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A Partial Theory of Takeover Bids

D. J. ASHTON and D. R. ATKINS*

ABSTRACT

There is a natural separation between production decisions affecting the firm as a whole and individual decisions by each shareholder about his portfolio of securities. The end result of these two types of decisions is normally referred to as a productive exchange equilibrium. At such an equilibrium, no individual wants to adjust his portfolio and no firm can muster majority support for a change in its production plans. This paper presents a partial theory of takeover bids in that it examines the role of a takeover bid as a mechanism by which a *simultaneous* change in shareholdings and production plans can be achieved. This enables a new production exchange equilibrium to be reached which is preferred by a majority of the shareholders but which is inaccessible without a contingent contract in the form of a takeover bid.

A POPULAR METHOD of gaining voting control over a public company is the takeover bid or tender offer. There are several reasons for this popularity, with the consequence that takeover bids often fulfill several different roles. This paper is a "partial" theory in that it concentrates on just one of these roles. This is the role of a takeover as a means of resolving disagreement between current or potential future shareholders about the production policies to be pursued by the firm.

There is a natural separation between production or policy decisions affecting the firm as a whole, possibly made through the medium of a shareholders' meeting, and individual decisions by each shareholder about his own portfolio of securities. The end result of these two types of decisions is normally referred to as a productive-exchange equilibrium. At such an equilibrium, no individual wants to adjust his portfolio, and no firm can muster majority support for a change in technology.[1] The takeover bid is a mechanism which allows the block on activity at such a productive-exchange equilibrium to be lifted.

Thus, within the framework afforded by takeover legislation, the dissenting shareholders can be compensated for their loss in utility following a change in the production policies of the firm by a redistribution of shares or wealth between all the participating shareholders. The essence, then, of a takeover, as considered in this paper, is that it is a contingent contract which enables a simultaneous

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[1] The rather odd phrase "a change in technology" should not be taken to be literally associated with technological innovation. In the "unanimity" literature this phrase is taken to mean a change in the probability distribution of future returns.

change in production policies and portfolio holdings, the share redistribution being conditional upon the requisite majority accepting the offer. The question addressed here is whether the takeover mechanism is impotent, allowing no redistributions unavailable by normal market trading. Or, put another way, is the set of takeover bid equilibria strictly contained in the set of productive-exchange equilibria? This paper explores some assumptions necessary for this to occur.

The often quoted motivational role of takeovers, whereby the threat of a takeover encourages management in the efficient use of resources, is not considered here; with the firm at a productive-exchange equilibrium, and assuming, as we do, no separation of control between management and owners, such inefficiencies are not present. We also restrict bids to individuals as the cleanest assumption to contain our analysis. Bids by syndicates, or as the law defines "others acting in concert," are close to bids by other firms as well and hence we would also have a theory of mergers. This would require a change in the production-exchange concept; if joint gains existed to mergers, presumably the same gains could partially be realized by suitably drafted contractual agreements instead of a merger. This would also obviate the loss of a market security and allow continued differential shareholding. Thus, we would then have to be concerned about multiple contracts, essentially cartels, among firms, which takes us too far from our primary purpose. There is a wide literature on takeovers but only a few, for example King [11], Leland [12], Grossman and Hart [7], try to identify the prerequisites for takeovers on theoretical grounds. One, however, Hart [9], is very close in spirit and we shall refer to it frequently.

I. An Informal and a Formal Model

The formal model and definitions of a takeover, to be presented later in this section, tend to be rendered somewhat opaque by the notational complexity. Therefore, before proceeding to the formal model and analysis, it is useful to try to develop an intuitive understanding of situations where takeover bids are likely to occur and of the nature of the takeover bid itself. This can fairly easily be achieved using an "Edgeworth Box" diagram, as in Figure 1. Consider the case of just two shareholders, the Target T and the Raider R, who between them share the one firm or risky asset and the one unit of "cash" or riskless asset in the economy. On the vertical of the box is measured the fractional shareholding of the Target, initially b^0. The remainder $(1 - b^0)$ is the Raider's initial shareholding in the firm. Similarly, along the base is Target's "cash" holding, initially a^0, with $1 - a^0$ for the Raider. Thus, isoquants such as ADC connect portfolio combinations of the risky and riskless asset to which the Target is indifferent. ABC represents a similar construct for the Raider. We further consider the case of just two technologies, an old or current technology for which we use a zero superscript and a new or alternative technology denoted by the superscript "1". For example, with this notation (a^0, b^0) is the current or initial portfolio of the Target shareholder. The contract curve with the current technology and endowments is the curve CC^0. For the sake of convenience, it will be assumed that the

