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Lending Policies of Financial Intermediaries Facing Credit and Funding Risk

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ABSTRACT

This paper compares the optimal lending decisions of financial intermediaries that differ in their risk exposure. All intermediaries are assumed to face a loan demand described by a random applicant arrival process with each applicant offering a unique risk-adjusted rate of return; loan demand is therefore uncertain in both quantity and quality. The intermediaries differ in terms of their risk exposure because of disparate funding practices. Intermediaries functioning as brokers minimize their exposure by borrowing funds only as demand is realized, whereas those behaving as asset-transformers borrow in advance of realizing loan demand, thereby maintaining a loanable funds inventory and sustaining the related exposure. The optimal sequential lending policy is shown to involve setting a credit standard that becomes stricter with the length of the intermediary's planning horizon and the volume of loans outstanding. Most importantly, it is shown that brokers adopt stricter credit standards than asset-transformers and therby reduce their volume of lending.

THE TURBULENCE OF FINANCIAL markets, widely attributed to contemporary inflation, is thought to have prompted financial intermediaries to alter their borrowing and lending practices in order to limit their risk exposure. For example, banks have reduced their fixed interest rate loans in favor of variable rate lending and have devoted increased resources to the treasury function in order to more closely match the durations of their assets and liabilities (see Boltz and Campbell [3], Flannery [7], and Wojnilower [13]). The effect of increased uncertainty in financial markets on the intermediary's choice of exposure is examined in Deshmukh, Greenbaum, and Kanatas [5]. In this paper we examine the effect of an adaptive reduction in risk exposure on the credit allocation decisions of financial intermediaries.[1]

The risk exposure of financial intermediaries can be understood in terms of their two basic functions, as explained by Niehans [10]. An intermediary may function as a "broker," merely joining borrowers and lenders with complementary needs. In this case, the intermediary maintains a perfectly hedged position by

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[1] For an analysis of the bank's forward leanding decision, see Deshmukh, Greenbaum, and Kanatas [6] and Ricart i Costa and Greenbaum [11].

precisely matching the properties of its assets and liabilities and its only *raison d'être* is the reduction of search costs associated with joining complementary transactors. An example of this brokerage function is provided by the "best-efforts" contract of investment bankers (see Baron and Holmstrom [2]). Alternatively, the intermediary may modify the attributes (such as liquidity, duration, divisibility, numeraire or default/withdrawal risk) of claims held by its clientele (see Gurley and Shaw [8], and Diamond and Dybvig [4]). It then functions as a qualitative "asset-transformer," necessitating a mismatched balance sheet and consequent risk exposure. For example, duration mismatching implies interest rate exposure, whereas currency mismatching involves exchange-rate risk. The "firm-commitment" contract of the investment banker illustrates asset-transformation.

Thus, a reduction in the risk exposure of a financial intermediary may be viewed as an adaptation away from asset-transformation and towards the brokerage function. We examine the effect of such a shift on the intermediary's lending policy. Specifically, we compare the optimal lending policies of financial intermediaries functioning as stylized brokers with those engaging in qualitative asset-tranformation, where asset-transformation involves both divisibility (lot-breaking) and a form of duration mismatching.[2] Our major finding is that brokers adopt stricter credit standards than asset-transformers. If inflation impels financial intermediaries to reduce their risk exposure by adapting towards the broker's mode of operation, then we have also established a link between inflation and the allocation of credit.

The model employed is novel both in its characterization of financial intermediation and in its description of the lending process. Both brokers and asset-transformers are assumed to face uncertain loan demand and default risk. However, our broker operates like a custom producer, borrowing funds only as needed, subsequent to the realization of loan demand. The asset-transformer, on the other hand, operates like an inventory producer, borrowing funds in advance of realizing loan demand. Thus, even though both types of intermediaries sustain default risk, the asset-transformer's loanable funds inventory results in an additional exposure which we view as a form of interest rate risk associated with duration mismatching. Therefore, a reduction in the initially chosen funds inventory implies both reduced risk and less asset-transformation.

The lending process is modeled as a sequence of credit decisions taken by an intermediary facing uncertainty as to both the quantity and quality of loan demand. Credit applicants are assumed to arrive randomly over time with each applicant offering the lender a unique risk-adjusted rate of return. The expected profitability of each potential loan depends on both the applicant's observable default risk and the offered contractual interest rate. Upon each applicant arrival, the lender evaluates the risk-adjusted rate of return, the volume of loans already made, and the anticipated future lending opportunities over the remainder of the lender's planning horizon. A decision is then taken to either approve or deny the credit application, taking into account the effect of past decisions, the immediate

[2] Since our broker sustains default risk, it differs from the pure broker who merely provides informational services without assuming risk. Thus, strictly speaking, we will be comparing the lending policies of two asset-transformers with differing degrees of risk exposure.

profitability of lending to the current applicant, as well as the impact of the current decision on uncertain future opportunities. Treating the intermediary's problem as involving sequential lending to heterogeneous borrowers, like a dynamic capital budgeting problem, captures a distinctive feature of deposit-type financial intermediation. Neither the dynamic nor the customer heterogeneity element is made explicit in extant models of bank lending behavior (see Baltensperger [1]).

