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## Capital Structure Equilibrium under Market Imperfections and Incompleteness

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### ABSTRACT

This paper generalizes Miller's supply-side equilibrium argument to other forms of capital market imperfections and incompleteness. If corporations possess a comparative advantage in dealing with these imperfections, they have an incentive to act as financial intermediaries. Corporations' attempts to profit from these intermediation activities dictate an optimal capital structure for the corporate sector as a whole, but in equilibrium the capital structure of any single firm is a matter of indifference. In addition, the positive role that corporate finance plays in completing the market restores standard perfect market results on asset pricing and the associated portfolio separation properties.

THE MODIGLIANI-MILLER (MM) [21] capital structure irrelevance result has been derived, explicitly or implicitly, in the context of a complete and perfect capital market. Either an Arrow-Debreu world is specifically assumed, as in Hirshleifer [14], or some variation is invoked that, for practical purposes, amounts to the same thing.<sup>1</sup> The crux of MM's own analysis, for example, is the availability to investors of costless "homemade" leverage. In subsequent generalizations, a similar role is played by assumptions of costless financial intermediation (Stiglitz [28]) or "equal access" (Fama [12]). The key element in each case has been investors' ability to costlessly duplicate any return patterns that might be created by corporate financing decisions.

Recently Miller [19] has shown that a more limited irrelevance result can hold even in the face of a specific market imperfection—taxes. In Miller's model, corporations have a tax-induced advantage in borrowing relative to low tax bracket investors. In equilibrium, this advantage is driven to zero at the margin (hence, the irrelevance result at the firm level), but the corporate sector as a whole acts much like a financial intermediary. Specifically, it supplies leverage

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<sup>1</sup> A particularly strong example of this view may be found in Milne [20]:

It has been said that for the Modigliani-Miller finance theorems to be true, there must exist perfect markets in assets. Once one has grasped the essential point that a perfect market must involve explicit (or transparently derivable) markets for a given set of returns, these theorems follow easily because the asset model is no more than a thinly-disguised Arrow-Debreu Model.

services to those investors who would perform such services less advantageously on their own accounts.

The purpose of this paper is to show that Miller's irrelevance result can be generalized to any kind of capital market imperfection or incompleteness. As long as corporations possess a comparative advantage in dealing with these imperfections, they will have an incentive to act as financial intermediaries. In the limiting case where perfectly competing corporations can costlessly intermediate, standard perfect market results on asset pricing will be restored.

The crucial effect of market imperfections, be they transaction costs, agency costs, or missing markets,<sup>2</sup> is that marginal rates of substitution are not at all driven to equality. The representation of this feature that we employ is a divergence between borrowing and lending rates. Corporations in our model tailor their capital structures to eliminate or at least reduce this divergence.

We are by no means the first to suggest that one role for corporate finance lies in completing the market. A number of authors, including Williams [31], Durand [10], Litzenberger and Sosin [18], Sosin [27], and Litzenberger [17] have previously alluded to this idea. However, the equilibrium implications of this role for corporate finance have not been fully pursued. We show that, if corporations can borrow and lend advantageously relative to individual investors, an equilibrium results in which corporate capital structure is determinate in the aggregate but not at the individual firm level.

We present a portfolio selection model in Section I and analyze investors' preferences for firms' capital structures and the resulting capital structure equilibrium. In Section II we examine the special case in which firms costlessly intermediate and show that their capital structure adjustments restore the standard asset pricing relationship. In Section III we summarize our arguments and discuss some possible generalizations.

## **I. Equilibrium with Divergent Borrowing and Lending Rates**

### *A. The Basic Framework*

Consider a single-period economy with  $J$  firms. Each investor,  $i$ , has an initial endowment of cash,  $Y_i^1$ , as well as initial shareholdings,  $\bar{\alpha}_j^i$ , ( $j = 1, \dots, J$ ), in the  $J$  firms. Investors can either consume or save their endowments, and savings take the form of either bond holdings,  $l^i$ , or holdings of fractional shares,  $\alpha_j^i$ , in the values,  $S_j$ , of the firms' shares.