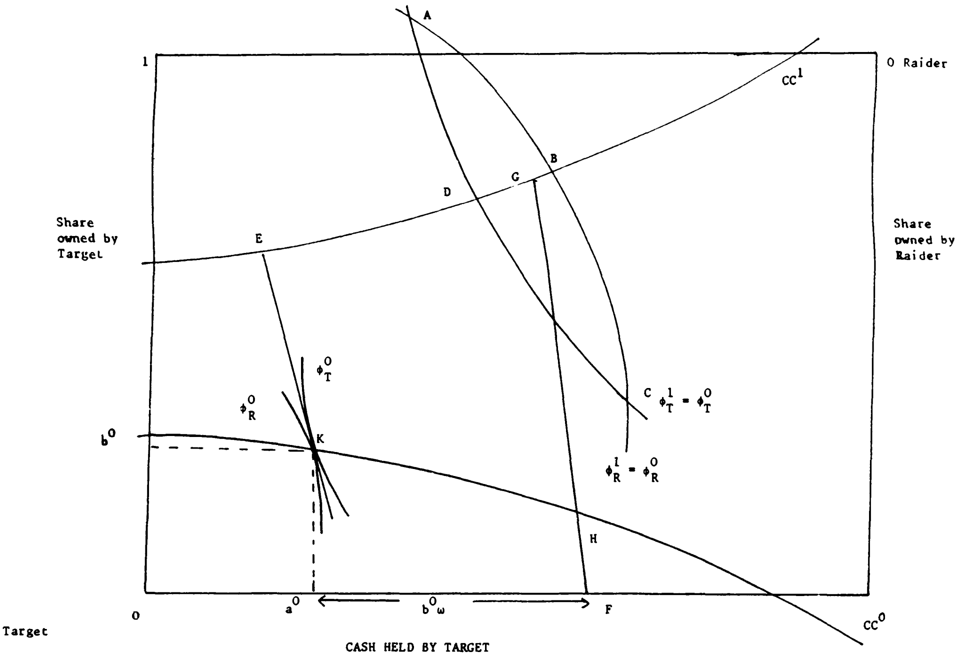

Figure 1. Takeovers in a Two Shareholders, One Firm Economy

initial portfolios constitute a share-exchange equilibrium and thus lie on the curve CC^0. At this equilibrium, K, the Target and the Raider have expected utilities ϕ_T^0 and ϕ_R^0, respectively.

If, however, the new technology were adopted, a completely new set of isoquants would be generated. Let ADC be the Target's isoquant under the new technology corresponding to an expected utility value ϕ_T^0. Similarly, ABC is the Raider's isoquant under the new technology, corresponding to an expected utility value of ϕ_R^0. Hence, the Raider is indifferent between any equilibrium portfolio on ABC with the firm operating the new technology and his initial portfolio at K with the firm pursuing the old technology. Clearly, all points with the lozenge $ABCD$ are preferred to K by both parties.

Thus, if simultaneous changes in shareholdings and technology were feasible, both shareholders would settle for some point in the lozenge $ABCD$. However, in the absence of a mechanism for achieving a simultaneous change, no change in either shareholding or production would necessarily take place. For example, suppose it was proposed, at say a shareholders' meeting, to adopt the new technology without any redistribution of the initial endowments. Following such a change in technology, it would be in the interests of both parties to readjust their portfolios. They would trade to (say) some point E, which is on the contract curve CC^1. At E, the Target shareholder, being to the left of ADC, would be worse off. If we temporarily assume that any vote to change the technology must be unanimous, then clearly such a proposal would fail, since it would lack the support of the Target shareholder.

However, if the Raider offered the Target an amount \$$\omega$ per share, for all of the Target's shares and if he were to accept, the Target would own no shares but an increased holding in the riskless asset of $a^0 + \omega b^0$. Such a transaction would move the Target and the Raider to point F. At point F, the Raider has gained control of the firm and would implement his preferred technology α^1. It would now be in both their interests to trade to G at a price p^1 say. Hence, following an appropriate transfer of wealth from the Raider to the Target, the result would be an adoption of the new technology leaving both parties better off. It is this simultaneous change in holdings and technology that will form the basis of our subsequent model and definition of a takeover.

Both Hart [9] and Drèze [4] emphasize that the need for this simultaneous change in shares and technology arises from the nonconvexities involved. Thus, if b is the Target's shareholding and $x(\alpha)$ is the second period outcome based on technology choice α, then T's second period income is $b\tilde{x}(\alpha)$, clearly nonconvex. In such an economy, optimization of shareholdings b and technologies α, separately, will not necessarily guarantee a global optimum. Takeover bids provide the contractual arrangements for simultaneous change in b and α which allow the move to a Pareto-preferred equilibrium. The offer of \$$\omega$ per share is only binding on the Raider if the bid produces the requisite control. Equally, the Target will only accept the bid and sell his voting rights if the wealth transfer adequately compensates him for any potential loss in utility resulting from the new technology.

In the case where changes in technology require a majority (as opposed to 100%), the outcome would be unchanged if the bid were such as to move the Target to a point on GF, H say, such that at this point the Raider held at least the requisite voting majority. Thus, in this case, the Target shareholder is indifferent between an offer (ω^1, s^1) which gives per share held ω^1 in cash and s^1 new shares $(0 \leq s^1 \leq 1)$, as long as $\omega^1 + s^1p^1 = \omega$, the previous "cash" only offer. Hence, we can talk about a (ω, s) offer and refer to $\omega + sp^1$ as the takeover price. Trading along FG shares the risk; it is the jump onto the new budget line by the transfer of wealth that has permitted the takeover to work. It should be noted, however, that this indifference along FG is possible because at the moment we are assuming that price perceptions about p^1 are common. Nevertheless, Figure 1 would suggest that takeover bids do provide release attractive to all parties from a productive-exchange equilibrium. It is an exploration of the circumstances which are necessary for such a takeover bid to exist which forms the substance of this paper. In order to investigate this, we need a more formal model.