Within the context of this stochastic dynamic model of lending, we show that the rational credit policy of a lender (broker or asset-transformer) involves setting a credit standard and that a longer planning horizon or a higher volume of loans outstanding imply stricter credit standards. We also show that, *ceteris paribus*, the broker's credit standard will be stricter (and hence the total volume of credit granted will be smaller) than that of the asset-transformer. Finally, we show that a smaller inventory of loanable funds or lending capacity will result in stricter credit standards. Thus, if the asset-transformer seeks to limit its exposure by reducing its loanable funds inventory or by adopting the broker's mode of operation, aggregate lending will diminish.

In Section I, we describe brokers and asset-transformers and the dynamic lending process and then formulate the lender's decision problem. In Section II, we examine the structure of the optimal lending policy and show how it depends on the intermediary's funding risk exposure. The concluding section summarizes.

I. The Model

Consider a lender who extends credit for a fixed time period, say τ, to risky loan applicants as they arrive randomly over time. Suppose that each applicant requests a one dollar loan and that applicants arrive according to a Poisson process at rate λ. In addition to the uncertainty regarding the number of loan applicants, there is also uncertainty regarding the profitability of each potential loan. Let $\rho \in [-1, \infty)$ denote the risk-adjusted rate of return from lending one dollar to a given applicant. For example, if r is the contractual interest rate, and if $\theta \in [0, 1]$ is the probability of default on both principal and interest, then

$$\rho = r - \theta(1 + r)$$

As a special case, r may be exogenously fixed and therefore the default risk becomes the sole distinguishing characteristic among applicants. As another special case, applicants may be indistinguishable in terms of their default risk, but offer different contractual interest rates. Still another example, consistent with Jaffee and Russell [9] and Stiglitz and Weiss [12], relates the default risk to the interest rate offered. Specifically, suppose that higher contractual interest rates induce applicants to undertake riskier projects so that θ is an increasing function of r.[3] Then the lender's optimal contractual interest rate yields

$$\rho = \mathrm{Max}_{r \geq 0}[r - \theta(r)(1 + r)]$$

[3] Both Stiglitz and Weiss [12] and Jaffee and Russell [9] assume θ is unobservable and increasing in r. As a result, the lender charges a common interest rate to all applicants while recognizing that adverse selection or incentive effects may lead to declines in the expected risk-adjusted rate of return as r is increased.

Although credit market idiosyncrasies can be expected to influence the lender's procedure for evaluating an applicant, our sole concern is with the resulting risk-adjusted rate of return, ρ. Therefore, we simply assume that the lender can evaluate the ρ of each loan applicant and that ρ varies across the applicant pool according to a probability density (or mass) function $f(\cdot)$ with the corresponding distribution function $F(\cdot)$. Then $\lambda F(\bar{\rho})$ is the probabilistic rate of applicant arrivals whose profitability does not exceed $\bar{\rho}$. Thus, the Poisson process of arrivals at rate λ describes the uncertainty in the quantity of loans demanded whereas the probability distribution $F(\cdot)$ captures the uncertainty regarding the quality of loan demand.

Without these two undertainties, the lender's decision problem would be trivial. For example, if the lender knows that exactly N applicants will arrive over the given planning period $[0, T]$ and that the applicants are characterized by $\rho_1, \cdots, \rho_N$, then the lender could simply rank the applicants according to their ρ's and satisfy the demand of the best L applicants with ρ's exceeding the lender's marginal cost of borrowing; the remaining $(N - L)$ unprofitable applicants would then be rejected. With the aforementioned uncertainties, however, the lender's decision on each applicant should take into account not only the immediate profitability of the loan, but also the subsequent loan decisions that will have to be made with uncertainty about the future number and quality of applicants. Thus, if a loan applicant arrives at time $t \in [0, T]$, the lender must consider the applicant's ρ, the cumulative loans already made, say L_t, (which affects the lender's marginal cost of granting additional loans), and the remaining time $(T - t)$ until the end of the planning horizon (during which additional lending opportunities may become available). With this information, the lender decides whether to approve or to deny the given application. A decision to lend is denoted by $d_t = 1$, and a negative decision by $d_t = 0$. Thus, after a decision at time t, total loans outstanding become $(L_t + d_t)$ and remain at this level until a decision is taken on the next applicant arrival.

A. The Mode of Intermediation and the Cost of Funds

The intermediary's funding cost and risk exposure depend on its mode of borrowing. Its exposure to loan demand uncertainty can be minimized by borrowing unit quantities only as needed, i.e., when loan demand is realized over time. By synchronously borrowing and lending for the same duration, τ, the intermediary operates without a cash inventory and thereby maintains a continually duration-matched balance sheet. We therefore refer to this kind of intermediary as a broker, even though it sustains default risk.

