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Economies of Scale and Economies of Scope in Multiproduct Financial Institutions: A Study of British Columbia Credit Unions

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ABSTRACT

This paper investigates the production technology facing computerized credit unions in Canada. A full system of translog cost equations is estimated in order to test for economies of scale, economies of scope, and other production characteristics in a multiproduct context. The regression results indicate that most of the credit unions in our sample experience significant increasing returns to scale as they expand their level of output. There is also evidence of cost complementarity or economies of scope in their mortgage and other lending activities. As a result, legislation which limits the ability of credit unions to grow and diversify will likely raise the operating costs of this important group of financial institutions. Additional structural tests of the most general translog specification suggest that none of the restrictive production conditions commonly imposed by other researchers using Cobb-Douglas and CES specifications provide a valid representation of credit union technology. The results of many earlier studies are therefore open to question.

THIS PAPER PRESENTS A number of structural tests of the production technology facing large computerized credit unions in Canada. A full system of translog cost equations is estimated using detailed cross-sectional data on British Columbia credit unions. The parameter estimates obtained from our multiproduct specification allow us to test for economies of scale, economies of scope, and factor substitution.

Unlike many earlier studies, the present paper introduces the various outputs as separate variables instead of crudely proxying them with a single output measure such as total assets, the total number of loans, or the number of depositors. By explicitly recognizing the multiproduct nature of the intermediation process, we are able to address more interesting production questions and remove some of the bias caused by misspecification and measurement error in simpler approaches. The flexible translog specification allows us to estimate the sensitivity of costs to proportionate increases in all outputs (economies of scale), as well as the cost savings firms realize by producing several outputs jointly rather than specializing in the production of one (economies of scope).

The test results reported in the latter part of the paper suggest that increasing returns to scale exist in almost all of the credit unions in our sample. Though

* Bank of Canada and University of Western Ontario, respectively. The views expressed in this paper are those of the authors; no responsibility for them should be attributed to the Bank of Canada. The authors would like to thank the Finance Workshop at the University of North Carolina, Jon Cockerline, Steve Poloz, Mariette Sabourin, and George J. Benston for their helpful suggestions and assistance.

the returns tend to diminish with asset size, this inverse relationship is very weak. Returns to scale remain significant for many of the largest credit unions, making it impossible to identify a minimum efficient firm size. Strong evidence of economies of scope or cost complementarity is also uncovered between mortgage lending and consumer lending. While there is some suggestion of similar complementarity on the investment side of credit union operations, these economies of scope do not appear to be statistically significant.

Such production information is valuable to managers who are interested in selecting an efficient firm size and an optimal output mix. It is also useful to regulators. The existence of significant returns to scale over a broad output range makes it impossible for a number of small competitive firms to operate efficiently in a given market. Cost complementarities arising from jointness in the production process favor "full service" institutions and argue against restrictions which encourage specialization.

The paper is divided into four sections. Section I contains the minimization model used to derive the system of cost equations. The cross-sectional data used to estimate the equations are described in Section II, and the regression results are reported in Section III. The paper concludes with a brief summary of our major findings.

I. The Model

Credit unions are assumed to minimize the costs associated with a given level of output.[1]

$$\text{Min. } C = \textstyle\sum_{j=1}^{m} p_j x_j \tag{1}$$

subject to a production constraint

$$F(y_1, \ldots, y_n; x_1, \ldots, x_m) = 0 \tag{2}$$

where

C = total cost of real capital, labor services, deposits, and shares;
p_j = unit price of jth factor input;
x_j = quantity of jth factor input; and
y_i = quantity of ith output (investments, mortgages, personal loans, commercial loans, and other services).

Researchers analyzing the production characteristics of financial institutions have traditionally imposed a specific functional form on F (the product transformation function) and then solved for the implied reduced form cost equation, min. $C^*(y; p)$.[2] The production functions they impose on F using this primal approach are of necessity very restrictive (e.g., the Cobb-Douglas and the CES). Output is often represented by a single scale variable and the production functions

[1] An alternative objective could have been profit maximization, yielding a function with input prices and output prices as its arguments. Unfortunately, profits have no unambiguous interpretation in a co-operative institution like a credit union. Also, the cost function provides a more convenient vehicle for the analysis of economies of scale and economies of scope.

[2] See for example, Bell-Murphy [2]; Benston [3]; Koot [13]; Murray-White [16]; and Wolken-Navratil [21].

are assumed to be homogeneous with a constant or unitary elasticity of substitution. As a consequence, the heterogeneous nature of most intermediation activity is either ignored, or inadequately captured in a series of appended control variables designed to proxy output proportions and other relevant production characteristics.

