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Author(s): Bill McDonald
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# Beta Nonstationarity and the Use of the Chen and Lee Estimator: A Note

BILL McDONALD*

THE PREPONDERANCE OF EMPIRICAL evidence indicating nonstationarity in the estimate of systematic risk (e.g., Blume [4], Levy [11], or Alexander and Chervany [2]), has encouraged attempts to provide a more dynamic means of estimating Beta (e.g., Ohlson and Rosenberg [12]). Most recently, Chen and Lee [7], hereafter CL, have proposed a mixed-Bayesian estimator that allows Beta to vary through time. Subsequent applications of this method appear in Chen [5] and Chen and Keown [6]. The purpose of this note is to carefully re-examine the interpretation and empirical properties of the CL estimator. The results indicate that the nonstationarity parameter is not, in most cases, statistically significant and the interpretation of this parameter is obscured by its similarity to other statistical forms.

## I. Models of Nonstationarity

In the finance literature, the use of the term "nonstationarity" in the current context has been generalized to encompass any type of parameter variation occurring in the estimation of Beta. The econometric classifications for this phenomenon are more precise, identifying stationary stochastic models and nonstationary stochastic models. The former refers to specifications where the parameters have constant mean and variance. The latter case is more general, and allows the parameter and its distributional characteristics to change over time. It is important to note that the CL estimator is a stationary stochastic model.

The widespread application of Beta emphasizes the importance of providing an accurate estimate of this parameter. The primary trade-off in the estimation process is between the stability and the dynamic properties of the parameter. An estimate of Beta using a large number of historical observations tends to be more stable. Estimates using only very recent data lack stability, but allow structural changes to be more rapidly reflected in the estimate. The dynamic characteristics of the estimator are, for example, critical in market event studies. Findings of "inefficiencies" could be attributable to changes in the model parameters not reflected in historical estimates. The ideal estimator would provide an instantaneous measure of systematic risk. Although it is mathematically feasible to approach this ideal, the properties of the estimate are undesirable. The stochastic parameter models attempt to incorporate the best of both types of estimators by

* College of Business Administration, University of Notre Dame, Indiana. This research was financed by a grant from the Business Partner's Fund. Data were provided by the Jesse H. Jones Research Data Base.

retaining a foundation of historical data, but allowing more current information or systematic variation to provide an adaptive estimate of Beta.

Although the nonstationarity of Beta is generally recognized, there has been little consistency in attempts to characterize the process. Francis and Fabozzi [8] model the process as a simple random coefficient, Kon and Lau [10] assume there are two regimes generating the process; others assume the variation to be a function of cross-sectional information. Previous empirical studies (e.g., Blume [4]) do not suggest that Beta has a constant mean and variance, and yet stationary stochastic parameter models are being proposed as appropriate OLS alternatives. The underlying structural and economic relationships do not suggest that the Beta for a given firm should vary around a single value over extended time intervals.

### *A. The Chen and Lee Estimator*

The CL estimator can be developed in terms of the single-index market model as:

$$R_{jt} = \beta_{jt} R_{mt} + \epsilon_{jt} \tag{1}$$

where

$$\beta_{jt} = \beta_{j0} + \mu_{jt} \tag{2}$$

and

$R_{jt}$ = excess return on security $j$;

$R_{mt}$ = excess return on the market portfolio

(A more detailed discussion of the model's assumptions and development is provided in Chen and Lee [7]). The model is essentially a heteroscedastic process where

$$R_{jt} = \beta_{j0} R_{mt} + \epsilon^*_{jt} \tag{3}$$

and

$$\mathrm{VAR}(\epsilon^*_{jt}) = \sigma^2_\epsilon + \sigma^2_\mu R^2_{mt} \tag{4}$$

which can be estimated by maximizing the likelihood function given by

$$L = \prod_{t=1}^{T} \left(\sqrt{2\pi}\,\sqrt{\sigma^2_\epsilon + \sigma^2_\mu R^2_{mt}}\right)^{-1} \exp\left\{-\frac{1}{2}\,\frac{(R_{jt} - \beta_{j0} R_{mt})^2}{\sigma^2_\epsilon + \sigma^2_\mu R^2_{mt}}\right\} \tag{5}$$

Noting that

$$\sigma^2_\epsilon + \sigma^2_\mu R^2_{mt} = \sigma^2_\epsilon (1 + \gamma R^2_{mt}) \tag{6}$$

where

$$\gamma = \sigma^2_\mu / \sigma^2_\epsilon \tag{7}$$

and substituting the maximum likelihood estimates of $\beta_{j0}$ and $\sigma^2_\epsilon$ in Equation (5), provides a log-likelihood for estimating $\gamma$ equivalent to CL:

$$\ln(\gamma) = -(T/2)\ln(2\pi) - (T/2)\ln(1/T)^* \left\{\left(\sum_t \frac{R_{jt}^2}{1+\gamma R_{mt}^2} - \sum_t \frac{R_{mt}R_{jt}}{1+\gamma R_{mt}^2}\right)^2 \Big/ \left(\sum_t \frac{R_{mt}^2}{1+\gamma R_{mt}^2}\right)\right\} - \left(\frac{1}{2}\right)\sum_t \ln(1+\gamma R_{mt}^2) - (T/2). \quad (8)$$

In a Bayesian context, CL go on to develop the estimate of $\beta_{jt}$ based on the prior estimate of $\beta_{j0}$:

$$\beta_{jt} = \frac{\beta_{j0} + \gamma R_{mt}R_{jt}}{1 + \gamma R_{mt}^2} \quad (9)$$

The important implications of the theoretical specification are readily apparent. The estimate of Beta can now be adjusted to reflect the relevant economic conditions for a specific time period. Temporal variations in Beta can now be identified and attempts can be made to characterize this process. Although the development of the CL estimator is straightforward, and its applications rich with opportunity, the remainder of this paper describes the interpretational and empirical deficiencies which limit the practical usefulness of this estimator.

