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TAXATION AND FINANCIAL DECISIONS

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The Interaction of Financing and Investment Decisions When the Firm has Unused Tax Credits††

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MUCH OF THE finance literature has assumed that firms are endowed with exogenous tax rates. Although these rates may be different from the full corporate tax rate because of tax losses, they are nevertheless assumed to be invariant with any individual investment or financing decision. In this paper, we describe and compare various mechanisms for exploiting the tax losses of the firm. These include both financial and real asset transactions. Given costs to financial transactions and an inelastic supply of real assets, we show that the "effective" tax rate for the firm with tax losses is less than the full tax rate and is endogenous to the firm's current and future set of real assets and financial transactions; the latter include financial leasing, acquisitions and lending decisions. As a result, the value of any asset can only be calculated simultaneously with the firm's optimal choice of both real assets and financial transactions.

In section one, we describe the equilibrium when tax motivated financial transactions are costless and unrestricted. In this case, all tax losses will be absorbed by tax-paying companies as long as aggregate corporate income is positive. In section two, we describe the costs and restrictions which prevent the achievement of this ideal. Section three converts the firm's real asset choice problem to an equivalent linear program. The main result is that the firm with tax losses should make choices as if income on projects is taxed when incurred and losses from projects receive tax rebates when incurred, but at "effective" tax rates which are specific to particular future states. The "effective" tax rates arise as shadow prices in the solution to the decision problem, and thereby are

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endogenous to the choice of real assets. In section four, we show how costly financial transactions constrain the firm's future tax position and interact with its real asset choice. In section five we develop an example involving two assets and two leases which illustrates the interaction of investment and financing decisions when the tax position is endogenous to the choice of real asset and lease transactions. The remaining sections then discuss the impact of uncertainty and some implications for public policy.

I. Equilibrium Without Restrictions and Costs to Tax Motivated Financial Transactions

Competitive Supply of Taxpaying Companies

Initially, we shall assume that taxpaying firms are large in number and are competitive in their supply of financial contracts. Assuming that aggregate corporate taxable income is positive, the gains to taxpaying firms from tax motivated transactions will be zero. For instance, one equilibrium in the market for corporate debt may be such that the interest rate after corporate taxes is equal to the (certainty equivalent) required return on equity; this is essentially Miller's [1977] equilibrium of debt and taxes. The equilibrium in the leasing market will be such that tax-motivated financial leases have a zero NPV to the taxpaying lessor, while in the acquisitions market prices will be set so that the company with the tax losses obtains all the tax benefits.

If there are no real resource costs or restrictions to at least one mechanism for transferring tax between companies, firms which are currently reporting tax losses (or expect to do so in the future) will always have access to transactions with positive net present value until they resume taxpaying. For example, they can issue equity and buy taxable income in the bond market, which will have a positive value as long as interest rates are greater than zero.

The equilibrium condition requires that all corporations expect to be taxpaying (or at least to have no unused tax losses) in all future periods. As a result, the appropriate capital budgeting rule is to evaluate all projects as if they are fully taxed with full rebate of tax on losses.

Competitive Supply of Non-Taxpaying Companies

The other extreme assumption is that all gains from tax motivated financial transactions accrue to taxpaying corporations. As a result, one equilibrium between the equity and bond markets would be such that the pre-tax interest rate is equal to the certainty equivalent return on equity; this is the M-M [1963] equilibrium.

With taxes as the only imperfection, no corporation pays taxes if it issues sufficient corporate debt to make interest charges always equal to taxable income from operations.[1] Interest charges provide a costless alternative mechanism for sheltering taxable income, so alternative tax shield substitutes have no value.

[1] This may require very high levels of leverage for firms with large taxable profits.

The capital budgeting rule requires that all projects should be evaluated on the basis of their pre-tax cash flows, using an unlevered equity required return as the discount rate.

