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Optimality of the Disclosure of Private Information in a Production-Exchange Economy

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ABSTRACT

This study provides an analysis of the effects on individual welfare of information revelation in a production-exchange economy. It is shown that the total effect of information revelation on an agent's utility level may be decomposed into three elements: a price effect, a shareholding effect, and a production effect. Through this decomposition, it is demonstrated that, although a Pareto improvement need not result from information revelation, there are perspectives from which the release can be considered beneficial.

THE MAJOR CONCLUSION OF previous studies[1] of information revelation in a heterogeneous beliefs economy is that the release will not result, in general, in an economic equilibrium Pareto superior to that which would have existed without the disclosure. This result, by itself, does not give an understanding of how information revelation affects agents' utility. Nor does it give much guidance to those regulatory bodies charged with deciding whether or not to mandate information disclosure. This study provides further insight into the effects on individual welfare of information revelation in a general production-exchange economy. It is shown that the total effect of information revelation on an agent's utility level may be decomposed into three elements: a price effect, a shareholding effect, and a production effect. The price effect is the change in the agent's utility caused by the change in prices resulting from the revelation of information, keeping shareholdings and production fixed. The shareholding effect is the change in his utility resulting from the change in his shareholdings caused by the release of information. The production effect is the change in his utility caused by the change in the production plan of each firm due to the information revelation. Conditions are given under which the signs of these three effects will be positive or negative for a given agent. The finding that the production effect is positive for some agents and negative for others is of particular interest since it is counter to a result of Arrow and Hirshleifer. They have argued that owners of production processes gain utility by having information revealed before production plans are fixed because they are then able to adjust those plans to account for the additional information. It is shown here that such a conclusion can only be made, in general, when owners of each production process have homogeneous beliefs. Finally, it is

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[1] See Arrow [1], Hakansson, Kunkel, and Ohlson [3], Hirshleifer [4], Kunkel [5], Marshall [6], Ng [7], Ohlson [8], and Verrecchia [11].

demonstrated that, abstracting from the effects of changing prices, no agents lose as a result of information release.

The paper is organized as follows. In Section I is a description of the economic setting. Section II analyzes the information revelation process and its effects on agents' welfare. Conclusions on the optimality of information revelation are presented in Section III. This is followed by a summary which concludes the paper.

I. Economic Setting

A single-good two-date economy is considered here. The date 0 state is known and there are S possible date 1 states. Firms take the single good as input at date 0 and transform it into output of the good at date 1. At date 0 individuals trade securities, consisting of claims to initial consumption and shares in the various firms. It is assumed that the market is complete[2] and that there is perfect competition in that managers and individuals act as if consumption-investment and production decisions do not affect state-contingent prices in the economy.

II. The Effects of Information Revelation in a General Production-Exchange Economy

At date 0 there is some private information in the economy. Its revelation, before trading and production decisions are made at date 0, consists of the public release of a signal, t, that causes each agent i to revise his prior subjective probability for the occurrence of each state s from π_s^i to π_s^{it}. In order to be able to analyze local, and thereby the full, revision of beliefs it is useful to define $\pi_s^i(\lambda)$ as follows:

$$\pi_s^i(\lambda) = \pi_s^i + \lambda(\pi_s^{it} - \pi_s^i)^3 \qquad \forall s \qquad 0 \leq \lambda \leq 1 \tag{1}$$

As λ moves from zero to one, $\pi_s^i(\lambda)$ moves from π_s^i to π_s^{it}. The effect of a local change in beliefs can then be examined by changing λ by an infinitesimal amount.