Both firms and individuals can issue riskless claims in the bond market. However, transaction costs cause borrowing and lending rates to diverge. For simplicity, the unit transaction cost,  $\lambda$ , faced by individuals is assumed constant across both individuals and transaction sizes. The gross return for lenders ( $l^i > 0$ ) in our model is  $R(1 - \lambda)l^i$ , while the amount borrowers ( $l^i < 0$ ) must repay is

<sup>2</sup> Ultimately, the distinction between market incompleteness and such imperfections as transaction costs or agency costs is somewhat fuzzy. Presumably transaction costs and agency costs are among the major reasons why some markets do not exist.

$R(1 + \lambda)l^i$ , where  $R$  is the gross interest rate that would prevail in a frictionless market.

Investors may also deal in the market for corporate stock, where purchase of an  $\alpha_j^i$  share of firm  $j$ 's stock entitles investor  $i$  to an  $\alpha_j^i$  share of end-of-period profits,  $\pi_j$ , given by

$$\tilde{\pi}_j = \tilde{\theta}_j \bar{X}_j - RD_j[(1 + \lambda^*)w_j + (1 - \lambda^*)(1 - w_j)]. \quad (1)$$

$\tilde{\theta}_j$  is a random variable, and letting  $E(\tilde{\theta}_j) = 1$ ,  $\bar{X}_j$  is the firm's expected operating cash flow.  $D_j$  is the firm's debt and all firms face the same transaction cost,  $\lambda^*$ , in the bond market. The variable  $w_j$  is equal to one if the firm is a borrower ( $D_j > 0$ ) and zero if the firm is a lender ( $D_j < 0$ ).

### B. Portfolio Equilibrium

We first analyze a portfolio equilibrium for investors, consisting of market prices,  $S_j$  and  $R$ , and portfolio holdings,  $l^i$  and  $\alpha_j^i$  (for all  $j$ ), such that both bond and share markets clear and each investor's expected utility is maximized. Equilibrium is predicated upon an exogenous value of  $D_j$  for each firm. Changes in the portfolio equilibrium resulting from a change in any firm's financing decision will be examined in Section I C.

Investors maximize their expected utility of current and future consumption, which amounts to choosing optimal values of  $l^i$  and  $\alpha_j^i$  (for all  $j$ ). Investor  $i$ 's problem is thus:

$$\max_{l^i, \alpha_j^i} E(\tilde{u}^i) = \int u^i[C_1^i, C_2^i(\tilde{\theta}_1, \dots, \tilde{\theta}_J)] dF(\tilde{\theta}_1, \dots, \tilde{\theta}_J), \quad (2)$$

where  $F(\tilde{\theta}_1, \dots, \tilde{\theta}_J)$  is the joint cumulative distribution of the  $\tilde{\theta}_j$ .<sup>3</sup> Present consumption,  $C_1^i$ , is given by

$$C_1^i = Y_1^i - l^i - \sum_j (\alpha_j^i - \bar{\alpha}_j^i) S_j \quad (3)$$

and future consumption,  $\tilde{C}_2^i$ , by

$$\begin{aligned} \tilde{C}_2^i = Rl^i[(1 + \lambda)v^i + (1 - \lambda)(1 - v^i)] \\ + \sum_j \alpha_j^i (\tilde{\theta}_j \bar{X}_j - RD_j[(1 + \lambda^*)w_j + (1 - \lambda^*)(1 - w_j)]), \end{aligned} \quad (4)$$

where  $v^i = 1$  if  $i$  is a net borrower,  $v^i = 0$  if  $i$  is a net lender, and  $0 < v^i < 1$  if  $i$  neither borrows nor lends.

Assuming that all investors act as price-takers, first-order conditions for optimal portfolios are

$$\frac{E(u_1^i)}{E(u_2^i)} = R((1 + \lambda)v^i + (1 - \lambda)(1 - v^i)), \quad (5)$$

<sup>3</sup> To simplify the notation, investors are assumed to agree on this distribution. This assumption is not critical to the results.

and (hereafter we drop the tildes for convenience)

$$\bar{X}_j \left( \frac{\int \frac{\partial u^i}{\partial C_2^i} \theta_j dF}{E(u_1^i)} \right) - \frac{E(u_2^i)}{E(u_1^i)} RD_j [(1 + \lambda^*)w_j + (1 - \lambda^*)(1 - w_j)] = S_j, \quad (6)$$

where

$$\frac{E(u_1^i)}{E(u_2^i)} = \int \frac{\partial u^i}{\partial C_1^i} dF \bigg/ \int \frac{\partial u^i}{\partial C_2^i} dF$$

is investor  $i$ 's marginal rate of substitution between current consumption and certain future consumption.