II. Costs and Restrictions to Tax-Motivated Transactions

The obvious costs to tax-motivated financial transactions are the resources necessary to perform and monitor transactions. For example, lending can involve agency, moral hazard, and bankruptcy costs.[2] In addition, there may be less obvious costs if the gain to the company is obtained from a transaction which results in a wealth transfer from shareholders to debtholders. For instance, if a Miller equilibrium holds between the equity and bond markets then it is profitable to issue equity and buy taxable bonds to shelter tax losses. However, the change in capital structure may reduce the risk of any outstanding (risky) bonds and increase their value at the expense of equity-holders. If the wealth transfer is larger than the present value of the tax gain, the financial transaction will not be profitable for shareholders to undertake. Such potential wealth transfers may explain why highly levered corporations in financial distress, recently for example Chrysler Corporation, are unable to take full advantage of large tax losses. The wealth transfers would be avoided if the outstanding debt could be purchased at a price which did not reflect any of the value of the tax losses.

Another cost to such tax motivated transactions will be the costs of buying bonds and selling equity. Corporations that are taxpaying will all have a marginal tax rate τ, the full corporate tax rate, whereas firms that have tax losses will, prior to any financial market transactions, face an effective marginal tax rate τ^*, where $\tau > \tau^* \geq 0$. The effective tax rate reflects the present value of the delay in tax payments caused by the tax losses. The tax rate τ^* may vary across firms if their profitability and the size of their respective tax losses also vary. Corporations with effective tax rates less than τ will have an incentive to engage in tax-induced financial market transactions; the gain is proportional to $(\tau - \tau^*)$. However, such gains may be offset by transactions costs.

Let us assume there are costs c per dollar to issuing and buying bonds with a value B, and the interest rate is i; the transactions costs are assumed to be a constant proportion of the amount of debt issued. A company that is non-taxpaying for N periods can buy an N-period bond, and the profit P from this transaction is:

$$P = Bi\tau A_{N,r} - NBi\tau D_{N+1,r} - Bc$$

where $A_{N,r}$ is the N-period annuity factor at r
$D_{N,r}$ is the N-period discount factor at r.

The profit before costs is the difference between the present value of the tax shield to the corporation issuing the bond and the present value of the tax paid in period $N + 1$ by the corporation which has the tax loss and buys the bonds. The discount rate, r, is the shareholders' required rate of return, which, in

[2] These costs are analyzed by Jensen and Mecking (1976), Myers (1977) and Warner (1977).

Miller's world is equal to the after-tax interest rate on bonds ($r = i(1 - \tau)$). We assume that the tax loss can be carried forward indefinitely.

If the level of transactions costs c, and the discount rate r are given,then we may compute the breakeven number of periods of non-taxpaying, N^*, where:

$$A_{N^*,r} - N^* D_{N^*+1,r} = \frac{c}{i\tau}$$

Only if the company forecasts a non-taxpaying period greater than N^* will there be an incentive to buy bonds. Since the purchase of bonds requires cash, which will have to be obtained by selling other assets or issuing equity, these costs should be included in c. In the subsequent example we assume $\tau = 50\%$, $i = 20\%$ and $r = 10\%$. Using the previous equation, we show below the maximum non-taxpaying period for different levels of transactions costs.

	Level of Transactions Costs (c)		
	2%	4%	6%
N^* (years)	1*	3*	4*

*rounded to the nearest year

As well as the explicit costs of the transactions, there are restrictions placed by the IRS on what type of tax motivated transactions are permissible. Such quantity restrictions may lengthen the period of non-taxpaying. In the U.S.A. and the U.K. there are restrictions on the types of financial leases that can be undertaken. Also, line-of-business restrictions exist on the use of tax losses after mergers. There do not, however, appear to be significant restrictions on corporate holdings of debt. Furthermore, trade credit can be viewed as a tax transfer vehicle, as described by Brick and Fung [1982].[3]

III. Real Asset Choice

Given that there are costs to tax motivated financial transactions, and that the firm has tax losses as a result, the firm may have an incentive to accept projects which have a negative NPV on a fully taxed basis if such projects can shelter the tax losses of other projects. In this respect, the size of the negative NPV is analogous to the costs of financial transactions considered previously. In this section we confine ourselves to the real asset choice. Although this should be considered simultaneously with financial market transactions, for ease of exposition we treat the problem in two stages. We assume:

1. The firm makes asset choices and pays taxes at discrete annual intervals. The current time is zero, and the future is described by a set of states. The state index, s, includes the date on which the state occurs, so that time is subsumed in the state definition.