Though Cobb-Douglas and CES production functions are convenient specifications for the purposes of modelling and estimation, they are incapable of generating anything other than smooth monotonically increasing or decreasing average cost curves. Returns to scale are forced to remain constant across all output levels and may therefore present a biased picture of the production/cost relationships at extreme points of the output range.

The translog cost function avoids all of these pitfalls,[3] as it allows us to enter the various outputs as separate variables, and does not force us to treat homogeneity and a constant elasticity of substitution as maintained hypotheses. In its most general form, the translog function provides a second order approximation to any twice-differentiable function. The production technology of a multiproduct financial institution can therefore be modelled with maximum flexibility, giving explicit recognition to each of the outputs. Provided certain regularity conditions and behavioral assumptions are met, one can obtain a complete representation of the underlying technology simply by analyzing the structure of the related cost function.[4] There is no need to assume a particular production relationship a priori and then impose it on the cost function. One can procede directly to a general representation of the cost function and use it to extract information about the true production function.

The translog cost function which is used in our analysis is essentially a Taylor series expansion in output quantities and input prices, and can be written as follows:

$$\ln C = \alpha_0 + \textstyle\sum_{i=1}^{n} \alpha_i \ln y_i + \sum_{j=1}^{m} \beta_j \ln p_j + ½ \sum_i^n \sum_k^n \sigma_{ik} \ln y_i \ln y_k$$

$$+ ½ \textstyle\sum_j^m \sum_h^m \gamma_{jh} lup_j \ln p_h + \sum_i^n \sum_j^m \delta_{ij} \ln y_i \ln p_j \qquad (3)$$

As a cost function, (3) should be linearly homogenous in all input prices, concave in p_j, and increasing in y_i and p_j. The homogeneity condition is satisfied when

$$\textstyle\sum_j^m \beta_j = 1$$

$$\textstyle\sum_j^m \delta_{ij} = 0$$

and

$$\textstyle\sum_j^m \gamma_{jh} = 0$$

$$\left(\text{or} \quad \textstyle\sum_j^m \gamma_{jh} = \sum_h^m \gamma_{jh} = \sum_j^m \sum_h^m \gamma_{jh} = 0 \quad \text{where the price response is symmetric, } \gamma_{jh} = \gamma_{hj}\right)$$

[3] A translog specification has been employed in other studies of financial institutions, but in a profit function framework. See D. J. Mullineaux [15]. A more detailed description of the translog cost function is contained in Diewert [11].

[4] Costs should be a positive homogeneous, nondecreasing, concave function of factor prices (Shepherd [20]). The firm is assumed to pursue cost-minimizing behavior and treat input prices and output quantities as exogenous elements in its decision process.

Equation (3) is quadratic in logarithms and linear in the unknown parameters, permitting easy estimation. It can be reduced to some of the more popular restrictive technologies, such as the Cobb-Douglas and the CES, by imposing zero restrictions on selected parameters.

For example, the underlying production function becomes homothetic when $\delta_{ij} = 0, i = 1, \ldots, n; j = 1, \ldots, m$. This condition implies that the marginal rates of substitution in production are independent of scale effects and depend solely on relative prices (i.e., the isoquants are symmetric about the origin of the production space). By setting all $\sigma_{ik} = 0$ and $\delta_{ij} = 0$, we can make the production function homogenous of degree $\sum_{i=1}^{n} \alpha_i$. If in addition we set $\gamma_{jh} = 0\ j, h = 1, \ldots, m$, the production function will have unitary elasticities of substitution among all the factor inputs.

The "overall" economies of scale realized in the general case when all outputs are increased by a common factor, λ, is obtained by differentiating (3) with respect to all y_i,

$$\eta = \frac{d \ln C}{\lambda} = \sum_i^n \frac{\partial \ln C}{\partial \ln y_i}$$

$$= \sum_i^n \alpha_i + \sum_i^n \sum_k^n \sigma_{ik} \ln y_k + \sum_i^n \sum_j^m \delta_{ij} \ln p_j \tag{4}$$

If η is greater than 1, credit unions will experience decreasing returns to scale as costs rise proportionately more than output. An η value equal to 1 indicates constant returns to scale, and a value less than 1 indicates increasing returns to scale. Instead of making η a function of a single scale variable, we allow it to respond to differences in output mix as well as differences in factor prices. Translog cost curves can therefore assume more realistic shapes and need not be restricted to the smooth monotonically increasing or decreasing paths imposed by the Cobb-Douglas and CES specifications.