*B. An Analogous Form*

A wealth of research exists that confirms the presence of heteroscedasticity in the single-index market model. (Bey and Pinches [2] provide an excellent synthesis of research on this topic). Previous findings of a heteroscedastic process do not necessarily support the CL specification. The CL model, in its estimation, is indistinguishable from a constant coefficient model with a heteroscedastic error process defined by $\epsilon_i^* = \epsilon_i + \mu_i x_i$. Unfortunately, there is no strong theoretical justification for assuming that the structural source of the heteroscedasticity is attributable to the variation in Beta. As indicated by Ohlson and Rosenberg [12], the variance specification of Equation (4) cannot distinguish between heteroscedasticity related to var $(R_{mt})$ and unequal variance caused by temporary shocks to Beta. They also note that the significance of $\sigma_\mu^2$ could be attributable to the omission of variables.

Previous studies of the market model and heteroscedasticity have not identified the functional form of the variance. Before the CL estimate can be empirically supported, evidence must be provided that indicates the variance structure of Equation (4) is predominant. Given the variety of possible heteroscedastic forms, such evidence would provide some credence to the CL specification.

## II. The Empirical Properties of the CL Estimator

To investigate the empirical properties of the CL estimator, the likelihood function of Equation (5) was estimated. The market return was based on the CRSP value-weighted index and the risk-free rate was represented by the yield on treasury-bills with one month to maturity. Consistent with previous applications of the CL estimator, logarithmic returns were used. The model was estimated over a five-year interval using monthly data. Initially two time intervals, 1971–75 and 1976–80, were tested to determine if the observed properties of the estimator persist through time. (This is especially important, since the form is heteroscedastic and previous studies have found the occurrence of heteroscedasticity to vary over time). After estimating the models, there was virtually no difference in the results from one period to the next. Therefore, to clarify the presentation, only the 1976–80 results are reported. All securities with complete data on the CRSP tape for the intervals studied were included in the sample, providing a data base of 979 securities.

The log-likelihood function of Equation (5) was maximized using gradient search methods under the restriction that $\gamma \geq 0$. Tests of the significance of $\gamma$ were based on the likelihood ratio method. Under general conditions the difference between the likelihood value of Equation (5) and a restricted function where $\gamma = 0$ is distributed as $\frac{1}{2}\chi^2_{(1)}$.

Table I presents the descriptive statistics associated with the 979 estimates of $\gamma$ for the 1976–80 sample. The average $\beta_{j0}$ was 1.031 and the average least-squares estimate of $\beta$ was 1.050.

**Table I**

Descriptive Statistics for the Estimates of $\gamma$

| | Full Sample | Significant Subset[a] |
|---|---|---|
| N | 979 | 102 |
| Mean | 29.331 | 162.173 |
| Standard Deviation | 83.854 | 203.784 |
| Minimum | 0 | 31.131 |
| Maximum | 1,342.17 | 1,342.17 |
| % Significant ($\alpha = .05$) | 10.415% | 100% |

[a] Sample subset consisting of all securities with significant $\gamma$ at .05 level.

The descriptive statistics associated with the estimate of $\gamma$ indicate that the significance of this parameter is more an exception than the rule. Chen and Lee report that estimates of $\gamma$ were "substantial;" however, they do not report on the statistical significance of this key parameter. The likelihood function was maximized at the lower bound of $\gamma = 0$ for 40 percent of the securities. For cases where $\gamma$ was significant the minimum and maximum values for $\beta_{jt}$ were also calculated. Almost half of the securities had a minimum $\beta_{jt}$ less than 0, and 30 percent of the securities had a maximum value greater than 2. Thus, even for cases where $\gamma$ was statistically significant, the time series estimates of the $\beta_{jt}$ were not well-behaved. No association was found between parameters of the least-squares model and the occurrence of a significant $\gamma$(e.g., $\gamma$ was not found to be significant for securities with lower $R^2$).

## III. Conclusions

Given the predominance of Beta and its application as an input to subsequent methodologies, the dynamic properties of this parameter must be accounted for in its estimation. The Chen and Lee estimator is a promising step in this direction; however, it has not proven to be empirically sound. Empirical tests of the key parameter indicate that there is little statistical significance associated with the extended specification. The insignificance of this key parameter places some doubt on conclusions based on this methodology (e.g., Chen [5] or Chen and Keown [6]). There are also interpretational difficulties in empirically distinguishing this specification from a simple heteroscedastic process.

Models based on a simple random coefficient process have not received strong empirical support in market model applications (see [1] or [12]). The empirical findings of Ohlson and Rosenberg [12] suggest that the stochastic component of systematic risk is much more complex than a simple random process. Future studies should consider these initial findings in attempting to characterize the stochastic nature of Beta.

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