[3] Professor Auerbach has pointed out in his discussion of this paper that if there were quantity restrictions on tax motivated transactions some firms with tax losses may remain permanently non-taxpaying. However, such quantity restrictions would have to include all financial transactions and real asset opportunities. Even with such binding constraints, we suspect only firms with relatively large tax losses and relatively small investment opportunities would be affected, and they would probably be in financial distress.

2. Once a year the state of the world is revealed, the firm pays taxes on its income, and the firm makes its real asset choice.
3. The equity market is complete and efficient, so that there is a pricing function $\rho(s)$, which is the price at time zero of \$1 of after tax corporate cash flow to be received in state s.
4. The firm seeks to maximize its current value.
5. The firm has a set of current and future real investment opportunities that are perfectly divisible, but in limited supply.
6. The tax rule is that losses can be carried forward without limit, but not carried back.

We can express the value of the firm as the net present value of its pretax cash flows minus the present value of its stream of future tax payments. Since pre-tax cash flows from projects are independent of the tax position of the firm, we will summarize all these pre-tax cash flows in a net present value for each project at the time at which it is undertaken. Since these pre-tax cash flows are being included as a component of the after-tax cash flow stream, their present values should be computed at after-tax rates.

We use the following notation:

r, s indices of future states. The state definition includes the date at which the state occurs, and the history up to that date.
(i, s) the ith real asset opportunity which can be undertaken in state s.
$\rho(s)$ the price at time zero of \$1 of cash flow after corporate tax generated in state s.
$\rho(r, s) = \rho(s)/\rho(r)$ the price in state r of \$1 in state s.
τ the corporate tax rate.
$Y(s)$ the taxable income reported by the corporation in state s.
$V(i, s)$ the NPV in state s of all *pre-tax* cash flows which occur in state s and beyond associated with project (i, s).
V_F the value of the firm.
V_0 the present value at time zero of all pre-tax cash flows associated with the existing operations of the firm.
$x(i, s)$ the proportion of project (i, s) undertaken.

The firm chooses the amounts of current and future projects which it plans to undertake. In doing so, it seeks to maximize its value:

$$V_F = V_0 + \textstyle\sum_i \sum_s x(i, s)V(i, s)\rho(s) - \tau \sum_s \mathrm{Max}(Y(s), 0)\rho(s)$$

The second term on the RHS is the NPV of all pre-tax cash flows from projects aggregated over all states and reduced to a time zero value. The third term is the present value of all future tax payments aggregated over all future states. The tax rule allows any loss to be carried forward to the next tax period, so that we can define taxable income, $Y(s)$, recursively:

$$Y(s) = I_0(s) + \textstyle\sum_i \sum_r x(i, r)I(r, i, s) - F(s - 1)$$

where,

$I_0(s)$ is the taxable income generated in state s by the existing operations of the firm.

$I(r, i, s)$ is the taxable income generated in state s by project i undertaken in state r.
$s - 1$ is the state which occurs one year before state s.
$F(r) = \text{Max}(-Y(r), 0)$ is the tax-loss carry-forward from state r.
$F(0)$ is the tax loss in period zero.

In the Appendix we convert this choice problem to an equivalent linear program.[4] We can solve such a choice problem numerically for a specific case, and the dual and complementary slackness conditions demonstrate interesting general features of the optimal choice.

The main result is that the firm should make choices as though income on projects were taxed when incurred and losses from projects received rebates when incurred, but at tax rates $\tau^*(s)$ which are specific to particular future states. These effective tax rates arise as shadow prices in the solution to the decision problem, and therefore, are endogenous to the choices made by the firm.

The effective tax rates, $\tau^*(s)$, will satisfy two sets of consistency conditions. The first is:

$$\begin{aligned} &\tau^*(s) = \tau \text{ if taxable income of the firm is positive in state } s \\ &\tau^*(s) = \textstyle\sum_{r\in s+1} \tau^*(r)\rho(s, r) \text{ if taxable income of the firm is negative} \end{aligned} \tag{1}$$

where $s + 1$ is the set of states that occurs one year after state s. This condition defines recursively the effective tax rate for any state in which the firm optimally should have a tax loss. The condition can be interpreted when we recognize that $\tau^*(s)$ is the shadow price on an extra dollar of tax loss in state s. Since this loss can be carried forward, the expected return to the carry-forward must equal the equity required rate of return.