Interproduct complementarities or economies of scope are determined by the relative values of the α_i and σ_{ik} parameters. Panzar and Willig [17] have shown that a multiproduct cost function exhibits economies of scope if

$$\frac{\partial^2 C}{\partial y_i \partial y_k} < 0 \qquad i \neq k;\ i, k = 1, \ldots, n \tag{5}$$

An approximate test of this condition is

$$\alpha_i \alpha_k + \sigma_{ik} < 0 \tag{6}$$

where the y_i and p_j variables have been normalized about their sample means.[5] Output combinations satisfying (6) enjoy cost complementarities or jointness in their production. A single firm can therefore provide them at a lower cost than other firms which specialize and attempt to produce and sell the outputs individually. Production functions which are subject to increasing returns to scale *and*

[5] Denny and Pinto [10].

economies of scope display "subadditivity." The presence of subadditivity over a broad output range would obviously preclude efficient atomistic competition among financial institutions.

II. The Data

The cross-sectional data used in the estimation of Equation (3) were taken from the audited year-end reports of credit unions operating in British Columbia over the 1976–77 period. Supplementary information on wage rates, rental prices, hours worked, and organizational structure was obtained through a separate questionnaire. Though the complete data file has information on more than 140 credit unions, only 61 were included in the regressions described below. The subset was restricted to those credit unions employing computers for at least 25 per cent of their posting activities in order to remove the influence of dissimilar technologies, and thereby isolate the effects of scale differences and output diversity on operating costs.

A. *Costs*

Total costs, C, include all labor and real capital expenses, as well as the interest and dividends paid to depositors and shareholders. Since credit union shares are readily encashable and pay a regular return, they are very close substitutes for other types of deposits. Dividends are treated as a cost, therefore, leaving undivided profits as the only excluded residual on the debit side of the income statement.

B. *Output*

Following Sealey and Lindley [18], we assume that economic output is best measured by the dollar value of earning assets. Other assets and liabilities are assumed to be inputs to the production process.

Credit union outputs are represented by three variables, y_1 = mortgage lending, y_2 = other loans, and y_3 = investments in excess of minimum liquidity requirements,[6] where each y_i is set equal to the average of its beginning and end-of-year dollar value. Additional runs were made substituting the number of accounts as an alternative measure of y_i, but the results were essentially unchanged.

Most credit union assets in British Columbia are devoted to residential mortgages. Though many credit unions also engage in personal lending, commercial lending, and some nonresidential real estate financing, these other activities seldom account for more than 30 per cent of the total dollar value of their assets. Total lending as a proportion of all assets ranges from 75 to 80 per cent with the remainder composed of invested funds and some fixed assets.

[6] Credit unions are required to hold at least 10 per cent of the value of their deposit liabilities as a reserve, in the form of cash deposits and other qualifying assets.

C. *Input prices*

Four input categories are identified: labor services, fixed capital and materials, demand deposits, and term deposits plus shares. A separate input price is assigned to each.[7]

A crude unit price of capital, P_1, is obtained by summing the major capital expenses such as rent, depreciation, and utilities, and dividing by the average dollar value of deposits in 1977. The unit cost of labor, P_2, is an average hourly rate based on employee benefits as well as the explicit salaries paid to workers. Further adjustments are made to reflect wage differences between full-time and part-time workers, and the volunteer assistance which some credit unions receive. This is accomplished by creating an effective wage rate, in which share weights are assigned to the wages of each labor group.

The price of demand deposits, P_3, is simply the effective rate of interest paid on each dollar of demand deposits. It is calculated by dividing the interest paid on demand deposits by the average dollar value of deposits outstanding in 1977. The last input price variable, P_4, is a combination of the interest paid on term deposits, RRSPs/RHOSPs, and member shares. It is also measured as an effective interest rate, obtained by taking a weighted average of the interest paid on each of the components.

D. *Other Control Variables*

Several additional variables are used to control for other distinguishing features which could affect credit union costs and otherwise bias our analysis of the production relationships. A "BRANCH" variable is added to incorporate any cost differences that might exist between unit and branch bank operations. Three alternative specifications are employed. BRANCH1 is set equal to the number of offices; BRANCH2 is set equal to the log of the number of offices; and BRANCH3 is a dummy variable set equal to one for multi-office credit unions and zero for single office credit unions. A second control variable, "RISK" is introduced to capture the effects of risk differences on credit unions' loan portfolios. It is proxied by the dollar value of the allowance for doubtful accounts, normalized by total dollar assets. The last control variable, "GROWTH," is measured as the log of total assets in 1977 divided by total assets in 1976. It is added to accommodate any aberrant cost behavior caused by short-run disequilibrium.

