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Bank Forward Lending: A Note

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IN DESHMUKH, GREENBAUM, AND KANATAS [1] [DGK hereafter], the authors explain the effects on banks' lending behavior of uncertainty regarding deposits and customer loan demand. Since all lending is assumed to be done via rudimentary fixed-rate loan commitments, the analysis focuses on how the probability distribution of deposits and customer takedowns affects the bank's choice of an optimal volume of loan commitments. The authors find that increased uncertainty, in the sense of a mean-preserving spread (see Rothschild and Stiglitz [5]), regarding deposits or customer takedowns will result in fewer loan commitments as well as reduced expected lending. A stochastic decrease (see Hadar and and Russell [4]), in deposits, which is equivalent to a stochastic increase in the bank's borrowing cost, similarly leads to reduced commitment sales and expected bank lending. A stochastic increase in customer takedowns is found to have an ambiguous effect on the bank's commitment decision.

The force of these results is limited because all bank lending is assumed to be done via loan commitments. Although most bank lending, particularly to businesses, is indeed done via commitments (see Hanweck [4]), the possibility of spot-market lending can be expected to influence the bank's commitment decision. For example, if the bank initially chooses its commitment volume, and the customer subsequently determines the fraction of commitments to exercise, the bank commonly retains the option of purchasing and/or selling additional loans in the spot market after the customer's takedown is realized. Moreover, the bank's opportunity to transact in the spot market after realizing the customer's takedown can be expected to influence its initial choice of commitments.

The purpose of this note is to examine whether the DGK results continue to hold in a setting where banks have spot-market lending opportunities. The model used here incorporates two significant modifications. Whereas both deposits, D, and the takedown fraction, θ, were random variables in DGK, we now have only one random variable, the spot interest rate s, realized after the bank chooses its commitment volume. The takedown fraction is consequently expressed as a deterministic function of the spot interest rate. Second, a more general bank borrowing cost function, $MC(L, s)$, depending on the bank's level of lending, L, and the spot interest rate obviates the need for a deposits variable. With our slightly altered formulation, we examine three cases:

(i) The original DGK case without spot-market lending;

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(ii) The case where banks are allowed to purchase or sell loans on the spot market at the realized spot interest rate; and
(iii) The case where the bank can only purchase (sell), but cannot sell (purchase) loans at the realized spot interest rate.

We find that in Case (iii) the bank's optimal commitment volume will be lower than in either Case (i) or Case (ii). The optimal commitment volume in case (ii) can be smaller or greater than in Case (i) depending on properties of the bank's borrowing cost function. We also find that in all cases, DGK's mean-preserving spread results continue to hold provided $\theta(s)$ is "almost" linear and we explain why the result fails otherwise. We also explain why stochastic increases in s have ambiguous effects on the commitment decision.

The following is organized in three sections: Section I sketches the model, Section II describes results, and Section III concludes.

I. The Model

A loan commitment is defined as an option to borrow a unit of funds one period hence at a fixed interest rate, r. The loan, if taken, will have a one-period term-to-maturity and no default risk. Let Q be the quantity of loan commitments, and $p(Q)$ be the commitment fee per unit charged by the bank. We assume

$$p'(Q) \leq 0 \qquad [2p'(Q) + Qp''(Q)] \leq 0 \qquad Q \geq 0 \tag{1}$$

The fraction of commitments exercised, $\theta(\cdot)$, is assumed to be a deterministic function of the spot interest rate wherein both the spot interest rate and the amount of loans are realized following the sale of the commitments.[1] Then

$$L = \theta(s)Q \qquad \theta'(s) \geq 0 \qquad \theta \in [0, 1] \tag{2}$$

The spot interest rate, $\tilde{s}$, is a random variable with a cumulative probability distribution $F(\cdot)$. Therefore $L(\tilde{s})$ is also a random variable.