The second consistency condition is that the optimal set of projects, when chosen on the basis of the effective tax rates $\tau^*(s)$, must obey the rule:

$$\text{Accept project } (i, s) \text{ if } V(i, s) - \textstyle\sum_r \tau^*(r)I(s, i, r)\rho(s, r) > 0 \tag{2}$$

Since the effective tax rates arise from the solution to a simultaneous choice of current and future real investments, these asset choices interact though their impact on the tax payments made by the firm. If the firm has a perfectly elastic supply of real assets at a zero fully taxed NPV, this endogenous tax rate will be equal to τ. This is analogous to a perfectly elastic supply of zero cost tax transfer opportunities, which we discuss in the next section.

We can compare the real asset choice of the firm with the tax loss with that of a fully taxed corporation; the difference arises through the tax rate appropriate to the choice. If a firm has tax losses, and condition (2) holds for a project, it will invest more than fully taxed corporations if the project satisfies the condition:

$$V(i, s) - \tau \textstyle\sum_r I(s, i, r)\rho(s, r) \leq 0$$

For firms which expect to be temporarily non-taxpaying the incentive to invest in particular assets (relative to taxpaying firms) depends upon the amount of depreciation over the forecast period of non-taxpaying. Thus the impact of tax

[4] Berry and Dyson (1979) examine project interdependencies induced by the tax system using mixed integer programming as the project selection procedure.

losses on real investment depends upon the types of depreciation schedules and the length of the forecast non-taxpaying period to which the loss gives rise.

IV. Financial Transactions

The real asset opportunities available to the firm, when considered alone, may result in a very low effective tax rate. This will be the case if, with its optimal package of current and future real assets, the date at which the firm expects to resume paying taxes is far in the future.

In such circumstances, financial tax-transfer markets, such as lending, leasing and acquisition, may have positive net present values, net of all costs involved, when evaluated against the effective tax rates resulting from the optimal real asset choice. The firm will then undertake financial transactions until the effective tax rate is driven to the point where the most tax-efficient of these transactions has a net present value of zero. The supply of these potential financial transactions is perfectly elastic to the individual firm, whereas the real asset opportunities are assumed to be in inelastic supply. The exception to this is leasing, where only real assets may be leased and they are in inelastic supply.

We denote a tax-transfer opportunity in state s by (j, s). Associated with each opportunity is a set of pre-tax cash flows, a vector of taxable income generated, and a cost to each participant of the sort described in section two. If there is a competitive supply of financial tax-transfer opportunities such as acquisitions and lending transactions, the tax position of the firm with the carry-forward must satisfy, for all admissible transactions of this kind, the condition:[5]

$$\textstyle\sum_q (\tau - \tau^*(q))\rho(s, q)I(s, j, q) < c(j, s) \tag{3}$$

where: $I(s, j, q)$ is the taxable income generated in state q to the tax-loss firm, per dollar of transaction (s, j).

$c(j, s)$ is the total real resource cost to both parties per dollar of the transaction.[6]

Since all variables in this inequality other than $\tau^*(q)$ are given exogenously, the financial markets serve to place limits on the possible values taken by the implicit tax rate. The smaller the transaction costs $c(j, s)$, the closer the effective tax rate $\tau^*(q)$ is to the full corporate rate. Real asset opportunities are evaluated relative to these implicit tax rates, so tax motivated financial transactions constrain real asset choice.

Equation (3) places a limit on how low the effective tax rate can be in optimality. For leasing, the level of costs in the leasing market does not necessarily constrain the effective tax rate, since leasing is restricted to the real assets acquired. For leasing, equation (3) gives an optimal decision rule rather than a constraint on the effective tax rate. The opportunity to lease changes the real asset selection criterion, since the NPV of a real asset must now include the

[5] In a world of uncertainty, firms expecting tax losses may wish to acquire other firms whose taxable profits are imperfectly correlated with their own.