Alternatively, the intermediary may borrow in advance of realizing the loan demand and then make loans from this inventory as applicants are approved over time. By borrowing *en bloc* and lending in unit quantities, the intermediary breaks lots, thereby altering the divisibility of claims. Since funds are borrowed at time 0 and loans are made at different times over $[0, T]$, the result is a continually duration-mismatched balance sheet with both cash and τ-period loans financed exclusively with τ-period borrowings. The cash inventory, absent in the broker's mode of borrowing, gives rise to the risk of ending the period with excess funds. This additional exposure can be interpreted as a form of interest rate risk in that unexpectedly low demand may necessitate the acceptance of a low (possibly zero) rate of return on surplus funds; similarly, an unexpectedly high demand may require additional borrowings at a high (possibly infinite) cost. Thus, quantity (demand) uncertainty translates into interest rate risk even if the penalty interest rates are known with certainty. We refer to the intermediary that sustains this added exposure as an asset-transformer.[4] Thus, our broker and asset-transformer both sustain default risk but only the latter maintains a cash inventory, implying a (duration and divisibility) mismatched balance sheet with consequent interest rate risk.

Let $\bar{L}$ denote an upper bound on the intermediary's total borrowing and lending over the planning period $[0, T]$. This limit may be viewed as a capacity constraint on the intermediary's scale of operation deriving either from a regulatory restriction or from the short-term fixity of financial capital. Thus, both the asset-transformer and the broker face a limit $\bar{L}$ on the total funds borrowed giving rise to an opportunity cost of lending. However, the asset-transformer borrows up to capacity at $t = 0$, whereas the broker borrows in unit quantities only as loans are approved and funds are needed through time. Let $C_A(\cdot)$ and $C_B(\cdot)$ denote the cost-of-funds function for the asset-transformer and the broker, respectively, and assume that the broker's marginal cost of borrowing, $[C_B(L_t + 1) - C_B(L_t)]$, is strictly positive and nondecreasing as a result of its growing default risk deriving from the debt-financed loan growth, L_t, over time. Although the asset-transformer's borrowing cost may be convex increasing in the size of its funds inventory owing to the same default consideration, its borrowing cost is sustained before the lending process commences. The asset-transformer's incentive to hold a funds inventory may arise from a borrowing cost reduction provided by its source of funds in exchange for reduced demand uncertainty; in addition, the asset-transformer mitigates any fixed transactions costs of borrowing.

Note that the definition of the capacity constraint, $\bar{L}$, and the borrowing cost functions, $C_A(\cdot)$ and $C_B(\cdot)$, require that the intermediary's borrowing and lending be carried out within a given time interval, $[0, T]$. The fixed planning horizon should be viewed as an analytical convenience designed to simplify the definition of costs and of risk exposure. Without a fixed T, borrowing costs at any time would depend on net loans at that time, necessitating the tracking of loan repayments and an explicit inventory carrying cost would be required in order to capture the funding risk. We assume for simplicity that there are no loan repayments (i.e., $\tau > T$) or discounting in $[0, T]$. Finally, for consistency and ease of comparison we assume that the funding costs of both intermediaries are paid at the end of the planning period.

[4] Some prefer to distinguish between brokers and asset-transformers on the basis that the former transact on behalf of clients, as for example a mutual fund manager, whereas the latter transact on their own accounts, as in the case of a commercial bank. However, the more general distinction involves risk exposure. For example, a commercial bank with a perfectly matched balance sheet is completely hedged and therefore restricts itself to the provision of search services; it behaves the same as the intermediary transacting on behalf of clients.

Thus, our asset-transformer sustains a fixed cost $C_A(\bar{L})$ of borrowing an inventory of funds at time $t = 0$ without additional borrowing costs as loans are made from this inventory over the time interval $[0, T]$, but there is a possibility of excess supply or demand. In contrast, the broker sustains neither a fixed borrowing cost nor any exposure to excess supply, but it does bear a higher marginal funding cost as loans are made.

B. The Lending Decision

Since the asset-transformer and the broker differ only in terms of their borrowing cost functions, we can analyze their optimal lending decisions within a single formulation. Let $C(L_T)$ denote the lender's borrowing cost paid at time T, where L_T denotes total loans made in $[0, T]$. Then for $L_T \leq \bar{L}$, the asset-transformer's cost of funds is

$$C(L_T) = C_A(\bar{L}) \tag{2.1}$$

whereas the broker's cost function is given by

$$C(L_T) = C_B(L_T) \tag{2.2}$$

and by assumption $C(\cdot)$ is nondecreasing and convex.

A lending policy is a rule, δ, that specifies an approve/deny decision on each loan application on the basis of information available at the time of the applicant arrival. If an applicant offering a risk-adjusted rate of return ρ arrives at time t, and if the loans outstanding at that time are $L_t = L$, then $\delta(t, L, \rho) = 1$ corresponds to a loan approval, whereas $\delta(t, L, \rho) = 0$ means that the application is rejected and the capacity constraint requires $\delta(t, \bar{L}, \rho) \equiv 0$.

Let $V(t, L, \rho)$ be the maximum expected net profit from time t until T, given that L loans have been made when an applicant with ρ arrives at time t. Thus, V is the optimal value function and the lender's problem is to determine the optimal lending policy δ^* that yields V. In the remainder of this section, we describe the dynamic programming functional equation satisfied by V.