The bank's marginal cost of borrowing, $MC(L, s)$, is assumed to be continuous nondecreasing and convex in both arguments and such that there exists $\bar{L} > 0$ for almost all s and

$$MC(L, s) \leq s \quad \text{whenever} \quad L \leq \bar{L} \tag{3}$$

Condition (3) rules out the pathological corner solution wherein all values of L result in a marginal borrowing cost in excess of the spot interest rate. Notice that the cost function used in DGK is a special linear case of $MC(L, s)$.[2]

[1] Unlike the more conventional option pricing models, θ is continuously variable over [0, 1]. For a detailed explanation of this point, see Greenbaum and Venezia [2].

[2] In DGK, deposits are available at a constant subsidized interest rate, c_1, and were we to retain the distinction between deposits and purchased money it would be natural to assume that

$$D = D(s) \qquad D'(s) \leq 0 \quad \text{and} \quad D'' \leq 0$$

DGK assume that funds in excess of deposits (purchased money) are available at a rising marginal cost, linear in the quantity of purchased money borrowed, $(L - D)$. Thus

$$MC(L, s) = \begin{cases} c_1 & \text{if } L \leq D(s) \\ c_1 + c_2[L - D(s)] & \text{if } L > D(s) \end{cases}$$

If the bank operates exclusively in the forward market, it chooses Q to maximize expected profits given by

$$\pi_N(Q) = E\left[p(Q)Q + r\theta(\tilde{s})Q - \int_0^L MC(t, \tilde{s})\, dt\right] \tag{4}$$

where N denotes this special case and $E[\cdot]$ is the expectation operator.

When the bank is allowed to sell or purchase loans in the spot market as well as sell commitments, $A(s) \gtreqless 0$, the additional loans purchased (sold), is defined by[3]

$$MC(L + A, \tilde{s}) = \tilde{s} \tag{5}$$

Notice that if s exceeds the bank's marginal borrowing cost, then it can profit by purchasing additional loans and if s is less than $MC(L, s)$, the bank can profit by selling loans. For this case, the bank chooses Q to maximize expected profits, $\pi_C(Q)$, given by

$$\pi_C(Q) = E\left[p(Q)Q + r\theta(\tilde{s})Q - \int_0^L MC(t, \tilde{s})\, dt + \int_0^A (\tilde{s} - MC(L + t, \tilde{s}))\, dt\right] \tag{6}$$

where L is defined in (2) and A is defined in (5).

If the bank is able to purchase (sell) additional loans, but cannot sell (purchase) them, the loans bought in the spot market will be given by

$$B = \max\{0, A\} \tag{7}$$

The bank then chooses Q to maximize $\pi_R(Q)$ given by

$$\pi_R(Q) = E\left[p(Q)Q + r\theta(\tilde{s})Q - \int_0^L MC(t, \tilde{s})\, dt + \int_0^B (\tilde{s} - MC(L + t, \tilde{s}))\, dt\right] \tag{8}$$

subject to (5) and (7).

Notice that $MC(L, s)$ is nondecreasing convex in both arguments and in order to satisfy (3) we only need

$$Pr(\tilde{s} \leq c_1) = \int_0^{c_1} dF(s) = 0$$

where Pr denotes probability.

[3] If $MC(L, s)$ is not strictly increasing in L for given s, then A obtained by solving (5) may not be unique. Since the marginal profit is zero for values of A where s is equal to marginal cost, without loss of generality we can choose

$$A = \inf\{X: MC(L + X, s) = s\}$$

II. A. Optimal Loan Commitments: No Spot-Market Trading

Optimal Q can be found by differentiating (4). The first-order condition is given by

$$\pi_N'(Q^*) = p + p'Q^* + rE[\theta(\tilde{s})] - E[MC(L^*, \tilde{s})\theta(\tilde{s})] = 0 \tag{9}$$

where $L^* = \theta(\tilde{s})Q^*$. By *defining*

$$g(s, Q) \equiv [r - MC(L, s)]\theta(s) \tag{10}$$

(9) can be rewritten as

$$p + p'Q^* + E[g(\tilde{s}, Q^*)] = 0 \tag{11}$$

The partial derivatives of g with respect to s are given by

$$\frac{\partial g}{\partial s} = [r - MC(L, s)]\theta'(s) - \theta(s)\left[\frac{\partial MC}{\partial s} + \frac{\partial MC}{\partial L} Q\theta'(s)\right] \tag{12}$$

$$\frac{\partial^2 g}{\partial s^2} = [r - MC(L, s)]\theta''(s) - 2\theta'(s)\left[\frac{\partial MC}{\partial s} + \frac{\partial MC}{\partial L} Q\theta'(s)\right]$$
$$-\theta(s)\left[\frac{\partial^2 MC}{\partial s^2} + \frac{\partial^2 MC}{\partial L^2} Q^2(\theta'(s))^2 + \frac{\partial MC}{\partial L} Q\theta''(s)\right] \tag{13}$$