[6] The total cost here must include any loss of economic rents which result from the acquisitions. Assets where rents are not specific to their operator will therefore, be more efficient transfer vehicles.

value of the most profitable lease that can be written against it. The value of the effective tax rate to use in evaluating the lease will be given by equation (3) for the most efficient tax-transfer market. The criterion for using leasing as a tax-transfer vehicle depends upon the cost of leasing, $c(j, s)$, relative to the lease's ability to increase taxable income through the sale of tax depreciation. Evaluation of the lease implicitly involves a comparison with the alternative tax-transfer vehicle that is most tax-efficient per unit of cost.

V. Analysis Under Certainty: An Illustration

In this section, we provide an example of a new firm with two prospective assets, each of which may be purchased for cash or leased. The characteristics of the two projects and two leases are described in Table 1. The example demonstrates the following principles: 1) the firm's tax position is endogenous to the choice of assets purchased or leased, and the firm's tax position changes with its choice; 2) the value of the firm is not equal to the sum of the values of the assets evaluated individually; and 3) the value of the lease-purchase decision can only be obtained when all other asset and financing choices have been evaluated and the optimum solution calculated.

We have chosen the purchase-lease case because all the literature on financial leasing assumes that the "effective" after tax interest rates are exogenous to the decision to lease or purchase. For a firm that is only temporarily non-taxpaying, such an assumption must be unrealistic either for an individual lease or a combination of leases. The assumption that the tax rates are exogenous may lead to an over-valuation of the lease as we demonstrate in our example.[7]

In Table 2, we have summarized the NPV's of the individual asset and lease decisions and particular combinations. As expected, the NPV of purchase of asset A (−4.68) plus the NPV of purchase of asset B (5.82) does not equal the NPV of purchase of assets A and B combined (3.30). The reason is that the tax losses which occur in particular periods for one asset are in part sheltered by the taxable profits of the other asset. The NPV's of the individual leased assets can be added to obtain the NPV of the combinations only because the firm is taxpaying in all periods if it acquires each asset via a lease.

In Table 3, we have provided the set of cash flows relating to the leases of assets A and B combined. Using the loan balance method described by Franks and Hodges (1978), we have determined the loan in period zero that would provide a pattern of repayments, interest and taxes that would equal the net cash flows of the two leases combined in each subsequent period.[8] The initial loan represents the NPV of the lease decision.

In Table 4 we have provided the set of cash flows relating to the purchase of assets A and B combined. An initial loan is calculated (by iteration) with a

[7] Discussions with Stewart Hodges were helpful in specifying this problem.

[8] Note that we have compared leasing with buy and borrow. Such a comparison is appropriate in an M-M world (see Myers, Dill and Bautista (1976)). Brealey and Young (1980) argue that, in a Miller world, leasing should be compared with equity financing, since debt is more expensive than equity for a firm with a tax loss.

Table 1

Project Characteristics

Asset	Capital Cost	Life (years)	Annual Net Operating Cash Flows	Annual Lease Payment	Pre Tax Borrowing Rate (%)	Tax Depreciation (First Year)
A	50	5	12.572	11.530	12.0	100%
B	50	3	24.914	18.169	12.0	100%

Table 2

Summary of Net Present Values

	Lease	Purchase	Purchase Minus Lease
Asset A alone	2.19	(4.68)	(6.87)
Asset B alone	9.01	5.82	(3.19)
Assets A + B combined	11.21	3.30	(7.91)

Table 3

Cash Flows Associated With Acquiring Both Assets A and B With a Lease

Purchase Cashflows: Period	0	1	2	3	4	5
Net Operating Cash Flows		37.49	37.49	37.49	12.57	12.57
Rentals		−29.70	−29.70	−29.70	−11.53	−11.53
		7.79	7.79	7.79	1.04	1.04
Taxes (50%)		−3.89	−3.89	−3.89	−0.52	−0.52
		3.89	3.89	3.89	0.52	0.52
Loan Balance at 12% pre-tax	11.21	7.99	4.57	0.95	0.49	0.00
NPV = 11.21						