Suppose at time t, with $L_t = L$, an applicant arrives with characteristic ρ. If the lender approves the application (i.e., $d = 1$), ρ is the expected return, whereas denial ($d = 0$) results in zero immediate return. In either case, cumulative loans become $(L + d)$ after the decision, and the lender then waits an exponentially distributed random time, $\tilde{s}$, until the next applicant arrival. If the lender employs an optimal policy following the next arrival, maximum expected profit from then on will be $V(t + \tilde{s}, L + d, \tilde{\rho})$. Therefore, the optimal decision on the present applicant should maximize current return, ρd, plus the maximum expected future profit, $E[V(t + \tilde{s}, L + d, \tilde{\rho})] \equiv U(t, L + d)$, from the next applicant forward. Consequently, the lender's total maximum expected profit from the next applicant onward must satisfy, for all $t \in [0, T]$, $L < \bar{L}$ and $\rho \in [-1, \infty)$,

$$V(t, L, \rho) = \underset{d}{\text{Max}}\{\rho d + U(t, L + d)\}$$

or, more explicitly,

$$V(t, L, \rho) = \text{Max}\{\rho + U(t, L + 1), U(t, L)\} \tag{2.3}$$

where

$$U(t, L) = \int_0^{T-t} \int_{-1}^{\infty} V(t+s, L, \rho) f(\rho) \lambda e^{-\lambda s}\, d\rho\, ds - C(L) e^{-\lambda(T-t)} \quad (2.4)$$

It is therefore optimal to approve the loan application if and only if

$$\rho + U(t, L+1) \geq U(t, L) \quad (2.5)$$

Note that

$$\rho^*(t, L) = [U(t, L) - U(t, L+1)] \quad (2.6)$$

is the expected opportunity cost of lending to the current applicant in terms of the potentially high ρ's presented by future arrivals or the possibly higher borrowing costs (for the broker). Thus, (2.5) is the usual profit-maximizing condition with explicit account of intertemporal considerations and future uncertainties. Therefore, we can express the optimal lending policy, δ^*, in terms of a critical rate of return (or a credit standard) ρ^*, so that it is optimal to lend to the applicant if and only if the offered ρ exceeds $\rho^*(t, L)$.

As a special case, suppose the loan interest rate is arbitrarily fixed at $\bar{r}$ and applicants differ in their observable default risk θ. Then $\rho = \bar{r} - \theta(1 + \bar{r})$, and the optimal lending policy establishes a critical default risk standard

$$\theta^*(t, L) = \{\bar{r} - [U(t, L) - U(t, L+1)]\}/(1 + \bar{r})$$

so that loans are made only to applicants whose default risks are less than $\theta^*(t, L)$. More generally, describing the lender's decision in terms of a minimum acceptable ρ^*, we see how lending to disparate loan applicants on the basis of risk-adjusted return may be (mis)interpreted as "credit rationing."

II. Optimal Lending Policies

In order to analyze the form of the optimal lending policy, δ^*, or equivalently the optimal credit standard, ρ^*, we prove the following properties of the lender's optimal value functions, V and U, in the appendix. For expositional simplicity, results are presented in differential notation even though the functions involved may not be differentiable.

Lemma 1. If the cost of funds function $C(\cdot)$ is convex nondecreasing, then the optimal value functions $V(t, L, \rho)$ and $U(t, L)$ are nonincreasing and complementary in (t, L), nondecreasing in ρ and concave in L. That is, for all $t \in [0, T]$, $L < \bar{L}$ and $\rho \in [-1, \infty)$, we have

$$\frac{\partial V(t, L, \rho)}{\partial \rho} \geq 0 \qquad \text{(monotonicity in } \rho) \quad (3.1)$$

$$\left.\begin{aligned} \frac{\partial V(t, L, \rho)}{\partial t} &\leq 0 \quad (3.2) \\ \frac{\partial U(t, L)}{\partial t} &\leq 0 \quad (3.3) \end{aligned}\right\} \text{(monotonicity in } t)$$

$$\left.\begin{array}{l} 0 \le V(t, L-1, \rho) - V(t, L, \rho) \\ \quad \le V(t, L, \rho) - V(t, L+1, \rho) \\ 0 \le U(t, L-1) - U(t, L) \leqslant U(t, L) - U(t, L+1) \end{array}\right\} \text{(concavity in } L) \qquad \begin{array}{r}(3.4)\\ \\ (3.5)\end{array}$$

$$\left.\begin{array}{l} \dfrac{\partial}{\partial t}[V(t, L, \rho) - V(t, L+1, \rho)] \le 0 \\ \\ \dfrac{\partial}{\partial t}[U(t, L) - U(t, L+1)] \leqslant 0 \end{array}\right\} \text{(complementarity in } (t, L)) \qquad \begin{array}{r}(3.6)\\ \\ (3.7)\end{array}$$

Condition (3.1) states that total expected profits are increasing in the profitability of the current customer. Conditions (3.2) and (3.3) indicate that the closer the end of the planning horizon, the lower is the expected profit from lending, since fewer customer arrivals and decision opportunities remain. According to (3.4) and (3.5), the marginal opportunity cost of lending is nonnegative and nondecreasing in the funds already lent. Committing fewer loans by time t provides the broker with a lower cost of funds and both the asset-transformer and the broker with greater flexibility with regard to future loan decisions. Hence expected total profits from t onwards are improved. However, this marginal advantage is less pronounced for lower values of the funds already committed. Finally, (3.6) and (3.7) indicate that the marginal advantage of reduced past lending declines as the end of the planning period approaches. Future lending opportunities depend upon both the remaining time as well as the availability (or cost) of funds; as the remaining time diminishes, the marginal value of a dollar not lent also diminishes. Thus, in the long-run it is better to have more time left and less funds committed, although there are diminishing benefits to having committed fewer funds and the value of uncommitted funds diminishes with the passage of time.