The means and extreme points of all the variables are reported in Table I.

III. Empirical Results

A. *Estimation procedure*

Equation (3) is linear in its unknown parameters and is therefore amenable to ordinary least squares estimation. While OLS provides a simple means of deriving

[7] Wolken-Navratil [21] discuss the estimation bias that can result when factor price variables are omitted in tests for economies of scale. Wages are the only factor price they include in their regressions, however.

Table I

Mean Values and Ranges for the Cost Function Variables

Variable	Smallest Value	Mean	Highest Value
Total cost—C	\$ 69,917	\$ 2,944,010	\$ 41,373,300
Capital share—S_1	0.030	0.090	0.272
Labor share—S_2	0.057	0.179	0.420
Demand deposit share—S_3	0.015	0.212	0.494
Term deposit share—S_4	0.294	0.518	0.852
Mortgage lending—Y_1	\$ 72,139	\$20,984,100	\$334,099,000
Other lending—Y_2	\$176,146	\$ 3,185,900	\$ 30,196,100
Investments—Y_3	\$ 23,084	\$ 1,880,660	\$ 18,019,100
Price of capital—P_1	\$ 0.001[a]	\$ 0.005	\$ 0.010
Price of labor—P_2	\$ 2.28/hr.	\$ 6.06/hr.	\$ 9.59/hr.
Price of demand deposits—P_3	1.33%	5.13%	8.82%
Price of term deposits—P_4	1.84%	8.65%	11.57%
Number of offices BRANCH1	1	2.3	10
Portfolio risk—RISK	\$ 0.000[b]	\$ 0.002	\$ 0.013
Disequilibrium—GROWTH	4.67%	31.26%	74.71%

Notes: [a] Capital cost per dollar of total assets. [b] Allowances for doubtful accounts per dollar of total loans.

unbiased estimates,[8] it fails to incorporate any extra information which might be extracted from a restricted system of cost equations. For this reason it was deemed desirable to augment the single cost equation with a set of derived cost share equations and estimate the system simultaneously.

It is possible to derive a system of cost share equations directly from the translog cost function by differentiating (3) with respect to P_j,

$$\frac{\partial \ln C}{\partial \ln p_j} = \frac{p_j x_j}{C} = S_j = \beta_j + \textstyle\sum_h \gamma_{jh} \ln p_h + \sum_i \sigma_{ij} \ln y_i; \qquad j = 1, \ldots, m \tag{7}$$

where S_j is the cost share of the jth input (Shepherd's lemma).[9] Since the cost share equations do not introduce any additional unknown parameters into the estimation, we can gain additional degrees of freedom by estimating the cost function and the derived cost share equations as a multivariate regression system. The resulting systems estimates should be more efficient than the single equation estimates generated by the cost function alone.

Since the sum of the cost shares must equal one by definition, one of the equations in (7) is redundant. Any attempt to estimate the complete system will produce a singular disturbance covariance matrix. Fortunately, maximum likelihood estimates should be invariant to the equation we choose to exclude.[10] An iterative Zellner procedure is used on the cost system, allowing for nonzero correlations in the off diagonal elements of the disturbance covariance matrix.

Equations (8)–(11) comprise the full cost system. Cross-equation restrictions

[8] This assumes that the specification is correct and that the disturbance term is well behaved.

[9] W. E. Diewert [11].

[10] Note: the normality assumptions required to obtain maximum likelihood estimates from the iterative Zellner procedure may be violated due to the bounded values of the S_j variables. This is tested below.

on the parameter values are imposed to ensure that the linearity conditions and balance sheet constraints identified in Section I are met.