Consider a two-period economy in an uncertain environment, "now" period 0 and "then" period 1. The state of the world $\theta \in \Theta$ will be known then but is a random variable now. Θ will usually be R^1 or a part thereof and will be the support for homogeneous beliefs about the outcomes, represented by the probability distributions $f(\theta)$. Takeovers, as defined here, are more likely with heterogeneous beliefs, for example the Raider being much more optimistic about the prospects of a new technology than existing shareholders. Thus, it is more interesting to question whether takeovers still exist with homogeneous beliefs. This is also in line with takeover bid legislation concerning disclosure, which results in a certain resolution of diverging beliefs. In the U.S. the Williams Act and subsequent amendments have added to the Securities Exchange Act of 1934 concerning disclosure of "material information," as also has the voluntary U.K. City Code on Takeovers and Mergers. Disclosure also forms a major part of the comprehensive new Companies (Acquisition of Shares) Act of 1980 in Australia, superseding earlier legislation such as the 1971 Amendments of the Companies Act. However, it must be admitted that such resolution is very partial and disclosure remains the weakest part of all such legislation, particularly since the possibility of effective enforcement is fairly low.

Shareholders $i \in I = 1, \cdots, n$ are Von-Neumann-Morgenstern expected utility maximizers with utilities $U_i(x)$ over wealth x in period 1. We follow Diamond [3] and Mossin [14], unlike Hart [9] and Drèze [4], in ignoring first-period consumption which we take as exogenously given. Investment in firms then becomes an alternative to investment in a riskless asset. For simplicity the riskless rate will be taken as 1. We also assume $U_i'(x) > 0$ and strict risk-aversion $U_i''(x) < 0$, because in a world with unlimited short positions in the riskless asset a risk neutral person will simply own all the shares. Individual i will hold a portfolio of a_i in the riskless asset and shares b_{ij} in firm $j \in J = 1, \cdots, m$ where $\sum_i b_{ij} = 1$. Firm j can make a production decision to undertake technology $\alpha_j \in \Lambda_j$ which would imply an output of $x_j(s, \alpha_j)$ in state s in period 1 and an input investment $I_j(\alpha_j)$ now. Individuals may go short in the riskless asset and, in general, long or short in shares, although this might be tightened to either $0 \leq b_{ij}$ or $b_{ij} \leq 1$ or both as required. Each market will clear at a single price and initially shareholder i holds $(a_i^0, b_{i1}^0, \cdots, b_{im}^0)$ where

$$\sum_j I_j + \sum_i a_i^0 = M$$

We define

$$\phi_i(a_i, b_i, \alpha) = E_{s \in S} U_i(a_i + b_i \cdot x)$$

where b_i, α, and x are vectors with the obvious dimensions and transpositions are understood. Following Hart, we define a stock market allocation and a constrained Pareto optimum.

Definition: A stock market allocation is a triple (a, b, α) where

i. $a \in R^N$
ii. $b \in R^{NM}$ $\sum_i b_{ij} = 1$ for all $j \in J$
iii. $\alpha \in \prod_j \Lambda_j$
iv. $\sum_j I_j(\alpha_j) + \sum_i a_i = M$

Definition: The triple (a, b, α) is a constrained Pareto optimal stock market allocation if $\nexists$ (a^0, b^0, α^0) such that $\phi_i(a^0, b^0, \alpha^0) \geq \phi_i(a, b, \alpha)$ for all $i \in I$ and strict for at least one $i \in I$.

A competitive exchange equilibrium sustained by prices p is defined in the usual way with shareholders maximizing ϕ subject to their budget constraints with markets clearing at p.

Definition: *A productive-exchange equilibrium* is a competitive exchange equilibrium such that there does not exist $k \in J$ and competitive exchange equilibrium (a^1, b^1, α^1) sustained by prices p^1 such that

i. $\alpha_j^1 = \alpha_j^0$ for all $j \neq k$
ii. (a^1, b^1, α^1) dominates (a^0, b^0, α^0)
iii. $a_i^1 + \sum_j p_j^1 b_{ij}^1 \leq a_i^0 + \sum_j p_j^1 b_{ij}^0 - b_{ik}^0(I_k(\alpha_k^1) - I_k(\alpha_k^0))$

This definition can be altered to deal with a majority decision rather than a unanimous one.

In this definition we make additional investment proportional to initial shareholdings, whereas in Hart [9] it comes from current consumption on a "pooled" basis. Also, unlike Hart [9], improvement is measured after final markets clear; it is quite possible for individuals to be worse off immediately post-decision, but the decision is made on the basis of subsequent trading. This should really be termed a perceived equilibrium as there has been no assumption about common expectations regarding the prices p^1. At this stage to include assumptions to ensure that aggregation was possible, or simply to make common price expectations, would limit the generality of the issue too much.

Definition: A stock market allocation (a^0, b^0, α^0) is a *Takeover equilibrium* (*TE*) if for no shareholder $i^* \in I$ and firm $j^* \in J$ does there exist a pair $(\omega, s) \in R^2$ and a competitive exchange equilibrium (a^1, b^1, α^1) sustained by prices p^1 such that

i. $\alpha_j^1 = \alpha_j^0$ for all $j \neq j^*$
ii. $\sum_{i \in A} b_{ij^*}^0 \geq \delta$ where $A = (i \mid \phi_i(a_i^1, b_i^1, \alpha^1) \geq \phi_i(a_i^0, b_i^0, \alpha^0))$
iii. $b_{i^*j^*}^1 \geq \epsilon$
iv. $0 \leq s \leq 1$
v. $a_{i^*}^1 + \sum_j p_j^1 b_{i^*j}^1 \leq (1 - s\sum_{i \neq i^*} b_{ij^*}^0)(I_{j^*}^0 - I_{j^*}^1) + a_{i^*}^0 - \omega + \omega b_{i^*j^*}^0$
$+ \sum_{j \neq j^*} p_j^1 b_{i^*j^*}^0 + (1 - s\sum_{i \neq i^*} b_{ij^*}^0) p_{j^*}^1$
vi. $a_i^1 + \sum_j p_j^1 b_{ij}^1 \leq s b_{ij^*}^0 (I_{j^*}^0 - I_{j^*}^1) + a_i^0 + \omega b_{ij^*}^0$
$+ s p_{j^*}^1 b_{ij^*}^0 + \sum_{j \neq j^*} p_j^1 b_{ij}^0$ for all $i \neq i^*$

This definition requires some explanation.