Unlike the familiar perfect market case, marginal rates of substitution are not driven to equality in (5) for all investors. Rather, these rates depend on the investors' propensities to borrow or lend. Condition (6) represents the investor's subjective marginal valuation of firm  $j$ 's stock. Because trading in shares is competitive and unrestricted, these valuations are in agreement for all investors.<sup>4</sup> Nevertheless, because of divergent borrowing and lending rates, investors may not agree about the two separate components on the left-hand side of (6).<sup>5</sup> In particular, all investors are not equally risk averse at the margin.

It should be noted that investors achieve their optimal leverage positions in a portfolio equilibrium both directly, through personal borrowing and lending, and indirectly, through shareholdings in positively or negatively-levered firms. Thus corporate and personal leverage are substitutable, albeit imperfectly so, if  $\lambda$  and  $\lambda^*$  are unequal. If  $\lambda$  were equal to  $\lambda^*$  and all firms had zero net leverage (i.e.,  $D_j = 0$  for all  $j$ ), investors would construct their preferred net leverage position purely on their own accounts. For future reference, denote the optimal portfolio in this special case by  $(l^*, \alpha_j^*)$ . In general, however, firms will have some nonzero amount of leverage which investors will seek to offset or augment on their own accounts. One case of special interest is that in which firms have a cost advantage in borrowing and lending (i.e.,  $\lambda^* < \lambda$ ). Such an advantage might stem from economies of scale in monitoring and record-keeping, for example. Under these circumstances, investors' preferences for corporate leverage will be based both on their personal leverage preferences and on firms' cost advantage.

<sup>4</sup> In general, of course, share trading could also be subject to transaction costs and this could cause divergences in investors' share valuations in much the same fashion that short-selling restrictions cause such divergences in Litzenberger and Sosin [18]. We have chosen to focus on the bond market. This necessitates, however, the implicit assumption that investors cannot create a risk-free security through trading in stock, for in that case the bond market would be dominated and would disappear. See also footnote 5 in this connection.

<sup>5</sup>  $E(u_1^i)/E(u_2^i)$  can be driven into equality if short sales are unrestricted and if there exists an unlevered risk-free firm. Similar possibilities exist through combined purchases and short sales of the stock of two firms identical in all respects except their level of debt. These particular forms of short-selling are ruled out by assumption. This restriction seems justified in that short-selling has many characteristics in common with borrowing. The same forces that make borrowing costly for investors would also be expected to make short-selling costly. If all short-selling were ruled out, an inequality would replace the equality in expression (6), but the analysis would continue to hold for investors with positive shareholdings ( $\alpha_j^i > 0$ ).

### C. Investor Preferences for Firms' Capital Structures

We turn now to the question of what corporate capital structures investors would perceive as optimal. Starting from an initial portfolio equilibrium, we want to analyze how changes in a given firm's capital structure affect investors' welfare. The first step is to differentiate a representative investor's expected utility with respect to a change in firm  $i$ 's debt and to evaluate this derivative at a portfolio equilibrium by constraining  $\alpha_j^i = \bar{\alpha}_j^i$ .<sup>6</sup> This results in<sup>7</sup>

$$\frac{d \int u^i}{dD_j} dF = E(u_1^i) \left( 1 - \frac{E(u_2^i)}{E(u_1^i)} R[(1 + \lambda^*)w_j + (1 - \lambda^*)(1 - w_j)] \right) \bar{\alpha}_j^i, \quad (7)$$

or, substituting from (5),

$$\frac{d \int u^i}{dD_j} dF = E(u_1^i) \left( 1 - \frac{(1 + \lambda^*)w_j + (1 - \lambda^*)(1 - w_j)}{(1 + \lambda)v^i + (1 - \lambda)(1 - v^i)} \right) \bar{\alpha}_j^i. \quad (8)$$

