claimant wealth unchanged and therefore will be wealth neutral if $D_{ij} = D_i + D_j$, given that $V_{ij} = V_i + V_j$.

¹ Later we show these conditions for debt claims; equity fair exchange conditions are commonly known.

Defining the redistribution effect as $\Delta D = D_{ij} - (D_i + D_j)$, neutrality requires $\Delta D = 0$. Combining (1) for both i and j with (3),

$$\Delta D = r^{-1}\{(rV_i - C_i)[F^*(C'_{ij}) - F^*(C'_i)] + (rV_j - C_j)[F^*(C'_{ij}) - F^*(C'_j)] + \sigma_i f^*(C'_i) + \sigma_j f^*(C'_j) - (\sigma_i^2 + \sigma_j^2 + 2\rho_{ij}\sigma_i\sigma_j)^{1/2}f^*(C'_{ij})\}. \quad (4)$$

Since $\sigma_i + \sigma_j \geq (\sigma_i^2 + \sigma_j^2 + 2\rho_{ij}\sigma_i\sigma_j)^{1/2}$, substituting into (4) and reducing gives

$$\Delta D \geq r^{-1}\{\sigma_i[(f^*(C'_i) + C'_i F^*(C'_i)) - (f^*(C'_{ij}) + C'_i F^*(C'_{ij}))] + \sigma_j[(f^*(C'_j) + C'_j F^*(C'_j)) - (f^*(C'_{ij}) + C'_j F^*(C'_{ij}))]\} \quad (5)$$

By the lemma in Appendix A $\Delta D > 0$ for all $C'_i \neq C'_j \neq C'_{ij}$, and $\Delta D = 0$ for $C'_i = C'_j = C'_{ij}$ in (5). Thus, the literature of the redistribution effect in merger is correct in arguing $\Delta D \geq 0$, but in a number of cases is incorrect in portraying neutrality, $\Delta D = 0$, as requiring only $\rho_{ij} = 1$ [2], [3], [5]. The requirement is more stringent under the assumption of normality.² For $\Delta D = 0$,

$$\frac{C_i - rV_i}{\sigma_i} = \frac{C_j - rV_j}{\sigma_j} = \frac{(C_i + C_j) - r(V_i + V_j)}{\sigma_{ij}}$$

A necessary condition for the above equalities to hold is that $\rho_{ij} = 1$, however, this is not a sufficient condition. The above conditions require, in addition, that j be a scale multiple of i in mean ($\bar{X}_j = \alpha\bar{X}_i$), in standard deviation ($\sigma_j = \alpha\sigma_i$), and in debt promise ($C_j = \alpha C_i$).³

The coinsurance literature demonstrates that $\Delta D > 0$ for $\rho_{ij} < 1$ in conglomerate merger (that, unless neutralized, debt claimant wealth increases at the expense of equity). The above analysis shows that for normally distributed cash flows $\Delta D > 0$ is not restricted to $\rho_{ij} < 1$, and occurs *generally* with the minor exception of perfect scalar (including debt promise) mergers. We include a numerical example of $\Delta D > 0$, $\rho_{ij} = 1$ merger in Appendix B.

II. Debt Wealth in Merger

The above demonstrated that debt value *in toto* must either increase or remain constant in conglomerate merger. That result begs the question of separate (premerger) debt claimant wealth change. *If premerger debt claimants are merely allowed to retain premerger claims upon the merged firm cash flow, there will generally be a further redistribution of wealth among debt claimants.*

Suppose that $C_1 = C_2$ but $\sigma_2 > \sigma_1$ in a merger. Here $D_{12} > D_1 + D_2$, so a redistribution from equity (E) to debt takes place ($V_{12} = V_1 + V_2$, but $E_{12} < E_1 + E_2$). Since the debt promises are equal, without further adjustment,

$$D_1^a = D_2^a \frac{C_1 \text{ or } 2}{C_1 + C_2} (D_{12})$$

² Care must be taken in comparing our result to others with different distributional assumptions.

³ $\Delta D = 0$ can be induced, of course, by $C'_{ij} > C'_i + C'_j$, a leverage increase. But that is tangential to the point of this argument.