$$
\begin{aligned}
\ln C = {} & \alpha_0 + \alpha_1 \ln y_1 + \alpha_2 \ln y_2 + \alpha_3 \ln y_3 + \beta_1 \ln p_1 + \beta_2 \ln p_2 \\
& + \beta_3 \ln p_3 + (1 - \beta_1 - \beta_2 - \beta_3) \ln p_4 \\
& + ½\sigma_{11} \ln y_1 \ln y_1 + ½\sigma_{22} \ln y_2 \ln y_2 + ½\, \sigma_{33} \ln y_3 \ln y_3 \\
& + \sigma_{12} \ln y_1 \ln y_2 + \sigma_{13} \ln y_1 \ln y_3 + \sigma_{23} \ln y_2 \ln y_3 \\
& - (\gamma_{12} + \gamma_{13} + \gamma_{14})/2 \cdot \ln p_1 \ln p_1 \\
& - (\gamma_{12} + \gamma_{23} + \gamma_{24})/2 \cdot \ln p_2 \ln p_2 \\
& - (\gamma_{13} + \gamma_{23} + \gamma_{34})/2 \cdot \ln p_3 \ln p_3 \\
& - (\gamma_{14} + \gamma_{24} + \gamma_{34})/2 \cdot \ln p_4 \ln p_4 \\
& + \gamma_{12} \ln p_1 \ln p_2 + \gamma_{13} \ln p_1 \ln p_3 \\
& + \gamma_{14} \ln p_1 \ln p_4 + \gamma_{23} \ln p_2 \ln p_3 \\
& + \gamma_{24} \ln p_2 \ln p_4 + \gamma_{34} \ln p_3 \ln p_4 \\
& - (\delta_{12} + \delta_{13} + \delta_{14}) \ln y_1 \ln p_1 \\
& + \delta_{12} \ln y_1 \ln p_2 + \delta_{13} \ln y_1 \ln p_3 + \delta_{14} \ln y_1 \ln p_4 \\
& - (\delta_{22} + \delta_{23} + \delta_{24}) \ln y_2 \ln p_1 \\
& + \delta_{22} \ln y_2 \ln p_2 + \delta_{23} \ln y_2 \ln p_3 + \delta_{24} \ln y_2 \ln p_4 \\
& - (\delta_{32} + \delta_{33} + \delta_{34}) \ln y_3 \ln p_1 + \delta_{32} \ln y_3 \ln p_2 + \delta_{33} \ln y_3 \ln p_3 \\
& + \delta_{34} \ln y_3 \ln p_4 + \varepsilon_1
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
S_1 = {} & \beta_1 - (\gamma_{12} + \gamma_{13} + \gamma_{14}) \ln p_1 + \gamma_{12} \ln p_2 \\
& + \gamma_{13} \ln p_3 + \gamma_{14} \ln p_4 \\
& - (\delta_{12} + \delta_{13} + \delta_{14}) \ln y_1 - (\delta_{22} + \delta_{23} + \delta_{24}) \ln y_2 \\
& - (\delta_{32} + \delta_{33} + \delta_{34}) \ln y_3 + \varepsilon_2
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
S_2 = {} & \beta_2 + \gamma_{12} \ln p_1 - (\gamma_{12} + \gamma_{23} + \gamma_{24}) \ln p_2 \\
& + \gamma_{23} \ln p_3 + \gamma_{24} \ln p_4 + \delta_{12} \ln y_1 + \delta_{22} \ln y_2 \\
& + \delta_{32} \ln y_3 + \varepsilon_3
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
S_3 = {} & \beta_3 + \gamma_{13} \ln p_1 + \gamma_{23} \ln p_2 \\
& - (\gamma_{13} + \gamma_{23} + \gamma_{34}) \ln p_3 + \gamma_{34} \ln p_4 \\
& + \delta_{13} \ln y_1 + \delta_{23} \ln y_2 \\
& + \delta_{33} \ln y_3 + \varepsilon_4
\end{aligned}
\tag{11}
$$

where the ε_is are normally distributed random variables with zero means.

The additional structural restrictions associated with homothetic, homogenous, and/or unitary elastic production functions are tested by imposing zero restrictions on the parameters (see Section I) and comparing the resulting likelihood ratios.

B. *Results*

Parameter estimates and goodness of fit statistics are reported in Table II for both the single cost equation and the cost system.[11] Most of the estimates are statistically significant and carry their expected signs, despite the large number of variables included in the regressions. All of the control variables exerted a negative influence on costs, but none were statistically significant.[12] The cost system estimates are preferred to the single equation estimates because of their greater efficiency. Though the differences in their t-statistics are often very small, they are nevertheless potentially important when we come to make statistical inferences. Note, for example, the differences observed in the t-statistics on the factor price coefficients.[13]

The residuals were tested for normality, homoscedasticity, and independence. In no case were we able to reject the null hypothesis. Normality was tested with the Shapiro-Wilk analysis of variance, while heteroscedasticity was tested by first plotting the residuals (with the credit unions ranked by asset size) and then regressing the log of the squared residuals on the log of total assets. The Durbin-Watson statistic was used to check for autocorrelation.[14]

The Allen partial elasticities of substitution,[15] reported in Table III, were calculated as follows:

$$\Omega_{jh} = (\gamma_{jh} + S_j S_h)/S_j S_h$$

$$\Omega_{jj} = (\gamma_{jj} + S_j(S_j - 1))/S_j^2$$

They indicate that labor and capital are strong substitutes. The other factor inputs seem to bear very weak substitute and complementary relationships with one another. This is not surprising, as it is not clear how labor and capital could effectively substitute for deposits.[16] The estimated negative elasticity of substitution between demand deposits and term deposits might be capturing a portfolio diversification effect.