Since (8) is positive for  $v^i$ ,  $w_j = 1$ , investors who borrow on their own accounts prefer that firms in which they hold shares do more borrowing. Likewise, investors who neither borrow nor lend, and whose marginal rates of substitution lie between  $R(1 + \lambda^*)$  and  $R(1 + \lambda)$ , prefer that a borrowing firm do more borrowing. For investors who are lenders or whose marginal rates of substitution lie between  $R(1 - \lambda)$  and  $R(1 - \lambda)^*$ , the analysis is reversed and more lending by firms is preferred.

How will firms respond to these conflicting desires on the part of final shareholders? If we shift our attention back to the initial shareholders, the assumptions of this model satisfy DeAngelo's [8] sufficient conditions for unanimity, and thus all initial shareholders will support value-maximizing capital

<sup>6</sup> Constraining initial and final shareholders to be the same will be recognized from the unanimity literature (see Baron [3], DeAngelo [8]) as the *ex post* (instead of the *ex ante*) methodology. We will argue below that our model is fully consistent with an *ex ante* analysis, under which initial shareholders have a unanimous preference for value-maximizing capital structures. The difficulty with employing the *ex ante* analysis directly in this context is that it provides little insight into what these value-maximizing capital structures look like.

<sup>7</sup>

$$\begin{aligned} \frac{d \left( \int u^i dF \right)}{dD_j} &= [E(u_2^i)R((1 + \lambda)v^i + (1 - \lambda)(1 - v^i)) - E(u_1^i)] \frac{dI^i}{dD_j} \\ &+ \sum_j \left[ X_j \int \frac{\partial u^i \theta_j dF}{\partial C_2^i} - E(u_2^i)RD_j((1 + \lambda^*)w_j + (1 - \lambda^*)(1 - w_j)) - E(u_1^i)S_j \right] \frac{d\alpha_j^i}{dD_j} \\ &- \left[ E(u_2^i)R((1 + \lambda^*)w_j + (1 - \lambda^*)(1 - w_j)) + E(u_1^i) \frac{dS_j}{dD_j} \right] \alpha_j^i + E(u_1^i) \left( \frac{dS_j}{dD_j} + 1 \right) \bar{\alpha}_j^i. \end{aligned}$$

Substituting (5) and (6) in the above and setting  $\alpha_j^i = \bar{\alpha}_j^i$  yields (7).

structures.<sup>8</sup> Which capital structures will maximize value? Investors with specific desires for borrowing or lending by firms will compete with one another to bid up the values of those firms that satisfy these desires. Value-maximizing firms will thus adjust their capital structures to serve the needs of different investor clienteles. Some firms will increase their borrowing, while others will increase their lending. If it is feasible, these capital structure adjustments will continue until all individual borrowing and lending has been displaced by corporate borrowing and lending.

**PROPOSITION 1.** *If stochastic constant returns to scale technology is sufficiently widespread, then under the conditions of this section, value-maximizing firms will adjust their capital structures until all borrowing and lending is done at the corporate level. At this equilibrium, individuals' marginal rates of substitution will satisfy  $R(1 - \lambda^*) \leq E(u_1^i)/E(u_2^i) \leq R(1 + \lambda^*)$  for all  $i$ .*

*Proof:* Suppose initially that investors face the transaction cost  $\lambda^*$  and that all firms have zero net leverage. Investors will choose optimal portfolios,  $(l^{i*}, \alpha_j^{i*})$ , which reflect their individual rates of time preference and attitudes toward risk. Let this portfolio equilibrium stand as an unconstrained optimum and note from (5) that, at the margin,  $R(1 - \lambda^*) \leq E(u_1^i)/E(u_2^i) \leq R(1 + \lambda^*)$  for all  $i$ .

Now suppose that the transaction cost faced by investors increases to  $\lambda$  while that for firms remains at  $\lambda^*$ . This causes investors to reshuffle their portfolios. Suppose further that the production technology of some firm  $j$  exhibits stochastic constant returns to scale.