The postmerger values of the debt claims, D^a , are equal. However, it will also be true that

$$(D_1^a - D_1) < (D_2^a - D_2)$$

The higher-variance premerger debt will increase in value more than the lower-variance premerger debt. It can also happen (see example in Appendix B) that

$$D_1 > D_1^a$$

The value of the lower-variance debt claim can actually decrease in a merger that increases total debt value. In such a case there is a redistribution of wealth from both equity and low-variance debt to high-variance debt.⁴

An equal-relative "within debt" redistribution can be accomplished by producing postmerger debt values bearing the premerger value relationship:

$$D_1^a = \frac{D_1}{D_1 + D_2} (D_{12})$$

$$D_2^a = \frac{D_2}{D_1 + D_2} (D_{12})$$

It is not clear, however, by what mechanism this would be accomplished.

III. Conclusion

We have shown, by deriving the conditions for wealth redistribution in merger, that perfect correlation of merging firm's cash flows is a necessary but not always a sufficient condition for wealth invariance by equity holders. In addition, if the market value of debt claims is to be unchanged, maintaining premerger debt promises, merging firms must be perfect scalar multiples.

Debt holder wealth can also be redistributed in merger, and we demonstrate that if debt is merely allowed to retain its premerger claims, such a redistribution will almost certainly occur. Although reallocating the wealth shift from equity among debt claimants on the basis of premerger value weight is conceptually simple, it is not clear how this would be implemented.

Appendix A

To show that when two firms (i and j) with jointly normal cash flows merge, the right-hand side of Equation (5) in the main text, H , is non-negative, i.e.,

$$H = r^{-1} \{ \sigma_i [(f^*(C'_i) + C'_i F^*(C'_i)) - (f^*(C'_{ij}) + C'_i F^*(C'_{ij}))] \\ + \sigma_j [(f^*(C'_j) + C'_j F^*(C'_j)) - (f^*(C'_{ij}) + C'_j F^*(C'_{ij}))] \} \geq 0 \quad (\text{A1})$$

we need the following lemma.

⁴ Similar redistributions between debt claims are possible for other parameter differences; the variance example is sufficient, however, to make the point.

Lemma:

$$h(x, y) = [f^*(x) + xF^*(x)] - [f^*(y) + yF^*(y)] \geq 0 \quad \text{for all } x, y.$$

Proof: $h(x, y)$ can be rewritten as

$$h(x, y) = [f^*(x) + xF^*(x)] - [f^*(y) + yF^*(y)] - (x - y)F^*(y)$$

or

$$h(x, y) = [g(x) - g(y)] - (x - y)F^*(y) \quad (\text{A2})$$

where

$$g(x) = f^*(x) + xF^*(x)$$

Taking the derivative of $g(x)$ with respect to x (applying Leibnitz' Rule for differentiating an integral function), $g'(x) = F^*(x)$. Since the distribution function $F^*(x)$ is a nondecreasing function, $g(x)$ is nondecreasing, continuous, and differentiable. By the mean value theorem, there exists a value $z \in (x, y)$ such that

$$g(x) - g(y) = (x - y)g'(z)$$

Using the above, (A2) can be rewritten as

$$h(x, y) = (x - y)[g'(z) - F^*(y)]$$

$$\text{or} \quad h(x, y) = (x - y)[F^*(z) - F^*(y)] > 0 \quad \text{for all } x \neq y$$

$$\text{and} \quad h(x, y) = 0 \quad \text{only if } x = y$$

Q.E.D.

Applying the lemma to (A1), it follows immediately that

$$H > 0 \quad \text{for all } C'_i \neq C'_j \neq C'_{ij}$$

$$\text{and} \quad H = 0 \quad \text{only when } C'_i = C'_j = C'_{ij}$$

Therefore

$$\Delta D > 0 \quad \text{for all } C'_i \neq C'_j \neq C'_{ij}$$

$$\text{and} \quad \Delta D = 0 \quad \text{when (i) } C'_i = C'_j = C'_{ij} \\ \text{and (ii) } \rho_{i,j} = 1$$

Appendix B

A Numerical Example of Redistribution Effects When Cash Flows of Merged Firms are Perfectly Correlated

Consider two firms i and j which merge to form firm ij . The cash flows of firms i and j are jointly normal and perfectly correlated ($\rho_{i,j} = 1$). The characteristics of the firms are described below.