Structural tests of the cost functions all rejected the more restrictive specifi-

[11] The results were robust with respect to the deletion of the largest and the smallest credit unions.

[12] These results are generally consistent with those of Murray and White [16], though they found that branching had a statistically significant effect on costs. This difference could be due to the larger argument list and the flexible functional form used in the present paper.

[13] For example, with respect to economies of scope, the approximate t-statistics based on the single equation estimates are all insignificant at the 5 per cent level. The cost system estimates indicate significant economies of scope between mortgage and other lending activities (see Table VI).

[14] Although this is a cross-sectional analysis, it is possible to obtain a crude test of the specification by ranking the credit unions according to asset size and examining the Durbin-Watson statistic for autocorrelation.

[15] Since the Eigen values of the Hessian matrix for (8) are all nonnegative, the concavity condition on the cost function is satisfied (at the point of approximation).

[16] Sealey and Lindley [18] provide a convincing explanation of the limited substitution possibilities among the various factor inputs.

Table II

Parameter Estimates from the Cost System and Single Equation Regressions[a]

	Constant	Output[c]			Factor Prices[d]				Cross Product of Outputs					
	α_0	α_1	α_2	α_3	β_1	β_2	β_3	β_4	σ_{11}	σ_{12}	σ_{13}	σ_{22}	σ_{23}	σ_{33}
Simultaneous system estimation of equations (8)–(11)	−0.032 (2.69)[b]	0.731 (34.25)	0.099 (4.61)	0.102 (7.27)	0.086 (39.10)	0.159 (24.16)	0.232 (21.30)	0.522 (37.88)	0.132 (9.40)	−0.124 (7.68)	−0.103 (3.67)	0.071 (4.36)	−0.024 (1.06)	0.049 (4.83)
Ordinary least squares estimation of equation (8)	−0.037 (3.17)	0.771 (34.65)	0.125 (5.99)	0.061 (4.10)	0.120 (3.14)	0.285 (4.06)	0.302 (4.70)	0.293 (3.80)	0.130 (7.86)	−0.121 (5.65)	−0.078 (2.96)	0.083 (4.20)	−0.042 (1.75)	0.035 (3.36)

Cross Products of Outputs and Factor Prices

δ_{11}	δ_{12}	δ_{13}	δ_{14}	δ_{21}	δ_{22}	δ_{23}	δ_{24}	δ_{31}	δ_{32}	δ_{33}	δ_{34}
−0.007 (2.80)	−0.031 (4.06)	−0.008 (0.59)	0.046 (2.78)	0.002 (0.70)	−0.152 (1.88)	−0.023 (1.64)	0.036 (2.05)	0.003 (1.21)	0.014 (2.12)	0.040 (3.69)	−0.056 (4.09)
0.077 (1.39)	−0.109 (1.23)	−0.009 (0.08)	0.041 (0.48)	0.025 (0.54)	−0.551 (0.77)	−0.294 (2.71)	0.325 (2.59)	0.083 (2.63)	0.139 (2.52)	0.152 (2.47)	−0.208 (2.82)

	Cross Products of Factor Prices									
	γ_{11}	γ_{12}	γ_{13}	γ_{14}	γ_{22}	γ_{23}	γ_{24}	γ_{33}	γ_{34}	γ_{44}
Simultaneous system estimation of equations (8)–(11)	0.025 (12.59)	0.011 (2.11)	−0.014 (2.53)	−0.047 (7.02)	0.020 (2.24)	0.004 (0.30)	−0.055 (2.88)	0.074 (5.61)	−0.139 (4.76)	0.121 (5.97)
Ordinary least squares estimation of Equation (8)	0.075 (2.10)	−0.190 (1.28)	0.171 (2.12)	0.131 (1.03)	0.027 (0.24)	−0.516 (1.90)	−0.653 (3.51)	0.381 (3.08)	−0.418 (3.61)	−0.052 (0.34)

Goodness of Fit Statistics

	Mean of Dependent	Std. Error	$\bar{R}^2$	$D\text{-}W$
Cost System:				
Cost Function	−0.619	0.085	.997	1.70
Capital cost share	0.086	0.015	.783	2.07
Labor cost share	0.175	0.049	.544	1.79
Demand deposit share	0.215	0.093	.262	2.10
Single Cost Equation	−0.619	0.041	.999	1.42

Notes: [a] BRANCH, RISK, and GROWTH estimates are omitted as they were all insignificant. [b] Absolute value of t-statistic. [c] Output subscripts denote: 1. Mortgage loans, 2. Other loans, 3. Investments. [d] Factor price subscripts denote: 1. Capital, 2. Labor, 3. Demand deposits, 4. Term deposits.