Consider some investor  $i$ , whose optimal portfolio has positive net leverage. From (8),  $i$  would prefer that firm  $j$  adopt a positive leverage position and would be willing to pay more for firm  $j$ 's shares if it did so. Given its technology, firm  $j$  thus has an incentive to split off a piece of itself that invests  $\alpha_j^{i*}$  as much as the original firm and produces  $\alpha_j^{i*}\theta_j\bar{X}_j$  in the future.

This piece would be solely owned by investor  $i$  and it would issue an amount of debt,  $D_j^{i*} = -l^{i*}/\alpha_j^{i*}$ . Investor  $i$  could thus achieve a total portfolio return equal to that in the unconstrained case without doing any borrowing on his own account. At this point,  $E(u_1^i)/E(u_2^i) = R(1 + \lambda^*)$  and thus  $i$  does not desire any further capital structure changes by firms in which he owns shares.

The corporate sector as a whole would have an incentive to make similar capital structure adjustments for all investors until the initial equilibrium is restored. It is necessary only that such adjustments be feasible. That will be the case if constant returns to scale technology is sufficiently widespread so that the splitting up of firms allows investors to put together any desired combinations

<sup>8</sup> The reader may object that if  $\lambda$  is a transaction cost, DeAngelo's [8] zero exchange cost condition is violated. The thrust of all of DeAngelo's conditions, however, is to ensure that firms' decisions affect individuals' consumption opportunities only through their personal wealth. As long as firms and individuals act as price-takers, and as long as there are many firms, none of which possess any market power in issuing securities, a capital structure change by any one of them does not destroy any previously available return patterns. Since  $\lambda^*$  is the same for all firms in our model, they behave competitively and their decisions affect investors' consumption opportunities only through personal wealth.

of risky asset holdings and net leverage solely through shareholdings.<sup>9</sup> This completes the proof.

Capital structure equilibrium in this environment is very much like that described by Miller [19]. Firms compete with one another to serve investors' net portfolio leverage desires until all gains from further capital structure changes have been exhausted. All investors who formerly borrowed and lent on their own account wind up at a point of indifference toward further capital structure changes by firms. This implies that corporate borrowing and lending is determinate in the aggregate but not at the level of the individual firm.<sup>10</sup> Capital structure irrelevance at the firm level is somewhat weaker than in Miller's model, however. Equilibrium is maintained in the face of capital structure reshuffling by firms only so long as investors can continue to put together their optimal holdings of risky assets plus their optimal degree of net portfolio leverage solely through share purchases. Since time preferences and attitudes toward risk differ across investors, it is necessary that the financial intermediation services provided by firms be "tailor-made" to meet the requirements of each individual.

Similar results would emerge for the special case where  $\lambda^* = 0$ , in which return streams can be costlessly transformed by corporations. Again, corporate borrowing and lending would dominate personal borrowing and lending, and there would be an optimal amount of corporate debt. In this case, however, capital structure adjustments would be sufficient to drive marginal rates of substitution exactly into equality for all investors. The capital market would be fully spanned even in the face of "unequal access" on the part of individual investors.<sup>11</sup>

The assumption of constant returns to scale used to obtain these results may, of course, be quite restrictive. In practice, the corporate sector may not exhibit this technology in sufficient degree to allow all of the necessary capital structure adjustments. This possibility could in turn create a role for a separate financial intermediary sector. Even if pure financial intermediaries have no advantage over other corporations in borrowing and lending, they allow capital structure adjustments to be separated from the scale of real investment.

<sup>9</sup> The sufficient prevalence of constant returns to scale technology cannot be specified without knowing the range of investor preferences. If most investors desire little net leverage, it may be enough to have one such firm. Since the scale of any one risk class is determinate in general equilibrium, however, a single firm may not be able to issue enough debt to satisfy all investors' demands for leverage. At the other extreme, it is clearly more than sufficient to require that all firms exhibit constant returns to scale.

<sup>10</sup> These results may be contrasted with the case in which  $\lambda = \lambda^*$ , so that corporate and personal leverage are perfect substitutes. There would still be an optimal aggregate level of combined borrowing by firms and individuals, because anything more than this could not be undone without incurring costs. As long as this aggregate level of borrowing was maintained, however, it would not matter whether it was done by firms or individuals. Corporate capital structure would be unimportant not only at the firm level but also at the level of corporate sector, since any increase or reduction in borrowing by firms as a whole could be offset by individual investors. The traditional MM theorem can be viewed as the special case in which  $\lambda = \lambda^* = 0$ .