Characteristics	Firm <i>i</i>	Firm <i>j</i>	Firm <i>i, j</i>
Mean $\bar{X}$	1,000	1,000	2,000
Standard Deviation σ	450	900	1,350
Correlation with Market $\rho_{j,m}$	0.65	0.65	0.65
Promised Debt Payment C	600	600	1,200
One Plus Riskless Rate $r = 1.1$			
The Market Risk Premium $\lambda = 0.35$			

Using (2) and (1) to compute the value of the firm and the debt values respectively, we obtain:

	Firm <i>i</i>	Firm <i>j</i>	Firm <i>i, j</i>
Value of Firm (V)	816.02	722.96	1538.98
Value of Debt (D)	483.06	300.18	793.00
Value of Equity (E)	332.96	422.78	745.98

Obviously, the postmerger debt value is greater than the sum of the premerger values.

$$\begin{aligned}
 \Delta D &= D_{ij} - (D_i + D_j) \\
 &= 793.00 - (483.06 + 300.18) \\
 &= 9.76 > 0
 \end{aligned}$$

Since postmerger firm value is equal to the sum of premerger firm values, wealth is redistributed from equity holders to debt holders *in toto*.

$$\begin{aligned}
 \Delta E &= E_{ij} - (E_i + E_j) \\
 &= 745.98 - (332.96 + 422.78) \\
 &= -9.76 < 0
 \end{aligned}$$

In addition, the numerical results can also be used to illustrate wealth redistribution between debt claimants of the premerged firms.

(i) If the postmerger debt values are based upon the proportions of premerger promised payments:

Since the two firms premerger have equal promised payments, the values owned by the debt claims of firm *i* and *j* postmerger is

$$\begin{aligned}
 D_i^a &= 793.00/2 = 396.50 \\
 D_j^a &= 396.50
 \end{aligned}$$

Thus

$$\begin{aligned}
 \Delta D_i &= D_i^a - D_i = -86.56 \\
 \Delta D_j &= D_j^a - D_j = 96.32
 \end{aligned}$$

With postmerger debt claims based on relative sizes of premerger promised payments, debt claims of the lower-variance firm *i* actually lose value postmerger. Debt claims of the higher-variance firm *j* increase in value by more than the loss

of the lower-variance debt. The difference, 9.76, is redistributed from the equity holders of the two firms i and j .

If we further assume that postmerger equity values of firms i and j are based on premerger equity value proportions, then

$$E_i^a = \frac{E_i}{E_i + E_j} (E_{ij}) = 328.66$$

$$E_j^a = \frac{E_j}{E_i + E_j} (E_{ij}) = 417.32$$

$$\Delta E_i = E_i^a - E_i = -4.30$$

$$\Delta E_j = E_j^a - E_j = -5.46$$

The debt claim of the high-variance firm reaps the total benefit of value redistribution.

(ii) If the postmerger debt values are based upon proportions of premerger debt values:

In this arrangement

$$D_i^a = \frac{D_i}{D_i + D_j} (D_{ij}) = 489.08$$

$$D_j^a = \frac{D_j}{D_i + D_j} (D_{ij}) = 303.92$$

$$\Delta D_i = 6.02$$

$$\Delta D_j = 3.74$$

Debt values of both firms increase. However, with smaller premerger debt value, the debt value of the higher-variance firm j increases less than the lower-variance debt value.

REFERENCES

1. M. J. Brennan. "The Pricing of Contingent Claims in Discrete Time Models." *Journal of Finance* 34 (March 1979), 53-68.
2. D. Galai and R. W. Masulis. "The Option Pricing Model and the Risk Factor of Stock." *Journal of Financial Economics* 3 (1976), 53-81.
3. R. C. Higgins and L. D. Schall. "Corporate Bankruptcy and Conglomerate Merger." *Journal of Finance* 30 (March 1975), 93-113.
4. M. C. Jensen and W. H. Meckling. "Theory of the Firm: Managerial Behavior, Agency Costs, and Capital Structure." *Journal of Financial Economics* 3 (1975), 305-60.
5. E. H. Kim and J. J. McConnell. "Corporate Mergers and the Coinsurance of Corporate Debt." *Journal of Finance* 32 (May 1977), 349-65.
6. L. D. Schall. "Asset Valuation, Firm Investment and Firm Diversification." *Journal of Business* 44 (January 1972), 11-28.
7. R. L. Winkler, G. M. Roodman, and R. R. Britney. "The Determination of Partial Moments." *Management Science* 19 (1972), 290-96.