In order to examine this local effect, consider first an equilibrium reached with beliefs $\{\pi_s^i(\lambda)\}$, $\lambda < 1$, for each i. At this equilibrium individual i solves the problem:

$$\text{maximize}_{c_0^i,\{t_{ij}\}} U_i = w_i(c_0^i) + \textstyle\sum_s \pi_s^i(\lambda) y_i(c_s^i) \tag{2}$$

$$\text{subject to:} \quad c_0^i + \textstyle\sum_j t_{ij} v_j = \bar{c}_0^i + \sum_j \bar{t}_{ij}(v_j - q_{0j}) \tag{3}$$

$$c_s^i = \textstyle\sum_j t_{ij} q_{sj} \qquad \forall s \tag{4}$$

[2] The market is complete in the sense that agents are able to form any pattern of consumption across date 1 states. It is not complete in the broader sense of allowing agents to form any pattern of consumption across date 1 states and possible information signals.

[3] The form (1) is used because it clearly brings out the economic intuition of the effects of information revelation. Changing the specific form would have only the minor consequence of changing the signs of some effects for some agents in specific circumstances but would not alter the conclusions on the optimality of information revelation.

where

c_0^i is individual i's consumption at date 0;
$\bar{c}_0^i$ is individual i's endowment of date 0 consumption;
c_s^i is individual i's consumption at date 1, state s;
t_{ij} is individual i's percentage shareholding in firm j;
$\bar{t}_{ij}$ is individual i's percentage shareholding endowment in firm j;
q_{0j} is the initial input at date 0 of firm j;
q_{sj} is the output of firm j at date 1 in state s;
$v_j = \sum_s p_s q_{sj}$ is the value of firm j at date 0, where p_s is the price of a state contingent claim for state s consumption.[4]

It is assumed that U_i is strictly quasiconcave, $w_i'(c_0^i) \to \infty$ as $c_0^i \to 0$, and $y_i'(c_s^i) \to \infty$ as $c_s^i \to 0$ and $\pi_s^i > 0$ for all s.[5]

The first constraint (3) specifies that the value of the individual's purchases of initial consumption and securities equals the value of his endowment. Since v_j is the value of firm j at date 0 gross initial input q_{0j}, $v_j - q_{0j}$ is the net market value of firm j to the initial shareholders. The second set of constraints (4) specifies that the individual's consumption at date 1 equals his share of aggregate firm output.

The problem (2)–(4) yields first-order conditions for share purchases which can be written as:

$$v_j = \sum_s \pi_s^i(\lambda) \frac{u_{is}}{u_{i0}} q_{sj} \qquad \forall j \tag{5}$$

where

$$u_{i0} = w_i'(c_0^i), \qquad u_{is} = y_i'(c_s^i) \qquad \forall s$$

The solution to the individual's problem gives a maximum utility of $U_i^*(\lambda)$, evaluated using the posterior probability distribution $\{\pi_s^{it}\}$:

$$U_i^*(\lambda) = w_i[c_0^{i*}(\lambda)] + \sum_s \pi_s^{it} y_i[c_s^{i*}(\lambda)] \tag{6}$$

where the starred quantities represent optimal levels for the given λ.

Even though in this equilibrium $\{\pi_i^s(\lambda)\}$ and not $\{\pi_s^{it}\}$ are used by the individual, $\{\pi_s^{it}\}$ are the appropriate probabilities to use in evaluating his expected utility. Given that there is some private information, represented by signal t, in the economy, the best estimate, for agent i, of the probabilities of occurrence of the various states of nature is $\{\pi_s^{it}\}$ even if they are unknown to the agent; their use gives a more accurate measure of expected utility. If $\{\pi_s^i(\lambda)\}$ were used instead of $\{\pi_s^{it}\}$, then the real effect of information revelation on utility due to changed

[4] Although these state contingent claims are not formally traded, because there is a complete market their prices are implicitly determined in equilibrium.