1. The sequence of events is as follows. Person i^* offers cash ω and new shares s to every existing shareholder for every share held in firm j^*. Now a Takeover is a very particular type of contract, it is *non-price discriminatory* and it is *conditional.* Much of takeover legislation is concerned with these two aspects. First, it is non-price discriminatory; all shareholders receive a standard offer, at least within the same class of shares. Various legislation either prohibits Raider trading during the bid period or, if permitted, restricts other bargains to prices below the bid offer. A higher bargain results in a de facto bid revision and is typically available to all, including, retrospectively, to tenders already accepted. Second, the bid is usually conditional on a majority acceptance, δ in this case. In the U.K. there is usually 90% by a specified date in order to be able to compulsorily acquire minority holdings[2] but the Raider has the option to declare the bid

[2] Note this should strictly be 90% of $(1 - b_{i^*k^*}^0)$, and also a 75% of shareholders rule might also apply, but we shall ignore both these points.

unconditional when acceptances exceed 50% of voting rights. For a partial bid most legislation now applies a proration rule for oversubscribing. In this model, for simplicity, we have employed a compulsory acquisition of nontendered minority holdings. If less than δ acceptances are received the offer lapses, if more, then the Raider is obliged to fulfill the offer.

2. At this point the Raider, now in control, changes the production plan from $\alpha^0_{j^*}$ to $\alpha^1_{j^*}$ leaving all other plans unchanged. The new financing for this change is to be supplied by shareholders at the time, that is, after the (ω, s) transaction takes place. For this to be realistic, we have to require legislation that production plans and their consequences, such as additional financing, must be announced as part of the offer and adhered to subsequently. We defer until later a discussion of the moral hazard involved here whereby the Raider may, at that time, have an incentive to change to a plan quite different to his announced intentions. This is particularly important because disclosure of future plans is perhaps one of the weakest and least enforced portions of takeover legislation.

3. Trading then occurs at the new clearing prices. Notice that trading may also occur on stocks other than firm j^*, and that these prices may well have changed because of the events surrounding j^*.

4. Trading is subject to i^* retaining effective control defined by the exogenous variable ϵ. This of course implies that subsequent trading may not reach the exchange equilibrium expected, with a consequent distortion in the share price. We shall avoid this difficulty by basically ignoring it and set $\epsilon = 0$.

Before comparing this equilibrium concept to that of Hart, we will give a simplified version for the case of a cash-only bid ($s = 0$), unanimity required of all existing shareholders $\delta = 1$, no new investment needed ($I^1_{j^*} - I^0_{j^*} = 0$), and no concerns about subsequent control $\epsilon = 0$. This simplified version retains most of what we find interesting in the takeover situation and also removes much of the notational clutter.

Definition: A stock market allocation (a^0, b^0, α^0) is a *simple cash Takeover equilibrium* (*SCTE*) with respect to person 1 and firm 1 if there does not exist $\omega \in R$ and a competitive exchange equilibrium (a^1, b^1, α^1) such that

i. $\alpha^1_j = \alpha^0_j$ for all $j \neq 1$

ii. (a^1, b^1, α^1) Pareto dominates (a^0, b^0, α^0)

iii. $a^1_i + \sum_j p^1_j b^1_{ij} \leq a^0_i + \omega b^0_{i1} + \sum_{j \neq 1} p^1_j b^0_{ij}$ for all $i \neq 1$

iv. $a^1_1 + \sum_j p^1_j b^1_{1j} \leq a^0_1 - \omega + \omega b^0_{11} + p^1_1 + \sum_{j \neq 1} p^1_j b^0_{ij}$

For two people and one firm this is the case pictured previously in Figure 1 achieved by market trading along *FG*. Notice that with the TE any values of (ω^1, s^1) such that $\omega^1 + s^1 p^1 = \omega$ gives the same set of equations, that is, starting anywhere along *FG* will cause trading to occur to *G*. Takeovers are not risk sharing instruments per se, but simply wealth transfer instruments of a rather odd, i.e., proportional, kind. Thus Raider (2) has compensated Target (1) for his loss of utility occasioned by a change to technology α^1. Both now have new budget lines. Comparing our definition of equilibrium to that of Hart [9] we see several differences, apart from those occasioned by the model structure. First, he allows the takeover of several firms simultaneously, and hence will have a smaller set of equilibria. We could have done this but did not feel that it was of sufficient interest to justify the added complexity. Second, and more importantly, Hart [9] assumes that prices in all other markets *remain the same and that other markets need not clear.* The Raider however does announce the market clearing price of the target firm at which it will subsequently be traded. The core of the problem here is that either definition would only be entirely satisfactory if budget constraints were observable quantities to the individuals concerned. Leaving the price forecast as the individual's concern is, we consider, less ad hoc and in line with the casual observation that gaining good price forecasts is of great concern to target shareholders. It is however an assumption which rather stalls the analysis without some generally agreed methodology of price formation. Notice that although our equilibrium is a stochastic concept given a stochastic model of price formation, it is *not* a rational expectation (RE) equilibrium. We have implicitly made the assumption that trading is frozen on *all* markets, not just the firm in question, when the bid is announced and kept that way until after the offer has been settled, at which time markets open again. There is no opportunity for forming rational expectations about the bid. In a later section we will relax this assumption to consider the other cases in which some form of partial RE equilibrium could form. A takeover for which $\omega \neq p^1$ will be termed *proper.*

II. Some Examples

Do proper takeovers exist? The answer is yes.