Proposition 1:

If θ is a linear function of the spot interest rate in the support of the distribution of $\tilde{s}$, reduced uncertainty in $\tilde{s}$ in the sense of a mean-preserving spread results in an increase in the optimal volume of commitments and therefore in expected lending, $E[\theta(\tilde{s})]Q$.

Proof:

From (13), if $\theta'' = 0$ then

$$\frac{\partial^2 g}{\partial s^2} = -2\theta'(s)\left[\frac{\partial MC}{\partial s} + \frac{\partial MC}{\partial L} Q\theta'(s)\right]$$
$$- \theta(s)\left[\frac{\partial^2 MC}{\partial s^2} + \frac{\partial^2 MC}{\partial L^2} Q^2(\theta'(s))^2\right] \leq 0 \tag{14}$$

Therefore $g(\cdot, Q)$ is concave in s, and if $\tilde{s}$ changes to a more certain $\tilde{s}_1$ in the sense of Rothschild-Stiglitz [5], then

$$E[g(\tilde{s}_1, Q)] \geq E[g(\tilde{s}, Q)] \qquad \forall\ Q$$

Since $p + p'Q$ is decreasing in Q, we obtain $Q_1^* \geq Q^*$ and since $\theta(\cdot)$ is linear, we have $E[\theta(\tilde{s}_1)] = E[\theta(\tilde{s})]$ and consequently

$$E[L(\tilde{s}_1)] = Q_1^*E[\theta(\tilde{s}_1)] = Q_1^*E[\theta(\tilde{s})] \geq Q^*E[\theta(\tilde{s})] = E[L(\tilde{s})] \qquad \text{Q.E.D.}$$

Since this result also applies in DGK's case of a linear marginal borrowing cost, their three propositions remain valid. However, their results are unobtainable when $\theta(s)$ is nonlinear, due to the term $[r - MC(L, s)]\theta''(s)$ which we are unable to sign for all admissible values of $\tilde{s}$. Since all other terms in (13) are negative as long as $\theta(\cdot)$ is convex, we can expect the result to hold, however, for convex functions approximating the linear case.

Stochastic increases in $\tilde{s}$ have an ambiguous effect on Q^* paralleling DGK's result for stochastic increases in θ. The root of the problem is the term $[r - MC(L, s)]\theta'(s)$. As in DGK, variations in θ (now due to variations in $\tilde{s}$ affect both marginal revenue, $r\theta(s)$, and marginal cost, $MC(L, s)\theta(s)$, in the same direction resulting in an uncertain net effect on expected profits. If we assume that $\theta(s)$ is a positive constant, say $\bar{\theta}$, then

$$\frac{\partial g}{\partial s} = -\bar{\theta}\,\frac{\partial MC}{\partial s} \leq 0 \tag{15}$$

and stochastic increases in s will reduce the optimal commitment volume. For the case of DGK's linear marginal cost, this is the same as a stochastic decrease in deposits (since $D'(s) \leq 0$), which decreases Q^* as well as expected takedowns.

Having shown how the DGK results generalize, we now wish to examine the effects of spot-market lending by banks.