[5] This assumption rules out the possibility of bankruptcy by forcing every c_s^i to be greater than zero.

consumption and production decisions would be masked by the effect of agents simply changing their perceptions of the prospects for the future.[6]

With respect to production decisions, under the assumption of complete competitive markets shareholders unanimously support net market value maximization as the goal of the firm. Thus, each firm j solves the problem:

$$\text{maximize}_{q_{0j}} v_j - q_{0j} \tag{7}$$

$$= \sum_s \pi_s^i(\lambda) \frac{u_{is}}{u_{i0}} q_{sj} - q_{0j} \qquad \forall i \quad \text{using (5)} \tag{7'}$$

$$\text{subject to: } q_{sj} = Q_{sj}(q_{0j}) \qquad \forall s \tag{8}$$

The production function $Q_{sj}(q_{0j})$ is assumed to be concave with $Q_{sj}(0) = 0$ and $Q'_{sj}(0) > 0$.

This problem yields the first order condition:

$$\sum_s \pi_s^i(\lambda) \frac{u_{is}}{u_{i0}} \frac{dq_{sj}}{dq_{0j}} = 1 \qquad \forall i \tag{9}$$

The assumptions on the utility functions and on the production functions guarantee that equilibrium exists[7] and that the first order conditions indicate a maximum.[8]

The effect on agent i's utility level of a small change in beliefs is found by differentiating $U_i^*(\lambda)$ with respect to λ:

$$\frac{\partial U_i^*}{\partial \lambda} = u_{i0} \frac{\partial c_0^{i*}}{\pi \lambda} + \sum_s \pi_s^{it} u_{is} \frac{\partial c_s^{i*}}{\partial \lambda} \tag{10}$$

The functional notation, showing the variables' dependence on λ, has been suppressed. Note, again, the use of the posterior beliefs, $\{\pi_s^{it}\}$, in the utility function.

Substituting for c_0^{i*} in terms of the remaining choice variables and differentiating gives:

$$\frac{\partial U_i^*}{\partial \lambda} = u_{i0}\left(\sum_j \bar{t}_{ij}\left(\frac{dv_j^*}{d\lambda} - \frac{dq_{0j}^*}{d\lambda}\right) - \sum_j t_{ij}^* \frac{dv_j^*}{d\lambda} - \sum_j v_j^* \frac{dt_{ij}^*}{d\lambda}\right)$$
$$+ \sum_s \pi_s^{it} u_{is}\left(\sum_j t_{ij}^* \frac{dq_{sj}^*}{d\lambda} + \sum_j q_{sj}^* \frac{dt_{ij}^*}{d\lambda}\right) \tag{11}$$

To simplify this, note that

$$\frac{dv_j^*}{d\lambda} = \frac{\partial v_j^*}{\partial \lambda} + \frac{\partial v_j^*}{\partial q_{0j}^*} \frac{dq_{0j}^*}{d\lambda} + \sum_k \sum_i \frac{\partial v_j^*}{\partial t_{ik}^*} \frac{dt_{ik}^*}{d\lambda} \tag{12}$$

[6] This analysis assumes that the private information already exists in the economy; therefore the posterior beliefs, with information fully revealed, $\{\pi_s^{it}\}$, are appropriately used in (6) and in the ensuing analysis. If, however, the private information did not already exist in the economy, the beliefs $\{\pi_s^i(\lambda)\}$ would instead be used in (6).

[7] See Debreu [2].

[8] See Takayama [10].

Since the economy is at an equilibrium for each value of λ, the assumption of complete competitive markets implies that

$$\frac{\partial v_j^*}{\partial t_{ik}^*} = 0 \qquad \forall i, k \quad \text{and that} \quad \frac{\partial v_j^*}{\partial q_{0j}^*} = 1.$$