Table III
Allen Partial Elasticities of Substitution

	Capital	Labor	Demand Deposits	Term Deposits
Capital	−3.800			
Labor	1.745	−3.420		
Demand deposits	0.237	1.118	−0.437	
Term deposits	0.046	0.3997	−0.234	−0.030

Table IV
Cost System Tests of the Production Structure

	Likelihood	Statistic[a]	Degrees of Freedom	Critical Value[b]
Regular cost function	483.10	—	31	—
Homotheticity	443.76	78.68	12	21.03
Unitary elasticity of substitution	421.37	123.46	10	18.31
Homogeneity	386.81	192.58	18	28.87
Cobb-Douglas	332.50	301.20	28	41.35
Constant returns to scale	467.57	77.07	1	3.84
No economies of scope				
(1) $\alpha_1\alpha_2 + \sigma_{12} = 0$	444.57	77.07	1	3.84
$\alpha_1\alpha_3 + \sigma_{13} = 0$	483.06	0.08[c]	1	3.84
$\alpha_2\alpha_3 + \sigma_{23} = 0$	482.82	0.56[c]	1	3.84

Notes: [a] The test statistic is calculated as $-2\log(L_1/L_0)$ and is distributed as a χ^2. [b] χ^2 for 0.05 confidence level. [c] Cannot reject the null hypothesis.

cations (see Table IV).[17] There is no statistical support for a production structure which is homothetic, let alone homogeneous. The constant returns to scale hypothesis is decisively rejected, as is the unitary elasticity of substitution hypothesis. In short, none of the assumptions underlying the more popular production structures offer a valid representation of British Columbia credit union technology.

Increasing returns to scale are observed in all of the credit unions, though in some cases they are not statistically significant (see Table V). There is some evidence of an inverse relationship between returns to scale (measured as EOS $= 1 - \eta$) and asset size, but the correlation is only −0.40. Any systematic relationship which exists between asset size and EOS appears to be dominated by differences in the credit unions' output mix.

One of the major advantages of the explicit multiproduct modelling permitted by the translog specification is the ability to estimate any cost complementarities which might exist among the various outputs. Significant economies of scope were detected between mortgage lending and other lending activity when we used the full cost system estimates (see the likelihood ratio tests in Table IV and the approximate t-statistics reported in Table VI). Credit unions which offer a full

[17] The likelihood ratio tests reported in Table IV are all for the system estimates. Results for the single equation estimates were qualitatively identical except for the economies of scope tests.

Table V

Economies of Scale Estimates Ordered by Total Assets

Observation	Economies of Scale[a]	Total[c] Assets	Observation	Economies of Scale	Total Assets
1.	0.054 (0.034)[b]	434,500,000	31.	0.052 (0.015)	15,203,838
2.	0.010 (0.030)	147,600,000	32.	0.030 (0.019)	13,404,366
3.	0.088 (0.028)	134,400,000	33.	0.082 (0.010)	13,204,810
4.	0.089 (0.026)	109,300,000	34.	0.062 (0.009)	12,779,356
5.	0.044 (0.017)	85,621,860	35.	0.061 (0.010)	12,613,082
6.	0.063 (0.019)	85,058,093	36.	0.104 (0.016)	11,747,620
7.	0.032 (0.015)	59,401,529	37.	0.065 (0.011)	11,687,640
8.	0.018 (0.022)	55,384,233	38.	0.084 (0.013)	11,649,055
9.	0.074 (0.023)	47,157,202	39.	0.058 (0.016)	10,559,892
10.	0.048 (0.014)	42,061,938	40.	0.030 (0.018)	9,011,117
11.	0.076 (0.020)	39,291,826	41.	0.098 (0.013)	8,797,880
12.	0.032 (0.016)	38,367,040	42.	0.074 (0.010)	8,705,477
13.	0.063 (0.013)	37,335,038	43.	0.036 (0.017)	8,267,558
14.	0.073 (0.015)	35,952,294	44.	0.041 (0.016)	8,218,049
15.	0.055 (0.012)	32,435,468	45.	0.078 (0.010)	8,014,086
16.	0.118 (0.029)	31,064,936	46.	0.066 (0.010)	7,648,576
17.	0.070 (0.013)	30,410,176	47.	0.063 (0.015)	7,611,151
18.	0.078 (0.014)	28,866,454	48.	0.084 (0.012)	6,595,473
19.	0.051 (0.012)	28,581,516	49.	0.087 (0.012)	6,439,953
20.	0.080 (0.013)	25,400,559	50.	0.071 (0.012)	6,363,657
21.	0.092 (0.014)	22,607,489	51.	0.076 (0.012)	6,037,132
22.	0.055 (0.080)	22,316,279	52.	0.049 (0.017)	5,663,738
23.	0.019 (0.016)	20,840,528	53.	0.036 (0.019)	5,365,987
24.	0.085 (0.014)	20,521,807	54.	0.062 (0.015)	4,760,150
25.	0.035 (0.018)	20,058,398	55.	0.067 (0.017)	4,739,503
26.	0.118 (0.026)	19,968,711	56.	0.041 (0.028)	4,460,497