Example 1

There are three possible future states, one firm and two shareholders. Outcomes are given in Table I and the results can be easily checked.

After a "myopic" change to α^1 at a shareholders' meeting but without market trading we have

$$EU_1 = 2.4 \text{ (better)} \qquad EU_2 = 0.43 \text{ (worse)}$$

Table I

An Example of a Takeover

	States		
Yields	1	2	3
Riskless asset	1	1	1
Firm with decision α^0	0	4	0
Firm with decision α^1	0	0	300
Homogeneous beliefs	0.1	0.8	0.1

	Person 1	Person 2
Utility of x	$\sqrt{x}$	$\ln(1 + x)$
Initial cash	$a_1 = 1$	$a_2 = 0 \quad M = 1$
Initial shares	$b_1 = 3/4$	$b_2 = 1/4$
Initial utility	$EU_1 = 1.8$	$EU_2 = 0.8 \ln 2 = 0.5545$

Allowing markets to clear after a change to α^1

$$\mathrm{EU}_1 = 2.42 \text{ (better)} \qquad \mathrm{EU}_2 = 0.55 \text{ (still not enough)}$$

But suppose person 1 offers person 2 a deal (ω, s) and then a change to α^1 plus subsequently trading at the market clearing price $p^1 = 1.69$ where $\omega + p^1 s = 2$

New cash position	$a_1 = 0.64$	$a_2 = 0.36$
New share position	$b_1 = 0.92$	$b_2 = 0.08$
At equilibrium	$\mathrm{EU}_1 = 2.38$	$\mathrm{EU}_2 = 0.60$

and so 2 has been compensated by 1 by a takeover which is proper because $\omega + p^1 s \neq p^1$. We assume in this example that some unspecified mechanism exists for comparing perceptions, and so price forecasts are common.

Are there circumstances in which takeovers will never exist even with our economy which has no provision for contracts on contingent future commodities?

While it is difficult to provide a precise and complete characterization of economies in which takeover bids, as defined in this paper, would not exist, it is not difficult to identify fairly general circumstances which would lead to the absence of such takeovers. This in turn enables us at least to be able to identify some specific examples of economies in which the takeover bid is a vacuous instrument. Circumstances leading to the absence of takeovers can readily be identified by reconsideration of Figure 1. The crucial feature of the diagram which gave rise to the takeover in the choice between two technologies was the existence of an "improvement" lozenge *ABCD* which was inaccessible by normal trading activity. Thus, in the case where the lozenge is not empty, then takeovers are irrelevant instruments if by merely changing the technology the shareholders are able to trade by pure share exchange into the improvement set. In this case of course, K is not a productive exchange equilibrium. In the case where the lozenge is empty, although K is a productive exchange equilibrium, the shareholders would unanimously reject any further change in technology. Thus, in both cases it is the post-trading unanimity which is the reason for the nonuse of a takeover instrument.

We could attempt to characterize unanimity conditions in terms of either restrictions on the class of utility functions or on the available technology set. However, characterizing unanimity conditions purely in terms of the choice of technologies is not easy. While there are trivial examples, such as where the choice between technologies is nothing more than the choice of location of the distribution of returns, a general solution to this problem requires the establishment of stochastic dominance relationships between equilibrium portfolios. Furthermore, the trading price linking any two equilibrium portfolios will normally depend on the technologies as well as shareholder attitudes to risk. This remains a relatively unexplained area of the stochastic dominance literature.

Conditions which need to be imposed solely on the utility functions to ensure unanimity and hence the absence of takeovers are more fully explored. For example, if all investors exhibit linear risk tolerance with the same cautiousness, i.e., $\dfrac{u'}{u''}$ has a constant derivative for all shareholders, then there is no need for takeovers. A direct proof of this is included in the Appendix although it is not difficult to deduce the result from other known results. For this class of utility functions we know that technological changes are unanimous, Rosing [15]. Portfolios are balanced. Furthermore, the contract curve is independent of the technology and hence no post-offer trading is necessary. While Rosing [15] and Brennan and Kraus [2] establish that this is the only set of utility functions which leads to unanimity for arbitrary distributions, the extensive and more recent unanimity literature develops unanimity conditions which are likely to be at least asymptotically true in large markets. For example, if we assume some sort of "competivity" assumptions such as price taking with respect to implicit prices of risk and then ensure agreement about prices by assuming that the new technology is spanned, Ekern and Wilson [5], we get agreement on the choice of technology and takeovers again become irrelevant. The most comprehensive contribution to these aspects of unanimity can be found in Grossman and Stiglitz [8] and it is well summarized in Baron [1]. We will treat these ideas more formally in the next section.

One further contender for no takeovers might be thought to be a world broadly consistent with the Capital Asset Pricing Model, where shareholders make decisions based on the means and variances of the portfolios derived from a multinormal state distribution.[3] Certainly there is unanimity about small changes in α and there are common price expectations given the assumption of constant market price of risk, Ekern and Wilson [5]. But with significant changes in α, individuals' implicit price of risk will vary and then such people are not necessarily value maximizers. Both of these points are illustrated in the second example, which establishes the existence of takeovers in such an economy.