With this, (11) simplifies to:

$$\frac{\partial U_i^*}{\partial \lambda} \bigg/ u_{i0} = \textstyle\sum_j (\bar{t}_{ij} - t_{ij}^*) \dfrac{\partial v_j^*}{\partial \lambda} + \sum_j \left(\sum_s \pi_s^{it} \dfrac{u_{is}}{u_{i0}} q_{sj}^* - v_j^* \right) \dfrac{dt_{ij}^*}{d\lambda}$$

$$+ \textstyle\sum_j \left(\sum_s \pi_s^{it} \dfrac{u_{is}}{u_{i0}} t_{ij}^* \dfrac{dq_{sj}^*}{d\lambda} - t_{ij}^* \dfrac{dq_{0j}^*}{d\lambda} \right) \qquad (13)$$

The derivative is made up of three terms called, respectively, the price effect, the shareholding effect, and the production effect. In order to understand why each agent's utility level changes with information revelation, each effect will be discussed in detail below.

A. The Price Effect

The price effect measures the increased spendable wealth of the agent given that the price of each firm j increases by $\frac{\partial v_j^*}{\partial \lambda}$ and that the agent is selling $(\bar{t}_{ij} - t_{ij}^*)$ shares. It is simple to show the following:

PROPOSITION 1. *The price effect is negative for at least one agent and positive for at least one agent unless it is zero for every agent.*

Proof: The price effect for agent i is

$$\textstyle\sum_j (\bar{t}_{ij} - t_{ij}^*) \dfrac{\partial v_j^*}{\partial \lambda}.$$

Because $\sum_i \bar{t}_{ij} = \sum_i t_{ij}^* = 1$, the sum of the price effects over all individuals,

$$\textstyle\sum_i \sum_j (\bar{t}_{ij} - t_{ij}^*) \dfrac{\partial v_j^*}{\partial \lambda},$$

equals zero. Unless the effect for every agent equals zero, it must be negative for at least one individual and positive for at least one individual. Q.E.D.

Changing prices, as a result of the information revelation, cause a redistribution among the agents of the economy's wealth. Unless everyone's wealth is unaffected, at least one agent's wealth increases and at least one agent's wealth decreases. Those who gain are the ones who are on balance, sellers of shares in firms whose prices have risen.

B. The Shareholding Effect

To understand the shareholding effect, consider, initially, that there is only one firm outstanding. Given λ and $\{\pi_s^i(\lambda)\}$, each agent i, in maximizing his utility, sets his shareholdings so as to equate his implicit valuation of the firm,

$$\textstyle\sum_s \pi_s^i(\lambda) \frac{u_{is}}{u_{i0}} q_s^*,$$

to the market price, v^*. (The j subscript, denoting the firm, has been dropped for convenience). However, given full use of the signal, the more accurate implicit valuation of the firm (unknown to the agent) is

$$\textstyle\sum_s \pi_s^{it} \frac{u_{is}}{u_{i0}} q_s^*.$$

If this valuation is greater than the cost of purchasing shares,

$$v^* \left(\text{or, equivalently, if } \textstyle\sum_s (\pi_s^{it} - \pi_s^i(\lambda)) \frac{u_{is}}{u_{i0}} q_s^* > 0\right),$$

then the agent's utility will be increased by his purchasing more shares. Conversely, if the market price is greater, the agent will gain if he decreases his purchases. An increase in λ will positively affect the agent's utility, then, if it causes him to appropriately change his shareholdings. However, the change in his purchases, $\frac{dt_i^*}{d\lambda}$, is not governed by the sign of

$$\textstyle\sum_s (\pi_s^{it} - \pi_s^i(\lambda)) \frac{u_{is}}{u_{i0}} q_s^*;$$

rather it is governed by the difference between the change in his implicit valuation caused by an increase in λ,

$$\textstyle\sum_s (\pi_s^{it} - \pi_s^i) \frac{u_{is}}{u_{i0}} q_s^*,\text{[9]} \quad \text{and} \quad \frac{\partial v^*}{\partial \lambda}.$$

Only if the implicit valuation increases by more than the increase in the cost of the shares in the marketplace will the agent decide to purchase additional shares. Since

$$\textstyle\sum_s (\pi_s^{it} - \pi_s^i(\lambda)) \frac{u_{is}}{u_{i0}} q_s^* \quad \text{and} \quad \sum_s (\pi_s^{it} - \pi_s^i) \frac{u_{is}}{u_{i0}} q_s^* - \frac{\partial v^*}{\partial \lambda}$$

need not have the same sign, this shareholding effect need not be positive for each agent.