Table 4

Cash Flows Associated With Acquiring Both Assets A and B With Cash

Purchase Cashflows: Period	0	1	2	3	4	5
Net Operating Cashflows		37.49	37.49	37.49	12.57	12.57
Purchase Cost	−100.00					
Loan	103.30					
Repayment		−25.09	−28.11	−31.48	−10.34	−8.28
Interest		−12.40	−9.38	−6.01	−2.23	−1.00
Tax						−3.29*
Loan Balance	103.30	78.21	50.10	18.62	8.28	0.00
Tax Loss	100.00	74.91	46.81	15.33	5.00	−6.57
NPV = 3.30						

* This tax payment is 50% of the taxable profit in year 5 after the deduction of the tax loss carry-forward.

pattern of repayments, interest and taxes, such that the net cash flows of purchase are equal to zero in each period (except for period zero). The NPV of purchase is the difference between the initial loan and the purchase cost of the assets.

In previous papers on leasing and taxes by Franks and Hodges (1978) and Myers, Dill and Bautista (1976), it was assumed that the lease-purchase choice does not alter the periods in which the firm does or did not pay taxes. In our analysis, the tax position is endogenous to the choices made. As a result, the failure to identify all current and future asset decisions may overstate the value of the lease, as it would in our example, if asset A and asset B were considered separately (see the third column of Table 2).

Our example is only illustrative since we have assumed that the most tax efficient mechanism is the purchase of new assets with leasing. We should also compare the profitability of leasing new assets with the marginal acquisition decision, for example.

VI. Uncertainty

The example we have given in section five has assumed a world of certainty to illustrate the importance of the endogeneity of the tax position of the firm for the lease versus buy decision. In principle, solutions to particular choice problems under uncertainty could be derived using the linear programming approach of the Appendix (or an equivalent integer programming approach if there are indivisibilities in the choices).

In much of our analysis we have assumed that tax motivated financial transactions are in competitive supply at the full corporate tax rate. However, in a world of uncertainty this may not be the case. Several authors including De Angelo and Masulis (1980) have raised the question of whether, for instance, the Miller equilibrium is sustainable when the world is uncertain and debt is risky. However, even if the equilibrium between the debt and equity markets is at a tax rate below the full corporate tax rate, there will still be an incentive to issue equity and to lend until the effective tax rate for that firm is raised to the equilibrium tax rate prevailing between the debt and equity markets (after adjusting for the real resource cost of the transaction).

The second impact of uncertainty is that it makes the effective tax rate vary across states. If transactions are performed before the firm knows whether it will be paying taxes in a particular tax period, that uncertainty will reduce the value of the transactions. If, however, transactions can be performed at the very end of the relevant tax period, when the firm knows its tax position without the transaction, then the efficiency of the tax-transfer will be greater.

Such last-minute transactions are not prevented by regulation, but may be expensive in terms of their costs relative to the tax benefits achieved. For instance, it may be unprofitable to enter into last-minute lending transactions since the income generated during the remainder of the tax year per dollar of transaction will be small. Without costs, however, these transactions would eliminate tax losses. We can, therefore, explain certain phenomena, such as the fact that some companies which pay taxes do not write financial leases, both by the uncertainty about their future tax position, and by the costs which prevent them making last minute manipulations of their tax position each year to eliminate any tax losses.

VII. Conclusions and Implications

Differences in tax positions of corporations arise not because they are exogenously endowed with different tax rates but because some are making tax losses and others generate taxable income. The behaviour of corporations with tax losses can be described as if they were subject to effective tax rates which are lower than the full corporate tax rate. These rates depend upon the length of time for which the firm expects to have a tax loss carry-forward. They cannot, however, take on arbitrarily low values since there are opportunities to increase taxable income through lending, leasing and acquisition markets.

The endogeneity of the effective tax rate for corporations with tax losses has implications for real asset decisions and the evaluation of such financial market transactions as leasing. For instance, leasing rather than purchasing assets for cash may change the tax position of the firm and the valuation decisions should include that impact.

Such tax losses can persist only if there are costs or restrictions to tax motivated transactions. There is a variety of costs and restrictions which limit the operation of these markets, and the nature of these costs determines what types of firms will have persistent tax losses, and what the equilibrium prices will be in the tax-transfer markets.