With this interpretation, we expect that the lender will be more lenient in evaluating loan applicants that arrive later in the planning period or when fewer loans have been made. This follows immediately from (3.5) and (3.7), which imply that the minimum acceptable rate of return, $\rho^*(t, L)$ given by (2.6), is decreasing in t and increasing in L. We summarize the structure of the optimal lending policy as follows:

PROPOSITION 1. The optimal lending policy δ^* is of the form

$$\delta^*(t, L, \rho) = \begin{cases} 1 & \text{if} \quad \rho \ge \rho^*(t, L) \\ 0 & \text{otherwise} \end{cases} \tag{3.8}$$

where $\rho^*(t, L)$, given by (2.6), is nonincreasing in $t \in [0, T]$ and nondecreasing in $L \le \bar{L}$.

A. Credit Standards of Brokers vs. Asset-Transformers

Although the form of the optimal lending policy given in Proposition 1 is the same for both the asset-transformer and the broker, their credit standards will differ. In particular, we show that the broker will adopt stricter credit standards and hence will lend to fewer applicants than the asset-transformer. In order to compare the lending policies of the two intermediaries, we distinguish the optimal value functions, credit standards and other data for the asset-transformer and the broker using superscripts A and B, respectively, and prove the following in the appendix:

Lemma 2. For all $t \in [0, T]$, $L < \bar{L}$ and $\rho \in [-1, \infty)$

$$[V^A(t, L, \rho) - V^A(t, L+1, \rho)] \leq [V^B(t, L, \rho) - V^B(t, L+1, \rho)] \tag{3.9}$$

and

$$[U^A(t, L) - U^A(t, L+1)] \leq [U^B(t, L) - U^B(t, L+1)] \tag{3.10}$$

Thus, the marginal cost of committing funds is lower for the asset-transformer than for the broker. Consequently, from (2.6), the asset-transformer will adopt a less restrictive lending policy and we have

PROPOSITION 2. For all $t \in [0, T]$ and $L \leq \bar{L}$,

$$\rho^{B*}(t, L) \geq \rho^{A*}(t, L) \tag{3.11}$$

To compare the volume of credit granted by asset-transformers and brokers, consider an applicant arriving at time t. Since $\rho^{B*}(t, L) \geq \rho^{A*}(t, L)$, we know that the broker will be more selective in granting credit than the asset-transformer, provided both lenders have committed the same volume of funds, L, by time t. However, since the asset-transformer follows a less selective policy from the outset, more loans may have been granted by time t. Consequently, at any time t, the asset-transformer's credit standard may in fact be more selective than that of the broker. However, this can happen only if the asset-transformer has already made more loans than the broker. Hence, the asset-transformer will always lend more in aggregate than the broker. Thus, if financial intermediaries seek to reduce their funding risk exposure by switching from the asset-transformer's to the broker's mode of operation, aggregate lending will decline.

B. The Firm Size and Credit Standards

Short of switching to the broker's mode of intermediation, the asset-transformer may limit its exposure by merely reducing its loanable funds inventory. Alternatively, the lending capacity of either type of intermediary may be reduced as a result of a capital loss or because of altered regulatory restrictions. These changes are manifested in terms of a reduced $\bar{L}$ increasing the cost of borrowing, and the previous analysis permits us to conclude

PROPOSITION 3. A reduction in the intermediary's lending capacity or a reduction in the asset-transformer's funds inventory results in stricter credit standards and reduced aggregate lending.

Reductions in either the asset-transformer's inventory or capacity, or the broker's capacity result in unambiguous increases in the marginal cost of lending. These cost increases are the basis for stricter credit standards and reduced lending. Thus, exogenous capacity reductions have the same impact on lending as do the asset-transformer's adaptation towards the broker's mode of operation.

III. Conclusion

This paper is part of an effort to model the adaptive behavior of financial intermediaries. In Deshmukh, Greenbaum, and Kanatas [5], we determined the intermediary's optimal funding practice, linking financial market conditions to the intermediary's choice of risk exposure. In this paper, we have examined the effect of funding risk on the intermediary's spot lending behavior.

We described the intermediary's lending behavior as an adaptive process involving uncertainty regarding the quantity of loan demand as well as the credit risk of heterogeneous loan applicants. In financing loans, the intermediary may borrow funds only as needed, thereby functioning as a stylized broker, without sustaining funding risk. Alternatively, it can borrow with a possible cost advantage in advance of realizing demand, but the intermediary then accepts the risk associated with an inventory and it thereby performs the role of an asset-transformer. Within this framework of credit and funding risks, we characterized the intermediary's optimal lending policy and showed how it is affected by its funding practices. If intermediaries are impelled to reduce their funding risk, credit standards will be tightened and lending will be reduced. Thus, the intermediary's funding and credit risk, often viewed as independent sources of uncertainty, are seen to be systematically linked. Moreover, if inflation and consequent turbulent financial markets prompt intermediaries to alter their funding risk, the allocation of credit will be affected by inflation.