There is an obvious special case where these two quantities do have the same sign: when $\frac{\partial v^*}{\partial \lambda} = 0$. With information revelation not affecting the price of the firm, the direction of change in the agent's shareholdings will be governed by the sign of

$$\textstyle\sum_s (\pi_s^{it} - \pi_s^i) \frac{u_{is}}{u_{i0}} q_s^*,$$

[9] That this is the change in his implicit valuation for a small increase in λ is verified by differentiating (5) with respect to λ, keeping production decisions fixed.

which, given the definition of λ, is identical to the sign of

$$\sum_s (\pi_s^{it} - \pi_s^i(\lambda)) \frac{u_{is}}{u_{i0}} q_s^*.$$

The agent will change his shareholdings, given the new information, in such a way as to increase his utility. More generally, given the above relationships, a sufficient condition for the shareholding effect to be nonnegative is that

$$\frac{\partial v^*}{\partial \lambda} \frac{dt_i^*}{d\lambda} \geq 0.$$

This condition ensures that

$$\sum_s (\pi_s^{it} - \pi_s^i(\lambda)) \frac{u_{is}}{u_{i0}} q_s^* \quad \text{and} \quad \frac{dt_i^*}{d\lambda}$$

have the same sign. This is formally shown in the proof to the following proposition, which allows for the existence of several firms:

PROPOSITION 2. *The shareholding effect is nonnegative for agent i if*

$$\sum_j \frac{\partial v_j^*}{\partial \lambda} \frac{dt_{ij}^*}{d\lambda} \geq 0.$$

Proof: See the appendix.

C. The Production Effect

To understand the production effect, rewrite (9) as follows:

$$\sum_s \pi_s^i(\lambda) \frac{u_{is}}{u_{i0}} dq_{sj}^* = dq_{0j}^* \tag{14}$$

Input is increased to the point where agent i's implicit valuation of incremental output (given $\{\pi_s^i(\lambda)\}$) is equal to the required incremental input. However, the more accurate implicit valuation of the incremental output, given full use of the signal, is

$$\sum_s \pi_s^{it} \frac{u_{is}}{u_{i0}} dq_{sj}^* \tag{15}$$

This will not, in general, equal the value of the incremental input, dq_{0j}^*. Whether it is greater or less than dq_{0j}^* depends on the relationship between $\{\pi_s^i(\lambda)\}$ and $\{\pi_s^{it}\}$. If (15) is greater than dq_{0j}^*, then the agent's utility level would increase from an increase in the production level; if it is less, then he would gain from a decrease in the production level. Since agents may have heterogeneous beliefs, (15) may not be the same for all agents even though (14) is the same; therefore, an increase in λ, because it causes market prices, and thereby the net market value production plan of each firm j, to change, may result in some agents gaining utility (a positive production effect) and others losing utility (a negative production effect).

A lack of unanimity arises in this complete market setting because agents' utility levels are being measured relative to the more accurate beliefs, beliefs that the agents are not using (unless there is full information revelation) when choosing each firm's production plan. Some agents have agreed to too high a production level, with respect to their more accurate information, while others have agreed to too low a level.

This problem disappears, as expected, when there are homogeneous beliefs, as the proof to the following proposition demonstrates:

PROPOSITION 3. *The production effect is nonnegative for each agent i if the agents have homogeneous beliefs and do not sell shares short.*

Proof: See the appendix.