Table V—*continued*

Observation	Economies of Scale[a]	Total[c] Assets	Observation	Economies of Scale	Total Assets
27.	0.081	18,721,608	57.	0.048	3,985,039
	(0.012)			(0.020)	
28.	0.066	18,226,324	58.	0.040	3,293,425
	(0.018)			(0.026)	
29.	0.072	17,651,254	59.	0.090	1,931,239
	(0.010)			(0.021)	
30.	0.040	17,565,101	60.	0.108	1,518,357
	(0.011)			(0.035)	
			61.	0.147	651,112
				(0.035)	

Notes: [a] Economies of scale were calculated holding factor prices constant at their sample mean. Differences in economies of scale among the credit unions, therefore, reflect only the differences in their output mix and size. [b] Approximate standard error. [c] Average of year-end numbers for 1976 and 1977.

Table VI

Economies of Scope: Parameter Estimates

		Cost System	Single Cost Equation
Mortgage and Other Lending	$-\alpha_1\alpha_2 + \sigma_{12}$	−0.0516	−0.0246
		(1.98)[a]	(0.82)
Investment and Mortgage Lending	$-\alpha_1\alpha_3 + \sigma_{13}$	−0.028	−0.0310
		(1.11)	(1.31)
Investment and Other Lending	$-\alpha_2\alpha_3 + \sigma_{23}$	−0.014	−0.0344
		(0.57)	(1.39)

[a] Approximate t-statistics

range of consumer loans and mortgages seem to realize cost savings vis à vis other firms specializing in the provision of one type of loan.

The same was not true for investment activities. Weak evidence of cost complementarity was found between investments and loans; however, the estimates were small and statistically insignificant. The single cost equation estimates for economies of scope were negative and statistically insignificant for all three output combinations (see Table VI).

IV. Summary

A system of cost equations was estimated in order to identify and measure any economies of scale and economies of scope present in the production technology of British Columbia credit unions. A translog cost function was employed to avoid imposing any unnecessary restrictions on the production/cost relationships. Likelihood ratio tests were then used along with selective parameter restrictions to test for homotheticity, homogeneity, unitary elasticities of substitution, economies of scope, and constant return to scale.

Our major conclusions were as follows:

(1) Restrictive Cobb-Douglas and CES production functions were decisively rejected in favor of a more general specification;

(2) There was no evidence to support the imposition of homotheticity, unitary elasticity of substitution, or constant returns to scale conditions on credit union production functions;

(3) The regression results indicate that economies of scale exist in most of the credit unions in our sample. These results are broadly consistent with those obtained in an earlier study of British Columbia credit unions by Murray and White [16]; and

(4) Strong evidence of cost complementarity or jointness in production was uncovered between mortgage and other lending activities in the full cost system regression. Likelihood ratio tests also rejected the "no economies of scope" hypothesis when σ_{12} was set equal to $-\alpha_1\alpha_2$ in a series of constrained estimations. There was weak evidence of cost complementarity between investment activities and lending but the results were not statistically significant.

These results have important policy implications. They indicate that large, multiproduct credit unions are more cost efficient than small, single-product credit unions. Regulations that inhibit growth and diversification are therefore inimical to market efficiency. Important caveats to these conclusions include recognition of our small sample size and the possibility of simultaneity bias,[18] neither of which is serious in our view.

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[18] Recall that output and factor prices were assumed to be exogenous.

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