REFERENCES

1. E. Baltensperger. "Alternative Approaches to the Theory of the Banking Firm." *Journal of Monetary Economics* 6 (January 1980), 1–37.
2. D. P. Baron and B. Holmstrom. "The Investment Banking Contract for New Issues Under Asymmetric Information: Delegation and the Incentive Problem." *Journal of Finance* 35 (December 1980), 1115–58.
3. P. Boltz and T. S. Campbell. "Innovations in Bank Loan Contracting: Recent Evidence." *Staff Studies* 104 (1979), Board of Governors of The Federal Reserve System.
4. D. Diamond and P. Dybvig. "Bank Runs, Deposit Insurance and Liquidity." *Journal of Political Economy.*
5. S. D. Deshmukh, S. I. Greenbaum, and G. Kanatas. "Interest Rate Uncertainty and the Financial Intermediary's Choice of Exposure." *Journal of Finance* 38 (March 1983), 141–147.
6. ———. "Bank Forward Lending In Alternative Funding Environments." *Journal of Finance* 37 (September 1982), 925–40.
7. M. J. Flannery. "Market Interest Rates and Commercial Bank Profitability: An Empirical Investigation." *Journal of Finance* 36 (December 1981), 1085–1101.
8. J. G. Gurley and E. S. Shaw. *Money in a Theory of Finance.* The Brookings Institution, 1960.
9. D. M. Jaffee and T. Russell. "Imperfect Information, Uncertainty and Credit Rationing." *Quarterly Journal of Economics* 90 (November 1976), 650–65.
10. J. Niehans. *The Theory of Money.* Baltimore MD: The Johns Hopkins University Press. 1978.
11. J. E. Ricart i Costa and S. I. Greenbaum. "Bank Forward Lending: A Note." *Journal of Finance,* 38 (September 1983), forthcoming.
12. J. Stiglitz and A. Weiss. "Credit Rationing in Markets with Imperfect Information." *American Economic Review* 71 (June 1981), 393–410.
13. A. M. Wojnilower. "The Central Role of Credit Crunches in Recent Financial History." *Brookings Papers on Economic Activity* (2:1980), 277–326.

APPENDIX

A. *Proof of Lemma 1*

Consider a sequence of artifically truncated problems in which no more than n customers are permitted to apply for loans, where $n = 0, 1, 2, \ldots$ We will show by induction on n that Lemma 1 holds for the optimal value functions in each of these truncated problems, so that in the limit, as $n \to \infty$, the desired properties will also hold in the original problem, wherein an unlimited number of applicants is permitted.

Let $V_n(t, L, \rho)$ denote the maximum expected profit over $[t, T]$ when the cumulative loan size is L, a customer with the characteristic ρ has arrived at time t and at most n more arrivals are permitted over $[t, T]$. Similarly, let $U_n(t, L)$ be the maximum expected profit from the next customer onwards if at most n more arrivals could eventually occur until T and the current state is (t, L). Then, analogous to (2.3) and (2.4), we may derive the following recursive relationships for $t \in [0, T]$, $L < \bar{L}$, and $n = 1, 2 \ldots$

$$V_n(t, L, \rho) = \mathrm{Max}\{\rho + U_n(t, L + 1), U_n(t, L)\} \tag{A.1}$$

where

$$U_n(t, L) = \int_0^{T-t} \int_{-1}^{\infty} V_{n-1}(t + s, L, \rho) f(\rho) \lambda e^{-\lambda s}\, d\rho\, ds - C(L) e^{-\lambda[T-t]} \tag{A.2}$$

with

$$V_0(t, L, \rho) = \mathrm{Max}\{\rho - C(L + 1), -C(L)\} \tag{A.3}$$

and

$$U_0(t, L) = -C(L) \tag{A.4}$$

As in (2.6), we may define for all $t \in [0, T]$, $L < \bar{L}$ and $n = 0, 1, 2, \ldots,$

$$\rho_n^*(t, L) = [U_n(t, L) - U_n(t, L + 1)] \tag{A.5}$$

so that the optimal strategy δ_n^* when at most n more arrivals could occur specifies $\delta_n^*(t, L, \rho) = 1$ if and only if $\rho \geq \rho_n^*(t, \mathrm{L})$. We shall prove, by induction on n, that Lemma 1 regarding V_n and U_n, and hence Proposition 1 regarding δ_n^* and ρ_n^*, hold for each $n = 0, 1, 2, \ldots$. Details are omitted for the sake of brevity; they are available from the authors upon request.

If $n = 0$, then $U_0(t, L)$ is independent of t and concave nonincreasing in L (since $C(L)$ is nondecreasing and convex in L) and hence satisfies (3.3), (3.5), and (3.7) with U replaced by U_0.