That the sign of the production effect can be negative runs counter to a conclusion reached by Arrow and Hirshleifer. They provided examples where, when each agent completely owns his own productive resources, information revelation before the production plan is fixed allows each agent to appropriately revise his plan so as to increase his utility level. They were able to obtain these positive results, however, only because their setting was restricted; by not allowing for multiowner firms, the possibility of disagreement among owners as to how the production plan should be changed and consequently ambiguity as to the sign of the production effect could not arise.

III. Conclusions on the Optimality of Information Release

As with prior research, the analysis here confirms that information revelation will not result, in general, in an equilibrium Pareto superior to that which would have existed without the release;[10] any proposal to release information would be

[10] For completeness, the following example is provided to demonstrate this conclusion. Consider an economy composed of two agents, 1 and 2, two states of nature, both states having positive revised and initial probabilities of occurrence, and two firms, 1 and 2. Also assume that the initial beliefs of agent 1 are identical with the revised beliefs. Then, by definition of the shareholding and production effects, they both equal zero for this agent.

It is now sufficient, in order to demonstrate that someone is hurt by the information revelation, to show that the price effect is negative for agent 1. To do so, further specify that, in this economy, $\bar{c}_0^1 = 0$, $\bar{t}_{11} = 1$ and $\bar{t}_{12} = 0$ and that firm 1 produces a positive output only in state 1 and firm 2 only in state 2. In order to have positive consumption in each state (as required by the form of the utility function), agent 1 must purchase some shares of firm 2. He can only do this by selling some shares of firm 1.

Therefore, $t_{11}^* < 1$ and $t_{12}^* > 0$, or $(\bar{t}_{11} - t_{11}^*) > 0$ and $(\bar{t}_{12} - t_{12}^*) < 0$.

Finally, assume that $(\pi_1^{2t} - \pi_1^2) < 0$ while $(\pi_2^{2t} - \pi_2^2) > 0$. From (A.5) this implies that

$$\frac{\partial v_1^*}{\partial \lambda} < 0 \quad \text{and} \quad \frac{\partial v_2^*}{\partial \lambda} > 0.$$

The price effect for agent 1,

$$(\bar{t}_{11} - t_{11}^*) \frac{\partial v_1^*}{\partial \lambda} + (\bar{t}_{12} - t_{12}^*) \frac{\partial v_2^*}{\partial \lambda},$$

is then negative.

rejected under the Pareto criterion. However, this analysis goes further, providing additional perspectives on the optimality of information revelation. It has shown that, from the viewpoint of production, if the assumption of homogeneous beliefs is reasonable, then all agents agree that the release of information before production plans are fixed results in productive resources being better directed. From another viewpoint, that of the consumption-investment decision, abstracting from the effects of changing prices, the analysis shows that no agent loses utility as a result of information revelation, as the following proposition formally demonstrates:

PROPOSITION 4. *Given unchanging prices, the release of information results in a Pareto indifferent or Pareto superior solution in the production-exchange economy.*

Proof: If $\frac{\partial v_j^*}{\partial \lambda} = 0 \; \forall j$, then the price effect is zero for all agents. The shareholding effect is nonnegative, as Proposition 2 shows. There are no production changes, as is readily seen by noting that the numerator of the right-hand side of Equation (A.6) is equal to

$$\frac{\partial\left(\frac{\partial v_j^*}{\partial \lambda}\right)}{\partial q_{0j}^*}.$$

The production effect is therefore also zero. This completes the proof for the production-exchange economy. Q.E.D.

Any time that market prices change, for whatever reason, including that of information revelation, there is a redistribution of wealth in the economy and a change in each agent's opportunity set. However, abstracting from this, information revelation has the positive effect of improving each agent's consumption-investment decisions. Given fixed prices, the proposition has shown that the additional information is used by each agent to revise his investment plan so as to increase his utility level.

IV. Summary

This research solidifies the theory of the optimality of information release. It develops the theory in a general production-exchange economy explaining how an information announcement affects each agent's utility level and why it need not lead to a Pareto superior solution. In particular, it is shown that, opposite to that previously supposed, production changes caused by the information release need not benefit everyone.