There are some implications for public policy that arise from our analysis. First, if firms have tax losses and find it profitable to invest in real assets which have negative NPV's on a fully taxed basis (because they are prevented or find it too costly to use financial markets for transferring tax losses between companies), real investment will be distorted as a result. If this were the case, the government should consider whether it is preferable to provide easier access to financial markets for transferring tax losses between companies. A second implication is for competition between firms. In our world, if the supply curve of investment opportunities is highly inelastic then it will be very costly for the firm to use real assets to absorb tax losses. Alternatively, tax-paying firms may be in a position to accept projects which non-taxpaying firms cannot. Since tax losses are so persistent and pervasive in the U.S. and U.K. economies, distortion of real asset markets may be persistent and require policy consideration by government.[9]

Appendix

Appendix Optimal Real Asset Decisions

To demonstrate the solution to the choice problem, we convert it to an equivalent linear program. We define $T(s)$ as the taxable income on which tax is

[9] DeAngelo and Masulis report that 27% of U.S. firms are non-taxpaying in any one year, while the equivalent figure is 30% in the U.K. (see U.K. White Paper (1982)).

actually paid in state s. Tax paid is therefore $\tau T(s)$. We allow $T(s)$ to be a choice variable, although it will always be chosen to be as low as possible. The choice problem is then equivalent to:

$$\text{Max } \textstyle\sum_i \sum_s x(i, s)V(i, s)\rho(s) - \tau \sum_s T(s)\rho(s)$$

$$\begin{aligned} \text{Subject to: } & x(i, s) \leq 1 \\ & Y(s) - T(s) \leq 0 \\ & x(i, s) \geq 0 \\ & T(s) \geq 0 \end{aligned}$$

The definition of $Y(s)$ is:

$$Y(s) = -T(s-1) + Y(s-1) + I_0(s) + \textstyle\sum_i \sum_r x(i, r)I(r, i, s)$$

since:

$$F(s-1) = T(s-1) - Y(s-1)$$

Iterating back to time zero gives:

$$Y(s) = -\textstyle\sum_{j=1}^{n(s)-1} T(s-j) - F(0) + \sum_{j=0}^{n(s)-1} I_0(s-j) + \sum_{j=0}^{n(s)-1} \sum_i \sum_r x(i, r)I(r, i, s-j)$$

where:

$s - j$ is the state which occurs j years before state s

$n(s)$ is the number of years before state s occurs.

The dual problem is:

$$\underset{\psi i(s), \phi(s)}{\text{Min.}} \textstyle\sum_i \sum_s \psi_i(s) + \sum_s \phi(s)[F_0 - \sum_{j=0}^{n(s)-1} I_0(s-j)]$$

subject to:

$$\begin{aligned} & \psi_i(s) + \textstyle\sum_q [\phi(q) + \sum_{r \in q^+} \phi(r)]I(s, i, q) \geq V(i, s)\rho(s) \\ & \phi(s) + \textstyle\sum_{r \in s^+} \phi(r) \leq \tau\rho(s) \\ & \psi_i(s) \geq 0 \\ & \phi(s) \geq 0 \end{aligned}$$

where $I(s, i, q) = 0$ if q occurs before s and, where, q^+ is the set of all states that follow q in any later year. We define $\tau^*(s)$ by:

$$\rho(s)\tau^*(s) = \phi^*(s) + \textstyle\sum_{r \in s^+} \phi^*(r)$$

The complementary slackness conditions are:

$$x_i^*(s)[V_i(s)\rho(s) - \psi_i^*(s) - \textstyle\sum_q \rho(q)\tau^*(q)I(s, i, q)] = 0$$

$$T^*(s)[\tau\rho(s) - \rho(s)\tau^*(s)] = 0$$

Then

$$T^*(s) > 0 \text{ implies } \tau^*(s) = \tau$$

$$T^*(s) = 0 \text{ and } Y(s) < 0 \text{ implies } \tau^*(s) = \tau \textstyle\sum_{r \in s+1} \rho(s, r)\tau^*(r)$$

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