Example 2

Consider an economy with current technology α^0 such that the return on market portfolio is normally distributed with a mean of 10 and a variance of 8. Further, there exists the possibility of riskless borrowing and lending with unit return. Assume further that the participants in this economy consist of a Raider R and eight "Target" shareholders T. The Raider has a utility function over wealth x given by

$$U_R = 100(2x - 0.1x^2 - 7.33)$$

and an initial investment of 0.308 in the riskless asset and a holding of 0.470 in the market portfolio. The eight Target shareholders have identical utility functions given by

$$U_T = 1000(0.485 - e^{-x} - 0.5e^{-8x})$$

and identical holdings of 0.0633 in the market portfolio and 0.0865 invested at the riskless rate. Here the choice of the form of the utility functions are dictated partially by the need for analytical tractability of the expressions for the expected

[3] We forget about quadratic utility as already covered in the LRT class above and restrict ourselves to distributions with finite variances.

Table II

A Takeover in a Mean Variance World

State	Shareholder Group	Investment at Riskless Rate	Holding in Market Portfolio	Expected Utility
Initial equilibrium technology α^0	Raider	0.3077	0.470	0
$P_m{}^* = 9.247$	Target	0.08654	0.0663	0
Technology α^1 prior to trading	Raider	0.3077	0.470	+25
	Target	0.08654	0.0663	−23
Technology α^1 post-trading	Raider	−0.2778	0.5606	+31
$P_m = 6.435$	Target	0.1597	0.05492	−1.5
Takeover bid equilibrium techology α^1	Raider	−0.2961	0.5609	+29
$P_m = 6.471$	Target	0.1620	0.0549	0

* P_m denotes the equilibrium price of the market portfolio.

utility and partially by the need to avoid utilities from the same linear risk tolerance group. It is easy to check that with these initial endowments the market is in exchange-equilibrium, while the choice of parameters for the utility functions ensures that the initial values for the expected utilities ϕ_R and ϕ_T are zero.

Table II illustrates the impact of a proposal to change the technology to the alternative technology α^1 under which the return on the market portfolio is normally distributed with mean 12 and variance 35.2. Myopically, that is merely changing the technology without trading, such a change would improve the utility of the Raider by 25 but reduce that of the remaining majority by 23 each. A change accompanied by trading to a new equilibrium would change these utilities by 31 and −1.5 respectively. Thus, such a change is neither ex ante nor ex post unanimous, and the initial equilibrium is a productive-exchange equilibrium. However, the Raider could purchase shares from the target shareholders, by borrowing at the riskless rate, for 3.446 representing a takeover price[4] of 6.50 and then change the technology. The new equilibrium position would leave the Raider a majority shareholder with an increase in his utility of 29 units. At this minimum bid price, the Target shareholders are no worse off after trading to the new equilibrium.[5] It is this new equilibrium which constitutes a takeover bid equilibrium. In this example, as in example one, we have assumed all actors have access to a common (and it turns out, correct) price forecast.

III. Comparing the Takeover Case to the "Unanimity" Literature

The arithmetic of the last example is somewhat involved and disguises the economic intuition. We can gain more insight by comparing the takeover situation to the familiar unanimity literature, utilizing a local approximation. It is then shown how the assumptions typically made to ensure unanimity also lead to the

[4] This is his minimum bid price. The Raider could bid as high as 7.74 before such a takeover becomes unattractive to him.

[5] The maximum bid price would leave the Raider no better off but would increase the utility of the Target shareholders by 28.

absence of takeovers. Thus if $x(\theta, \alpha)$ is the vector of outcomes across all firms given the choice of technology α, and the random variable θ, the return to any individual's portfolio is $R = (\theta, \alpha) = a + bx(\theta, \alpha)$ where b as before is a vector of stock holdings and a is investment in a riskless asset scaled so as to earn 1 in the next period. Now the expected utility function ϕ is a mapping from the space of continuous functions to the reals. Thus $\phi(u) = \int u(R(\theta, \alpha))\, d\theta$ and $\phi: C[\theta] \rightarrow R$.

The Frechet differentiability of ϕ ensures that

$$\Delta\phi = \phi - \phi_0 = E_\theta\{hu'(R^0)\} + \delta(h) \tag{1}$$

where $|\delta(h)| \leq \frac{1}{2} \sup |Eu''(R^0 + \epsilon h)|\, \| h^2 \|$ and where $0 < \epsilon < 1$

$$h(\theta) = R(\theta, \alpha^1) - R(\theta, \alpha) = R - R^0 = \Delta a + b \cdot x - b^0 x^0$$

In the above model, it should be remembered that the expectations operator is currently person-specific. The factors which are likely to produce agreement on expectations of price will be discussed later. To avoid the use of too many subscripts we will use as the vector of shares b for a Target shareholder a small "t" and similarly a small "r" for the Raider. From the budget line we know that for a Target shareholder,

$$\Delta a = p^t \cdot t^0 - p^t \cdot t - p_j^t t_j^0 + \omega t_j^0 + p_j^t t_j^0 s - s t_j^0 \Delta I_j$$

where the first two terms on the right are vector products with the price vector indicated by a dot "$\cdot$" and the remaining scalars associated with the bid (ω, s), for the firm j. ΔI_j is the incremental investment required by firm j to implement the new policy. It should be noted that the prices are the Target shareholders' view about future prices.

$$\Delta R = -p^t \cdot \Delta t + t_j^0(\omega + s p_j^t - p_j^t) + x^0 \cdot \Delta t + t \cdot \Delta x - s t_j^0 \Delta I_j + \delta(h) \tag{2}$$