The Pareto optimality criterion is very restrictive, ignoring the benefit received by some agents as long as one individual is harmed. The analysis here provides additional perspectives on the optimality of information revelation, demonstrating that the disclosure of information can be considered valuable. The ultimate decision as to whether or not it is worthwhile to mandate the release of information remains, of course, with society.

Appendix

A. *Proof of Proposition 2*: From Samuelson [9, p. 32, Equation (21)] the effect on agents i's maximum utility due to a change in the parameter λ is such that the following holds:

$$\sum_j \frac{\partial^2 \bar{U}_i^*}{\partial t_{ij}^* \partial \lambda} \frac{dt_{ij}^*}{d\lambda} \geq 0 \tag{A.1}$$

(The utility function differentiated here, $\bar{U}_i^*$, is that maximized by the agent—his perceived utility function. It is the utility function of Equation (6) with π_s^{it} replaced by $\pi_s^i(\lambda)$.)

Therefore:

$$\sum_j \left(\frac{-\partial v_j^*}{\partial \lambda} + \sum_s (\pi_s^{it} - \pi_s^i) \frac{u_{is}}{u_{i0}} q_{sj}^* \right) \frac{dt_{ij}^*}{d\lambda} \geq 0 \tag{A.2}$$

From the definition of λ:

$$\frac{\pi_s^{it} - \pi_s^i(\lambda)}{\pi_s^{it} - \pi_s^i} = 1 - \lambda \tag{A.3}$$

Using (5) and (A.3), (A.2) becomes:

$$\sum_j \left(\pi_s^{it} \frac{u_{is}}{u_{i0}} q_{sj}^* - v_j^* \right) \frac{dt_{ij}^*}{d\lambda} \geq \sum_j (1 - \lambda) \frac{\partial v_j^*}{\partial \lambda} \frac{dt_{ij}^*}{d\lambda} \tag{A.4}$$

The term on the left-hand side of (A.4) is the shareholding effect. If

$$\sum_j \frac{\partial v_j^*}{\partial \lambda} \frac{dt_{ij}^*}{d\lambda} \geq 0,$$

then the shareholding effect is nonnegative. Q.E.D.

B. *Proof of Proposition 3*: The first order condition (9) can be rewritten as

$$\sum_i \sum_s \pi_s^i(\lambda) \frac{u_{is}}{u_{i0}} t_{ij}^* \frac{dq_{sj}^*}{dq_{0j}^*} = 1 \qquad \forall j \tag{A.5}$$

Total differentiating (A.5) with respect to λ and q_{0j} and rearranging gives

$$\frac{dq_{0j}^*}{d\lambda} = \frac{-\sum_i \sum_s (\pi_s^{it} - \pi_s^i) \frac{u_{is}}{u_{i0}} t_{ij}^* \frac{dq_{sj}^*}{dq_{0j}^*}}{\sum_i \sum_s \pi_s^i(\lambda) \frac{u_{is}}{u_{i0}} t_{ij}^* \frac{d^2 q_{sj}^*}{dq_{0j}^{*2}}} \tag{A.6}$$

(Note that (A.5) is used instead of (9) here. (9) cannot be used without modifying the derivation because, as λ increases, there is an additional effect on the implicit prices of each agent i due to changes in t_{ij}. For (9) to be used, this change must be taken into account. This is unnecessary when (A.5) is used because, at any equilibrium, the market valuations by all investors is unaffected by small changes in individual shareholdings.)