B. *Unlimited Spot-Market Trading*

With unlimited buying and selling of spot loans, the bank's optimal commitment volume is Q_C^*. From (5) we obtain

$$\frac{\partial A}{\partial Q} = -\theta(s) \tag{16}$$

and setting the derivative of (6) equal to zero yields

$$\pi_C'(Q) = E\left[p + p'Q + r\theta(\tilde{s}) + \tilde{s}\,\frac{\partial A}{\partial Q} - MC(L + A, \tilde{s})\left(\theta(\tilde{s}) + \frac{\partial A}{\partial Q}\right)\right] = 0 \tag{17}$$

From (16) and (17)

$$p + p'Q + E[(r - \tilde{s})\theta(\tilde{s})] = 0 \tag{18}$$

or equivalently

$$p + p'Q + E[g(\tilde{s}, Q)] + E[\theta(\tilde{s})h(\tilde{s}, Q)] = 0 \tag{19}$$

where

$$h(s, Q) \equiv MC[L, s] - s$$

Proposition 2:

If the bank can freely purchase or sell loans in the spot market, $Q_C^* \lesseqqgtr Q_N^*$ according to whether $E[\theta(\tilde{s})h(\tilde{s}, Q)]$ is negative, zero, or positive, respectively.

Proof:

Proposition 2 follows from concavity of $\pi_N(Q)$ and the fact that (19) is

$$\pi_N'(Q) + E[\theta(\tilde{s})h(\tilde{s}, Q)] = 0 \qquad \text{Q.E.D.}$$

To understand the proposition, suppose that $\tilde{s}$ can assume only two values and $\theta(\tilde{s}) = 1$ for both. A high probability of realizing the high s makes $E[h(\tilde{s}, Q)] < 0$ and we choose $Q_C^* \leq Q_N^*$. A low probability results in $E[h(\tilde{s}, Q)] > 0$ and $Q_C^* >$

Q_N^*. Thus, if the bank expects the spot interest rate to exceed its marginal cost of funds determined at Q_N^*, the possibility of spot-market transactions will result in $Q_C^* > Q_N^*$. If the spot rate is expected to be lower than the bank's marginal cost of borrowing again determined at Q_N^*, the inequality is reversed. The factor $\theta(s)$ in the formula merely represents the effect on takedowns of unit increases in commitments.

In order to see if our Proposition 1 still holds, define

$$K(s) = (r - s)\theta(s) \tag{20}$$

so that (16) can be written as

$$p + p'Q + E[K(\tilde{s})] = 0 \tag{21}$$

From (20) we have

$$K'(s) = -\theta(s) + (r - s)\theta'(s) \tag{22}$$

and

$$K''(s) = -2\theta'(s) + (r - s)\theta''(s) \tag{23}$$

As in the previous case, except when θ is a constant, $K'(s)$ has an ambiguous sign because $(r - s)$ is positive for some s and negative for others. Thus stochastic increases in $\tilde{s}$ have an ambiguous effect. The same ambiguity obtains for the mean-preserving spread if θ is nonlinear in s. However, for the linear case $K(s)$ is concave in s and therefore we obtain

Proposition 3:

With unrestricted spot-market lending possibilities and with $\theta(s)$ linear in s, a reduction in uncertainty in the sense of a mean-preserving spread of the probability distribution of $\tilde{s}$ produces an increase in the optimal volume of commitments and hence in expected loans.

This result is independent of the assumption on the marginal cost function as long as for each s there exists an A that equates marginal cost to s. Thus we find that DGK generalizes to the case where unrestricted spot-market transactions are allowed.

C. *Limited Spot-Market Trading*

This case corresponds to the situation where the bank can augment its loans in the spot market, but it cannot sell loans spot. Since

$$\frac{\partial B}{\partial Q} = \begin{cases} 0 & \text{if } A < 0 \\ -\theta(s) & \text{if } A \geq 0 \end{cases}$$

from (8) we see that

$$p + p'Q + E[r\theta(\tilde{s})] - \int_{\{s:A\geq 0\}} s\theta(s)\, dF(s) - \int_{\{s:A<0\}} MC(L + B, s)\theta(s)\, dF(s) = 0 \tag{24}$$