From Equation (1)

$$\frac{\Delta\phi^t}{\lambda} = -p^t \cdot \Delta t + t_j^0(\omega + s p_j^t - p_j^t) + \Delta t \cdot p^0 + \frac{1}{\lambda} t \cdot E_\theta \Delta x u'(R^0) - s t_j^0 \Delta I_j + \delta(h)$$

where $\lambda = E_\theta u'(R^0)$ and thus finally

$$\frac{\Delta\phi^t}{\lambda} = -\Delta t \cdot \Delta p^t + t_j^0(\omega + s p_j^t - p_j^t) + t \cdot E\Delta x \cdot q - s t_j^0 \Delta I_j + \delta(h) \tag{3}$$

where $q = u'(r^0)/\lambda$ is the implicit price of risk at (a^0, b^0). Equation (3) is in fact a generalization of Equation (4) of Baron, to include the impact of noninfinitesimal changes in costs associated with implementing a new policy following a takeover bid. Baron's differential equation can be deduced by setting $\omega = 0$, $s = 1$, which is of course equivalent to merely changing technologies and letting the required change become infinitesimally small. Alternatively the Baron's equation can be deduced by setting $\omega + s(p_j^t - \Delta I_j) = p_j^t$ and again taking limits. It will be recalled from the end of Section I that in the absence of additional investment requirements that $\omega + sp_j \neq p_j$ for "proper" takeovers. Thus Equation (3) can be interpreted as a capital gain on shares, plus a wealth transfer due to the bid and the risk sharing adjustment.

At this point, if we also make competivity assumptions such as price taking with respect to constant implicit prices, then this is equivalent to setting $q^1 = q^0$ and assuming $\delta(h) \equiv 0$. This implies

$$\Delta p^t = E\Delta x q^0 \quad \text{and hence} \quad \frac{\Delta\phi}{\lambda} = t^0 \cdot E\Delta x q^0 + t_j^0(\omega + sp_j^t - p_j^t) - st_j^0\Delta I_j$$

The additional assumption that $\Delta x_k = 0$ for $k \neq j$ gives

$$\frac{\Delta\phi^t}{\lambda} = t_j^0 \cdot E\Delta x_j q^0 + t_j^0(\omega + sp_j^t - p_j^t) - st_j^0\Delta I_j$$

For the nontakeover case with $\omega = 0$, $s = 1$, this now implies perceived net value maximization

$$\frac{\Delta\phi^t}{\lambda} = t_j^0\Delta(p^t - I_j)$$

But for the takeover case it implies the sum of net value and net wealth transfer maximization

$$\frac{\Delta\phi^t}{\lambda} = t_j^0(\omega + sp_j^t - p_j^0) - st_j^0\Delta I \tag{4}$$

If in addition we ensure agreement about the expected future value by an assumption such as spanning, then these equations imply unanimity among all Target shareholders. Similarly for the Raider we have,

$$\frac{\Delta\phi^r}{\lambda} = -\Delta r \cdot \Delta p^r - (\omega + sp_j^r - p_j^r)(1 - r_j^0) + r \cdot E\Delta x q^0 - (1 - (1 - r^0)s)\Delta I_j$$

where $1 - (1 - r^0)s$ is the Raider's shareholding immediately following the acceptance of the bid. With the same competivity assumptions this simplifies to

$$\frac{\Delta\phi^r}{\lambda} = r_j^0(\omega + sp_j^r - p_j^0) - (\omega + sp_j^r - p_j^r) - (1 - (1 - r^0)s)\Delta I_j$$

or

$$\frac{\Delta\phi^r}{\lambda} = r_j^0[\omega + s(p_j^r - \Delta I_j) - p_j^0] - [\omega + s(p_j^r - \Delta I_j) - (p_j^r - \Delta I_j)] \tag{5}$$

Thus with long or short positions disallowed and common price expectations, proper takeovers will not exist. This is because we must have $\omega + s(p_j - \Delta I_j) > p_j^0$ for the takeover to be attractive to the Target, Equation (4), and $p_j^0 > p_j - \Delta I_j$; otherwise we would not have been at a production equilibrium. This implies $\omega + s(p_j - \Delta I_j) - p_j > r_j^0(\omega + s(p_j - \Delta I_j) - p_j^0)$ or from (5), $\Delta\phi^r < 0$. It is thus clear that the Raider would not proceed with the takeover. This illustrates our observation in the previous section that with post-trading unanimity the Raider would not proceed with the takeover.

If we retain the competivity assumptions but allow the Raider and the Target different price perceptions then takeovers are still possible. In this case, the condition that takeovers are still attractive to both parties is that $\Delta\phi^t$, $\Delta\phi^r$ are both positive. Hence using Equations (4) and (5) we can deduce that a necessary condition for a takeover is that

$$\textstyle\sum_{t_j^0 \in T} t_j^0 p_j^t + [1 - (1 - r^0)s]p_j^r > p_j^0 - \Delta I_j$$

where T is the initial set of Target shareholders. This condition is that the share-weighted perceived final value of the company after takeover is greater than the current value after allowing for post-takeover reorganization costs.