The denominator is negative from the assumption on the form of the production function. Therefore:

$$\frac{dq^*_{0j}}{d\lambda} \geq 0 \quad \text{iff} \quad \sum_i \sum_s (\pi^{it}_s - \pi^i_s) \frac{u_{is}}{u_{i0}} t^*_{ij} \frac{dq^*_{sj}}{dq^*_{0j}} \geq 0 \tag{A.7}$$

From (A.3) and (A.5), (A.7) is equivalent to:

$$\frac{dq^*_{0j}}{d\lambda} \left(\sum_i \sum_s \pi^{it}_s \frac{u_{is}}{u_{i0}} t^*_{ij} \frac{dq^*_{sj}}{dq^*_{0j}} - 1 \right) \geq 0 \tag{A.8}$$

The production effect for agent i can be written as:

$$\sum_j \frac{dq^*_{0j}}{d\lambda} t^*_{ij} \left(\sum_s \pi^{it}_s \frac{u_{is}}{u_{i0}} \frac{dq^*_{sj}}{dq^*_{0j}} - 1 \right) \tag{A.9}$$

(A.8) implies that (A.9) is nonnegative if

$$\sum_i \sum_s \pi^{it}_s \frac{u_{is}}{u_{i0}} t^*_{ij} \frac{dq^*_{sj}}{dq^*_{0j}} - 1$$

is of the same sign as

$$t^*_{ij} \left(\sum_s \pi^{it}_s \frac{u_{is}}{u_{i0}} \frac{dq^*_{sj}}{dq^*_{0j}} - 1 \right) \qquad \forall j.$$

Given that $t^*_{ij} \geq 0$, $\forall i, j$, these are of the same sign if, for each j,

$$\pi^{it}_s \frac{u_{is}}{u_{i0}} \frac{dq^*_{sj}}{dq^*_{0j}}$$

and therefore

$$\pi^i_s(\lambda) \frac{u_{is}}{u_{i0}}$$

is identical over all i for each s. Because, in complete markets,

$$\pi^i_s(\lambda) \frac{u_{is}}{u_{i0}}$$

is identical over all i for each s,

$$\pi^{it}_s \frac{u_{is}}{u_{i0}}$$

is guaranteed to be identical for all i for each s if $\pi^{it}_s = k\pi^i_s(\lambda)$, where k is independent of i. This is satisfied if there are homogeneous beliefs. The production effect for each agent is therefore nonnegative if there are homogeneous beliefs, given that $t^*_{ij} \geq 0$ $\forall i, j$. If this shareholding restriction is dropped, then having homogeneous beliefs no longer guarantees a nonnegative production effect for each agent. Q.E.D.

REFERENCES

1. Kenneth Arrow. "Risk Allocation and Information: Some Recent Theoretical Developments." W. A. Mackintosh Lecture 1978.
2. Gerald Debreu. *Theory of Value.* New Haven: Yale University Press, 1959.
3. Nils Hakansson, J. Gregory Kunkel, and James Ohlson. "Sufficient and Necessary Conditions for Information to Have Social Value in Pure Exchange." Journal of Finance 37 (December 1982), 1169–1181.
4. Jack Hirshleifer. "The Private and Social Value of Information and the Reward to Inventive Activity." *American Economic Review* 61 (September 1971), 561–74.
5. J. Gregory Kunkel. "Sufficient Conditions for Public Information to Have Social Value in a Production and Exchange Economy." Journal of Finance 37 (September 1982), 1005–1013.
6. John Marshall. "Private Incentives and Public Information." *American Economic Review* 64 (June 1974), 373–90.
7. David Ng. "Pareto-Optimality of Authentic Information." *Journal of Finance* 32 (December 1977), 1717–28.
8. James Ohlson. "Efficiency and the Social Value of Public Information in Exchange Economies." Unpublished working paper, University of California, Berkeley, April 1980.
9. Paul Samuelson. *Foundations of Economic Analysis.* Cambridge, MA: Harvard University Press, 1947.
10. Akira Takayama. *Mathematical Economics.* Hinsdale, IL: Dryden Press, 1974.
11. Robert Verrecchia. "The Impact of Publicly Disclosed Information." Ph.D. dissertation, Stanford University, 1976.