Since MC is nondecreasing in L, for a fixed Q we have

$$r(Q) = \{s{:}A \geq 0\} = \{s{:}MC(L, s) < s\} = \{s{:}h(s, Q) < 0\} \tag{25}$$

and notice that

$$B(s) = 0 \quad \text{for all} \quad s \in \{s{:}A < 0\}$$

Hence (24) can be rewritten as

$$p + p'Q + E[(r - MC(L, \tilde{s}))\theta(\tilde{s})] + \int_{r(Q)} h(s, Q)\theta(s)\, dF(s) = 0$$

or equivalently

$$p + p'Q + E[g(\tilde{s}, Q)] + \int_{r(Q)} h(s, Q)\theta(s)\, dF(s) = 0 \tag{26}$$

and

$$\pi_N'(Q) + \int_{r(Q)} h(s, Q)\theta(s)\, dF(s) = 0 \tag{27}$$

Using the concavity of $\pi_N(Q)$ and the fact that $h(s, Q) < 0$ for all $s \in r(Q)$, given a fixed Q we can show

Proposition 4:

If the bank can purchase but is unable to sell loans in the spot market, the optimal volume of commitments is smaller than in either of the previously described cases, i.e., $Q_R^* \leq Q_N^*$ and $Q_R^* \leq Q_C^*$.

Corollary:

If the bank can sell but is unable to buy loans in the spot market, then the results of Proposition 4 are reversed.

Proof:

All else in Subsection C remains unchanged except that $r^c(Q)$ replaces $r(Q)$, i.e.,

$$r^c(Q) = \{s{:}h(s, Q) \geq 0\} \qquad \text{Q.E.D.}$$

Again, DGK generalizes to the case where restricted spot lending is permitted. Thus we define

$$\psi(s, Q) \equiv g(s, Q) + \min\{h(s, Q), 0\} = \theta(s)[r - \max(s, MC(L, s))] \tag{28}$$

Then (26) can be rewritten as

$$p + p'Q + E[\psi(\tilde{s}, Q)] = 0 \tag{29}$$

Because of the term $[r - \max(s, MC(L, s))]$ we encounter the same ambiguity as earlier. Namely, $\dfrac{\partial\psi}{\partial s}$ is ambiguous unless $\theta(s)$ is a constant. Furthermore, ψ is concave if $\theta(s)$ is linear, but ambiguous otherwise. Hence, we have shown

Proposition 5:

With linear $\theta(s)$ and restricted spot-market lending possibilities, a reduction in uncertainty produces an increase in the optimal volume of commitments and expected loan takedowns.

III. Conclusion

DGK show that the advent of liability management, interpreted as a stochastic decrease in deposits together with a mean-preserving spread of the probability distribution of deposits, should lead to reduced expected bank lending given that all lending is done via fixed-rate loan commitments. In addition, they show that increased uncertainty about loan takedowns can be expected to reduce the volume of commitments sold. This note has examined the robustness of DGK's results in a setting where banks are not restricted to forward lending. Rather, banks are able to lend in the spot-market after their clients takedown decisions become known. Thus, in contrast to DGK's special case, banks are permitted to offset their errors in estimating customer takedowns. We find that even with spot-market lending opportunities, an increase in uncertainty (captured here in the spot interest rate) will result in reduced loan commitments and hence reduced expected loan takedowns provided that takedowns are (almost) linear in the spot interest rate. Also consonant with DGK, stochastic increases in the spot interest rate are shown to have inherently ambiguous effects on the bank's choice of optimal commitments. In general, we can conclude that DGK's results hold in a setting with spot-market trading possibilities and no contradiction is encountered.

REFERENCES

1. S. D. Deshmukh, S. I. Greenbaum, and G. Kanatas. "Bank Forward Lending In Alternative Funding Environments." *Journal of Finance* 37 (September 1982), 925–40.
2. S. I. Greenbaum and I. Venezia. "Optimal Takedowns Under Adaptive Loan Commitment Pricing." *BRC Working Paper* #86, September 1982.
3. J. Hadar and W. R. Russell. "Rules for Ordering Uncertain Prospects." *American Economic Review* 59 (January 1969), 25–34.
4. G. A. Hanweck. "Bank Loan Commitments." In *Below The Bottom Line: The use of Contingencies and Commitments by Commercial Banks*, ed. B. Wolkowitz, et. al. Staff Studies 113 (January 1982), Board of Governors of The Federal Reserve System, Washington, D.C., 20551.
5. M. Rothschild and J. E. Stiglitz, "Increasing Risk: I. A Definition." *Journal of Economic Theory* 2 (1970), 225–43.