A. The Target's Problem

Because we have made the assumption that no syndicates can form among Target shareholders, for example in the formation of a "white knight in shining armor" counter offer, the Target shareholders has the problem of evaluating the offer and voting accordingly. Even if he voted against what was ultimately a successful bid, the law dealing with minorities will by and large see him no worse off than if he voted for it, although of course his point has been made and he may even have grounds to force an independent valuation. If he voted for an ultimately successful bid then he has just wasted some time, except insofar as influencing future bids. This is not the same situation as the Free Rider problem of Grossman and Hart [7], where they are largely concerned with the division of benefits from eliminating a known inefficient management. As we stated in the introduction, we have ignored problems arising from the separation of ownership and control. In our model the Target shareholders are quite happy with (management's) choice of technology. The new technology offered by the Raider only becomes attractive in conjunction with his cash compensation. The empirical observation of a maintained share price increase after an unsuccessful bid will not occur in this two-period model because there is no opportunity for a revision of expectations regarding future bids. The sources of uncertainty facing him are the future share price and the probability of the bid's success, and as we have seen, in this model the latter plays no real role. However, this is in consequence of our very restrictive assumptions that (1) he is too small to affect the outcome by himself, and (2) that trading is frozen on all assets during the offer period, which is typically not the case unless some significant abuse of the takeover rules is suspected. In practice, this intermediate market gives the Targert shareholders some valuable signals. They will use this market in two ways simultaneously. First, they will insure themselves as much as possible about the remaining uncertainty by adopting a hedged portfolio, trading some of the Target firm's share as well as unaffected firms with similar cash returns. Second, they will observe the intermediate price in order to form some kind of Rational Expectations (RE) about the future price. Thus, traders in this market will include arbitrageurs who may never have been significant shareholders nor who intend to be, and after announcement of the offer, large dealers and brokers stand ready to purchase shares on the market discounted from the bid price by their expectations regarding a successful outcome. The uncertainty surrounding the final equilibrium price of the firm post-takeover arises from two sources. First, there is the difficulty associated with the pricing of any technology in an incomplete market. Second, there is the moral hazard problem surrounding the Raider's choice of technology. Although, for example the U.K. "City Code on Takeovers and Mergers" does require a statement of intention on the part of the Raider with respect to the future continuance of the business, the surrender of voting rights by the Target shareholders effectively leaves the Raider free to choose the technology. The associated moral hazard problem can in part be offset by a conditional share exchange in the form of a convertible. Here the Raider is at least offering a minimum "guaranteed" return irrespective of his choice of technology, together with the possibility of further participation by the target shareholders in those returns should the technology subsequently prove attractive. The relative popularity[6] of convertible issues in the financing of acquisitions is well evidenced in survey work, e.g., Shad [16].

B. The Raider's Problem

The Raider's problem is to choose ω high enough to ensure a majority acceptance and low enough to make a profit. This problem is essentially a mixture of an estimation problem concerning the target shareholders and a bargaining problem. It is the latter problem that takes it outside the scope of this paper. There does however seem to be evidence that the outcome is weighted in favor of the target shareholders, with no excess gain to the Raider, Mandelker [13] and Firth [6]. There are many plausible explanations of this, but again a thorough analysis would be inappropriate here. The most obvious explanation would be that either the Raider adopts a managerial objective, seldom are Raider firm shareholders consulted on an offer, or in our case of a single person Raider, he has to pitch ω to capture the marginal (100%) Target shareholder. This leaves the average Target shareholder with a windfall gain.

In conclusion, then, we have seen that takeover bid equilibria are strictly contained in the set of productive-exchange equilibria. We have seen that this can arise both because of lack of unanimity about value maximization or because of different models of price formation. Following in the spirit of work such as Hart [10], it seems likely that, once we move away from these small economies to larger ones, the latter becomes the dominant reason. The price formation problem is not just the individual's inability to price a new technology, but just as much the uncertainty associated with which technology to price, a problem caused by the Raider's moral hazard problem after gaining control.

[6] It is perhaps worth noting though in a takeover bid that conditional sharing rules such as convertibles might well prove attractive for reasons different from those associated with moral hazard. For instance, in the case where the Raider is more optimistic about the returns from the new technology than the Target shareholders, a conditional sharing rule with a guaranteed minimum return might prove sufficient inducement to the Target shareholders to ensure the success of the bid.

Appendix

The Redundancy of the Takeover Bid Mechanism Where All Shareholders Exhibit Linear Risk Tolerance with the Same Cautiousness

For such a group of shareholders operating in a closed economy, where the supply of the riskless asset is fixed, any technological change is unanimous. Moreover, if the market is initially in equilibrium, then merely changing the technology does not disturb the equilibrium.

Proof:

$$\text{If } U_i(x) = \ln(c_i + x) \quad \forall_i$$

or

$$\text{if } U_i(x) = \text{sign}(p) \cdot (c_i + x)^p \quad \forall_i$$

then the equilibrium conditions or equation of the contract curve is given by $\frac{b_i}{a_i + c_i} = k$ where k is a constant which is easily seen to be independent of i and the technology α. Hence for log utilities $\phi_i = E \ln(c_i + a_i + b_i x(\alpha)) = \ln(c_i + a_i) + E \ln(1 + kx(\alpha))$ and for power utilities $\phi_i = \text{sign}(p)(c_i + a_i)^p E(1 + kx(\alpha))^p$. In both cases the sign of changes in ϕ is independent of the technology. For utilities of the form $U_i(x) = -e^{-\alpha_i x}$, the equilibrium conditions give $\alpha_i b_i = k \ \forall_i$. Thus $\phi_i = -e^{-\alpha_i a_i} E(e^{-kx(\alpha)})$ and again the sign of changes in ϕ is independent of i. In terms of the properties of the LRT class mentioned in Section II then a proof of the absence of takeovers is immediate. Thus for this class, constrained Pareto optimal allocations are also Pareto optimal and in addition the set of these allocations is independent of the choice of technology. Hence, if the market is initially in equilibrium, and hence Pareto optimal, then any change of technology (α) remains Pareto optimal. Hence, no Raider can compensate any target shareholder without being worse off himself. Thus given perfect foresight, the Pareto optimality of unanimously supported (opposed) technological change implies a redundancy of the takeover bid mechanism discussed in the paper.

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