<sup>11</sup> This conclusion may be contrasted with Fama's [12] analysis under the "equal access" assumption. Corporations here play somewhat the same role that costlessly created financial intermediaries do in Stiglitz [28].

## II. Market Imperfections and Asset Pricing

The asset pricing literature has devoted enormous attention to the conditions under which portfolio separation occurs, giving rise to the standard linear asset pricing models.<sup>12</sup> The presence of market imperfections is generally felt to at least alter the pricing relationship, if not destroy portfolio separation altogether. This entire body of literature, however, takes security supplies as given and thus ignores the capacity of firms to adjust their capital structures in response to imperfections at the investor level.

Consider, for example, divergent borrowing and lending rates in the context of the mean-variance model. Brennan [6] has studied equilibrium security pricing for the case where investors have riskless borrowing and lending opportunities but the borrowing rate exceeds the lending rate. Vasicek [30] and Black [4] have analyzed the case in which investors have a riskless lending opportunity but no riskless borrowing is allowed. In each of these papers it is demonstrated that the equilibrium expected return on security  $j$  has the same linear structure as in the standard CAPM but that the intercept and market price of risk are affected by the borrowing and lending restrictions. The capital market line in all three papers becomes piecewise linear with efficient portfolios falling along different segments for borrowing and lending investors.<sup>13</sup>

If firms do not face these restrictions and transaction costs, however, their intermediation activities can restore the standard CAPM pricing relationship. Using the notation of Section I, let  $\lambda^* = 0$  and  $\lambda > 0$  and let investors maximize the mean-variance utility function

$$u^i = u^i(C_1^i, \bar{C}_2^i, (\sigma^i)^2), \quad (9)$$

where  $C_1^i$  is given by (3),  $\bar{C}_2^i$  is the expected value of  $\tilde{C}_2^i$  in (4), assuming that  $E(\hat{\theta}_j) = 1$  for all  $j$ , and  $(\sigma^i)^2$  is the variance of  $\tilde{C}_2^i$  in (4). Maximizing (9) with respect to  $l^i$  and  $\alpha_j^i$  gives the first-order conditions

$$\frac{\partial u^i}{\partial C_1^i} \bigg/ \frac{\partial u^i}{\partial \bar{C}_2^i} = R[(1 + \lambda)v^i + (1 - \lambda)(1 - v^i)], \quad (10)$$

and

$$\begin{aligned} \bar{X}_j - RD_j[(1 + \lambda^*)w_j + (1 - \lambda^*)(1 - w_j)] - a^i \sum_k \alpha_k^i \bar{X}_j \bar{X}_k \text{cov}(\theta_j, \theta_k) \\ = S_j R[(1 + \lambda)v^i + (1 - \lambda)(1 - v^i)], \end{aligned} \quad (11)$$

where,

$$a^i = -2 \frac{\partial u^i}{\partial (\sigma^i)^2} \bigg/ \frac{\partial u^i}{\partial \bar{C}_2^i}.$$

To express (11) in terms of expected rates of return on the firm's total assets ("unlevered" expected returns), let  $E(R_j^u) = \bar{X}_j/V_j$ , where  $V_j = D_j + S_j$ . Summing

<sup>12</sup> See, for example, Cass and Stiglitz [7] and Ross [24].

<sup>13</sup> In the models of Vasicek and Black, short sales of the zero beta portfolio take the place of borrowing.

