the tax advantage of municipals for financial institutions are directly testable. Data exists on taxes paid by banks and insurance companies which would allow calculation of the covariance between interest rates and the tax rates of these financial institutions. I would also suggest broadening the scope to include explicit consideration of the tax trading hypotheses of Constantinides/Ingersoll (1983) which may help explain observed differences in term structures of interest rates between municipals and corporates. And since both these arguments examine demand induced price effects, some thought should be given to insure that there are no confounding effects from the supply side.

Second, some care in interpretation should be exercised in going from Table 2 to Table 3. Table 2 characterizes the distribution of returns to individual municipal bonds while Table 3 compares risk characteristics of municipal and corporate bonds by examining the returns to portfolios. Although the variances of the returns to the portfolios are similar, this fact is not sufficient to conclude that the distribution characteristics of individual bonds are similar. For example, the average municipal bound variance might be less than the average corporate bond variance but the covariance among returns to municipal bonds be higher than that for corporates, yielding similar portfolio variances. Thus, it would seem instructive to examine corporate and municipal bond return distributions at a similar level of disaggregation.

Third, measured risk differences could simply reflect differences in characteristics unrelated to tax status. Skelton must rely on a bond rating service to pick out issues of similar risk. It would seem a strong argument that Moody's corporate and municipal bond ratings are comparable both across as well as within manuals. Thus Skelton does not provide the most powerful test of the hypothesis that "differences in tax status lead to differences in risk characteristics". The ideal test would compare tax exempt industrial revenue bonds with taxable corporate bonds secured by the same underlying cash flows.

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DISCUSSION

MICHAEL R. GIBBONS*: There are very few empirical studies on the stochastic characteristics of interest rates. Given the economic significance of bond prices, this paper provides some very useful results. Particularly important is the authors' emphasis on temporal aggregation issues. In other words, what would be the effects of observing interest rates with a sampling interval which differs from the underlying time interval of the true stochastic process? The implications of such temporal aggregation need to be explored if a tight linkage between theoretical models and empirical work is to be established. Many of the important models

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of financial economics are based on continuous time; yet, the empiricist must rely on discrete sampling intervals. Very little attention has been given to this discrepancy.

The beauty of continuous time modeling not only stems from its mathematical tractability, but it also forces the researcher to recognize the difference between the sampling and the underlying trading intervals. Theoretical models based on discrete time do not necessarily avoid temporal aggregation problems, but it is somehow easier for the empiricist to assume that the discrete trading interval corresponds to the sampling interval without thinking about the ramifications of such an assumption, if incorrect.

While the temporal aggregation issue is quite important, the technical complications that arise as a result force the authors to de-emphasize the economic significance of their work. Fitting a particular distribution to the data is a worthy objective, but selecting among alternative distributional assumptions requires some indication of the eventual use of the distribution which is chosen. No tractable distribution will ever fit the data perfectly, and an economic metric is required to select the appropriate assumption given the eventual application. For example, if the goal of the paper is to provide some stylized facts to guide theorists in building models of the term structure, the stochastic properties of real, not nominal, interest rates seem more relevant to me. Yet, all the empirical results in this paper are based on nominal interest rates.

Another possible objective is to extract forecasts of inflation from nominal interest rates. If this is the goal, then the authors should select that parameterization which provides a superior prediction about inflation. In the work by Fama and Gibbons [3], useful inflation forecasts from interest rates were based on a model which implied that the real interest rates on 30-day Treasury Bills were generated by an integrated moving average with order (1, 1) (see Box and Jenkins *[1] for a discussion of these models). Such a time series model* implies an infinite order autoregressive scheme, and it is not nested within the stochastic process considered by the authors. Nevertheless, the IMA(1, 1) model fits the data well in that it produces useful inflation forecasts.

The authors do establish one important stylized fact from their data analysis; interest rates do exhibit heteroscedasticity. Such a conclusion emerges from just an examination of time series plots of the data. The implications from these graphs are supported by some formal tests of heteroscedasticity although the statistical significance of these statistics in finite samples is unknown given the apparent non-normality of the data. In future research it would be interesting to compare models for the heteroscedasticity which rely on a one-time discontinuous shift in the variance (e.g., pre-1970 versus post-1970) and models in which variance is a function of the level of the interest rate. I suspect the data would be completely consistent with both approaches; yet, the implications of each are totally different.

One discouraging conclusion which emerges from the tables is the inability to measure precisely the parameters of the autoregressive component. The paper would have a greater impact if the authors could demonstrate that empirical results are drastically affected when stochastic differential equations are naively

approximated by difference equations. Unfortunately, the standard errors in Tables 1 and 2 are so large that no conclusion can be made on this issue.

This imprecision in the estimates leads me to believe that more structure will have to be added for a clear understanding of the stochastic properties of interest rates. One way to impose this additional structure is by using models of the entire term structure. Explicit closed form solutions for bond prices are available from the work of Cox, Ingersoll, and Ross [2]. Relying on such models of bond prices for all maturities has several advantages. First,the temporal aggregation issue is avoided since the models yield specific implications for bond prices at any point in time. Secondly, as in the option literature on implied standard deviations, one can observe bond prices and invert for the parameters of the diffusion model for the short-term rate as well as the state variables themselves (see Gibbons and Ramaswamy [4]). By inverting for the state variables (usually thought of as the short-term real rate and inflation), some measurement error problems can be circumvented. For example, one need not assume that the term permium on a 30-day Bill is small enough so as to treat it as an appropriate rate on a bond with an instant to maturity. The authors recognize this measurement error problem, but they can only assume it away.

Of course, there is a disadvantage to studying the stochastic properties of short-term rates by relying on a model for the term structure. Inappropriate conclusions may result from any mis-specification in the theoretical term structure model. But, by taking explicit account of a term structure model, the empirical work gains not only more precision but also a framework to judge the appropriateness of the statistical model. That is, the fit of a particular stochastic process for the short-term rate can now be measured by its implications for the pricing of longer term bonds.

Let me conclude by commenting on some of the technical aspects of the maximum likelihood estimator for the diffusion model using data sampled at discrete intervals. The authors indicate that the conditional density given in equation (5) is not necessarily valid for some values of the parameter space. In fact, there are at least four rows in Table 2 where the estimates are in the region of the parameter space for which the conditional density is incorrect. For these cases, the results in Table 2 should be interpreted with caution. Furthermore, the small sample behavior of the estimators for this complicated likelihood function would be interesting to explore.

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