Now suppose that, for some n, $U_n(t, L)$ is nonincreasing in t, concave nonincreasing in L and complementary in (t, L). We show that $V_n(t, L, \rho)$ is nonincreasing and complementary in (t, L), nondecreasing in ρ, and concave in L. Monotonicity of V_n in ρ follows trivially from (A.1). Also from (A.1), it is clear that $V_n(t, L, \rho)$ is nonincreasing in t and L since $U_n(t, L)$ has this property. To show concavity we need to show that

$$[V_n(t, L-1, \rho) - V_n(t, L, \rho)] \leq [V_n(t, L, \rho) - V_n(t, L+1, \rho)] \quad \text{(A.6)}$$

We prove this by examining all possible expressions for V_n in terms of U_n and by exploiting the monotonicity and concavity properties of U_n, namely that

$$0 \leq [U_n(t, L-1) - U_n(t, L)] \leq [U_n(t, L) - U_n(t, L+1)] \quad \text{(A.7)}$$

This analysis is considerably simplified by utilizing the structure of the optimal policy δ_n^* as given in Proposition 1 which holds by monotonicity and concavity of $U_n(t, L)$ in L. Thus, as in Proposition 1, $\delta_n^*(t, L, \rho)$ is nonincreasing in L. Therefore, $\delta_n^*(t, L+1, \rho) = 1$ implies $\delta_n^*(t, L, \rho) = 1$, which in turn implies that $\delta_n^*(t, L-1, \rho) = 1$. Similarly, $\delta_n^*(t, L-1, \rho) = 0$ implies $\delta_n^*(t, L, \rho) = 0$ and $\delta_n^*(t, L+1, \rho) = 0$. Therefore, in order to prove (A.6) we need to consider only the four following cases; others are ruled out since they contradict the above structure of δ_n^*.

Case (i): $\delta_n^*(t, L+1, \rho) = \delta_n^*(t, L, \rho) = \delta_n^*(t, L-1, \rho) = 1$

Case (ii): $\delta_n^*(t, L+1, \rho) = 0, \quad \delta_n^*(t, L, \rho) = \delta_n^*(t, L-1, \rho) = 1$

Case (iii): $\delta_n^*(t, L+1\ \rho) = \delta_n^*(t, L, \rho) = 0, \quad \delta_n^*(t, L-1, \rho) = 1$

Case (iv): $\delta_n^*(t, L+1, \rho) = \delta_n^*(t, L, \rho) = \delta_n^*(t, L-1, \rho) = 0$

In each of the four cases it can be verified that (A.1) together with (A.7) imply (A.6).

Next, we show that $V_n(t, L, \rho)$ is complementary in (t, L), i.e., that $[V_n(t, L, \rho) - V_n(t, L+1, \rho)]$ is nonincreasing in t, using complementarity of $U_n(t, L)$ in (t, L). Again, monotonicity of $\delta_n^*(t, L, \rho)$ enables us to restrict ourselves only to the following three cases.

Case (i): $\delta_n^*(t, L+1, \rho) = \delta_n^*(t, L, \rho) = 1$

Case (ii): $\delta_n^*(t, L+1, \rho) = 0, \quad \delta_n^*(t, L, \rho) = 1$

Case (iii): $\delta_n^*(t, L+1, \rho) = \delta_n^*(t, L, \rho) = 0$

and it can be verified that in each case, (A.1) together with complementarity of $U_n(t, L)$ implies complementarity of $V_n(t, L, \rho)$.

Thus, we have shown that if $U_n(t, L)$ is nonincreasing and complementary in (t, L) and concave in L then $V_n(t, L, \rho)$ also has these properties. Now, as in (A.2) we have

$$U_{n+1}(t, L) = \int_0^{T-t} \int_{-1}^{\infty} V_n(t+s, L, \rho) f(\rho) \lambda e^{-\lambda s}\, d\rho\, ds - C(L) e^{-\lambda[T-t]} \quad \text{(A.8)}$$

which can also be seen to have these properties. In summary, by induction on n, it can be shown that, for each $n = 0, 1, 2, \ldots,$ $V_n(t, L, \rho)$ and $U_n(t, L)$ are nonincreasing and complementary in (t, L) and concave in L.

Clearly, permitting more applicants (and hence more decision opportunities) cannot reduce the optimum net profit, so that $V_n \leq V_{n+1}$ and $U_n \leq U_{n+1}$ for all n. Letting $n \rightarrow \infty$ we get $V_n \uparrow V$ and $U_n \uparrow U$, where V and U are the optimal value functions defined in the original problem. Since the properties of V_n and U_n that are derived above continue to hold in the limit as $n \to \infty$, we have shown that V and U have the desired properties claimed in Lemma 1.

B. Proof of Lemma 2

As above, consider the sequence of problems in which no more than n applicants are permitted loan considerations. Now, for $L < \bar{L}$,

$$U_0^A(t, L) = -C_A(\bar{L}) \quad \text{and} \quad U_0^B(t, L) = -C_B(L)$$

so that

$$\begin{aligned} &[U_0^A(t, L) - U_0^A(t, L + 1)] \\ &= 0 \le [C_B(L + 1) - C_B(L)] \\ &= [U_0^B(t, L) - U_0^B(t, L + 1)] \end{aligned}$$