(11) over  $i$  then yields the capital market line expression

$$E(R_j^u) = \eta \operatorname{cov}(R_j^u, R_m^u) + \frac{RD_j}{V_j} [(1 + \lambda^*)w_j + (1 - \lambda^*)(1 - w_j)] \\ + R \left( 1 - \frac{D_j}{V_j} \right) \left( \frac{1}{\sum_i \frac{1}{a^i}} \right) \sum_i \frac{1}{a^i} [(1 + \lambda)v^i + (1 - \lambda)(1 - v^i)], \quad (12)$$

where

$$\eta = \sum_k V_k / \sum_i \frac{1}{a^i},$$

and  $R_m^u$  is the unlevered return on the entire market. Equation (12) is analogous to Brennan's [6] capital market line for divergent borrowing and lending rates, generalized to include the effects of corporate leverage. If investors could borrow and lend at the risk-free rate  $R$  (that is if  $\lambda = 0$ ) and if all firms were unlevered ( $D_j = 0$  for all  $j$ ), (12) would reduce the standard CAPM capital market line expression,

$$E(R_j^u) = \eta \operatorname{cov}(R_j^u, R_m^u) + R. \quad (13)$$

Alternatively, with  $\lambda > 0$  but  $\lambda^* = 0$ , we have seen in Section I that if stochastic constant returns are sufficiently widespread, capital structure adjustments by firms are capable of creating the same portfolio opportunity set that investors face when they can borrow and lend at  $R$  themselves. At a complete capital structure equilibrium, therefore, the marginal rates of substitution expressed in (10) are equal to  $R$  for all investors, and (12) becomes (13). Firms' intermediation activities thus restore the standard capital market line relationship, even if investors face divergent borrowing and lending rates on their own accounts.<sup>14</sup>

Furthermore, this analysis need not be confined to the mean-variance context. As we have seen, imperfections in the borrowing and lending market have been examined primarily for the case of the mean-variance model and it is thus natural to examine supply adjustments in this framework. The same analysis could be extended, however, to the more general utility function environments studied by Cass and Stiglitz [7] or the more general return distribution environments studied by Ross [24].

Although we relied on costless supply adjustments in restoring the capital market line, the primary point that emerges from the preceding discussion is that asset pricing models that ignore supply effects may be misspecified even if the firms respond at some cost. Whereas the literature in corporate finance has tended to focus on the extent to which investors' portfolio opportunities can undo corporate financing decisions, the investment literature has tended to focus on the pricing of fixed supplies of securities. In either case, the positive role that

<sup>14</sup> A similar point is made in a different context by Black and Scholes [5] when they argue that supply responses by firms will tend to eliminate any dividend yield effect from the ordinary CAPM equation, even in the face of personal tax considerations.

corporate financing decisions can play in completing the market, and hence restoring perfect market-type pricing relationships, has been overlooked.

### III. Conclusion

This paper has shown that Miller's supply side equilibrium argument can be generalized to other forms of capital market imperfections and incompleteness. What is needed is for firms to have a cost advantage over investors in dealing with such problems. Firms' attempts to profit from this advantage will dictate an optimal capital structure for the corporate sector, but as in the perfect markets case, competition drives out profits, and in equilibrium the capital structure of any one firm will be a matter of indifference.

An important direction for extending the paper would be to see if similar results can be obtained in the context of a richer capital market model. Without the benefit of rigorous analysis, we can nonetheless state some conjectures. First, if we do not restrict corporate and individual borrowing contracts to be free of default risk, risky debt can still play a role in completing the market up to a point. The capacity of any given corporation to issue debt with the desired return attributes, however, depends on the characteristics of its operating cash flows. As a firm becomes more levered, its debt will no longer have its desirable near-riskless property, because its return stream approaches that available to unlevered equity. However, this problem may be alleviated by issuing differentiated or subordinated debt. Mergers may also enhance the capacity of corporate debt to complete the market. If the process of merging were costless, this activity alone would not affect equilibrium market valuation, but would simply arise as part of the competition among firms to meet investors' desired return patterns. Nonetheless, there would be an optimal economy-wide configuration of firms with various sizes.

Second, although we have focused only on debt and equity securities, a truly complete market may necessitate the existence of other forms of securities to be supplied by firms, such as hybrid securities with option characteristics (e.g., warrants, callable debt, etc.). In a multiperiod environment, a complete market is presumed to have securities that span the available time-state space. When investors' desires include return patterns across time, then securities with multiple periods become relevant. Thus, corporate debt maturity structure, at least in an economy-wide sense, may evolve naturally as a means of completing the market, although the limited MM theorem may still obtain at the individual firm level.

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