Suppose that for some $n = 0, 1, 2, \ldots$, we have, analogous to (3.10),

$$[U_n^A(t, L) - U_n^A(t, L + 1)] \le [U_n^B(t, L) - U_n^B(t, L + 1)] \tag{A.9}$$

so that, as in (A.5), the critical rates of return for A and B, with at most n more applicants permitted, satisfy

$$\begin{aligned} \rho_n^{B*}(t, L) &= [U_n^B(t, L) - U_n^B(t, L + 1)] \\ &\ge [U_n^A(t, L) - U_n^A(t, L + 1)] = \rho_n^{A*}(t, L) \end{aligned} \tag{A.10}$$

which is analogous to (3.11). Now we employ (A.9) and (A.10) to show, as in (3.9), that for $L < \bar{L}$,

$$[V_n^A(t, L, \rho) - V_n^A(t, L + 1, \rho)] \le [V_n^B(t, L, \rho) - V_n^B(t, L + 1, \rho)] \tag{A.11}$$

To prove (A.11), we need to consider various cases, depending upon the value of ρ in relation to ρ_n^{A*} and ρ_n^{B*} with (t, L) and $(t, L + 1)$. From monotonicity of ρ_n^{A*} and ρ_n^{B*} in L and (A.10), we have the following relationships:

$$\rho_n^{B*}(t, L + 1) \ge \rho_n^{A*}(t, L + 1) \ge \rho_n^{A*}(t, L)$$

$$\rho_n^{B*}(t, L + 1) \ge \rho_n^{B*}(t, L) \ge \rho_n^{A*}(t, L)$$

Therefore, we have either

$$\rho_n^{B*}(t, L + 1) \ge \rho_n^{A*}(t, L + 1) \ge \rho_n^{B*}(t, L) \ge \rho_n^{A*}(t, L)$$

or

$$\rho_n^{B*}(t, L + 1) \ge \rho_n^{B*}(t, L) \ge \rho_n^{A*}(t, L + 1) \ge \rho_n^{A*}(t, L)$$

and we need only to consider the following six cases, depending on the value of ρ in relation to the above inequalities.

Case (i): $\quad \rho_n^{B*}(t, L + 1) \ge \rho_n^{A*}(t, L + 1) \ge \rho_n^{B*}(t, L) \ge \rho_n^{A*}(t, L) \ge \rho$

or

$$\rho_n^{B*}(t, L + 1) \ge \rho_n^{B*}(t, L) \ge \rho_n^{A*}(t, L + 1) \ge \rho_n^{A*}(t, L) \ge \rho$$

Case (ii): $\rho_n^{B*}(t, L+1) \geq \rho_n^{A*}(t, L+1) \geq \rho_n^{B*}(t, L) \geq \rho \geq \rho_n^{A*}(t, L)$

or

$$\rho_n^{B*}(t, L+1) \geq \rho_n^{B*}(t, L) \geq \rho_n^{A*}(t, L+1) \geq \rho \geq \rho_n^{A*}(t, L)$$

Case (iii): $\rho_n^{B*}(t, L+1) \geq \rho_n^{A*}(t, L+1) \geq \rho \geq \rho_n^{B*}(t, L) \geq \rho_n^{A*}(t, L)$

Case (iv): $\rho_n^{B*}(t, L+1) \geq \rho_n^{B*}(t, L) \geq \rho \geq \rho_n^{A*}(t, L+1) \geq \rho_n^{A*}(t, L)$

Case (v): $\rho_n^{B*}(t, L+1) \geq \rho \geq \rho_n^{A*}(t, L+1) \geq \rho_n^{B*}(t, L) \geq \rho_n^{A*}(t, L)$

or

$$\rho_n^{B*}(t, L+1) \geq \rho \geq \rho_n^{B*}(t, L) \geq \rho_n^{A*}(t, L+1) \geq \rho_n^{A*}(t, L)$$

Case (vi): $\rho \geq \rho_n^{B*}(t, L+1) \geq \rho_n^{A*}(t, L+1) \geq \rho_n^{B*}(t, L) \geq \rho_n^{A*}(t, L)$

or

$$\rho \geq \rho_n^{A*}(t, L+1) \geq \rho_n^{B*}(t, L) \geq \rho_n^{A*}(t, L+1) \geq \rho_n^{A*}(t, L)$$

In each case it can be verified that (A.9) implies (A.11). Then by definition of U_{n+1}^A and U_{n+1}^B as in (A.2), we have, for $L < \bar{L}$,

$$\begin{aligned}
&[U_{n+1}^A(t, L) - U_{n+1}^A(t, L+1)] \\
&= \int_0^{T-t} \int_{-1}^{\infty} [V_n^A(t+s, L, \rho) - V_n^A(t+s, L+1, \rho)] f(\rho) \lambda e^{-\lambda s} \, d\rho \, ds \\
&\leq \int_0^{T-t} \int_{-1}^{\infty} [V_n^B(t+s, L, \rho) - V_n^B(t+s, L+1, \rho)] f(\rho) \lambda e^{-\lambda s} \, d\rho \, ds \\
&\qquad - [C_B(L) - C_B(L+1)] e^{-\lambda[T-t]} \\
&= [U_{n+1}^B(t, L) - U_{n+1}^B(t, L+1)]
\end{aligned}$$

where the inequality follows from (A.11) and that the broker's marginal cost of funds is nonnegative. Thus, (A.9) holds with n replaced by $(n+1)$. Therefore, by induction, we have proved (A.9) and (A.11) for all $n \geq 0$. Taking the limits as $n \to \infty$ in (A.9) and (A.11), we have the desired results (3.9) and (3.10). From this, as in (A.10) we also have (3.11).