traded. We are led to believe that in the absence of these bills, the results obtained in the paper would not be assured. In his appendix, Solnik derives what he claims are the pricing relations in the absence of these bills. These differ from those reported in the main part of the paper. This we find troubling. For the APT to have any import it must be possible to mimic riskless bills with portfolios of risky assets. The theory should be the same whether the bills exist or not.

We now consider briefly the gains realized in making this extension of the APT. The general advantages of the APT over the CAPM are well rehearsed by now. The fact that the latter requires an aggregation of individual portfolios makes its application to a partially segmented international market impossible. The APT does not suffer in this regard. We need only require that within the segmented market investors have access to enough assets to construct well-diversified portfolios. What is interesting in the IAPT is the emergence of testable restrictions not found in the APT, namely the fact that the lambdas associated with asset pricing in one country have a particular relation to those governing the asset pricing in another [equation 10]. This relation is determined by the factor loadings associated with movement in the exchange rates and it may be that this relation holds with more precision than the pricing relations associated with particular assets. Again, a direct test of the particular restrictions imposed on the covariance matrix is warranted, with the usual caveat concerning joint tests of the model and its assumptions.

DISCUSSION

JORGE BRAGA DE MACEDO*: What determines the net foreign asset position of the residents of a particular country is an important question in international economics. Rene Stulz's paper seeks to answer it in a framework where two countries differ in their technology and asset preferences, but where they produce the same good. In the Ricardo example, Portugal and England trade in wine (or cloth) alone. This eliminates the role of the relative price of national outputs, the exchange rate and thus allows for the existence of an international riskless asset. It also severely reduces the international flavor of the model and is not convincingly justified. Note that complexity is hardly a reason since the author and others have successfully constructed related models with more than one good.[1] Several other simplifications, convincingly justified in the paper, need no further elaboration.[2]

One interesting feature of the model is the now popular assumption—introduced some years ago by Ray Fair—that the two countries are initially identical, so that we have a closed economy model à la Cox-Ingersoll-Ross. It turns out, however, that the results generated by Stulz's elegant model hold even when this initial symmetry is not imposed, except for one which requires cross-country borrowing in the safe asset. Before proceeding to prove this claim, reference should be made to the special case of Bernouilli utility solved in Section 4. Since

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[1] See in particular Bortz (1982) and Stulz (1982).

[2] Some of the issues in strategic behavior are explored in Eaton and Gersovitz (1982).

in that case the change in wealth is a decreasing function of the (constant) rate of time preference, symmetry is required for a steady-state to exist.

Unless otherwise stated, the author's notation is kept. Denote the investor by $i(= D, A)$ and the industry by $j(= I, F)$. Then the value of investment by investor i in industry j, V_{ij}, can be obtained from the first order conditions as in equation (6) of the paper, rewritten as:

$$V_{ij} = \frac{1}{\lambda_i \sigma_j^2} (\mu_j - R - \textstyle\sum_{l=1}^{S} \lambda_{il} \sigma_{jl}) \tag{1}$$

where $\lambda_i = -J^i_{ww}/J^i_w$ is the coefficient of risk aversion of investor i with respect to changes in wealth.

$\lambda_{il} = -J^i_{wl}/J^i_w$ is the coefficient of risk aversion of investor i with respect to changes in state variable $l = 1, \cdots, S$.

Note that wealth of investor i is obtained by summing over j, $W_i = \sum_j V_{ij}$, so that $\gamma_i = \lambda_i W_i$ is the relative risk-aversion of investor i. Net foreign investment in the country of investor i, N_i, is given by the total investments in industry j, $\sum_i V_{ij}$, less the wealth of investor i, so that the terms where $i = j$ cancel and we are left with the difference between foreign investments at home, V_{AI}, and domestic investments abroad, V_{DF}. Equation (10) of the paper can then be rewritten as:

$$N^D = -N^A = V_{AI} - V_{DF} \tag{2}$$

Define the mean return of investment in industry j, adjusted for the state-variable risk by investor i as:

$$\mu_{ij} = \mu_j - \textstyle\sum_{l=1}^{S} \lambda_{il} \sigma_{jl} \tag{3}$$

Similarly, the average return for industry j, weighted by the risk-aversion of the two investors, is defined as:

$$\mu_j^{\tilde{\lambda}} = \tilde{\lambda} \mu_{Aj} + (1 - \tilde{\lambda}) \mu_{Dj} \tag{4}$$

where

$$\tilde{\lambda} = \lambda_D/(\lambda_A + \lambda_D)$$

From (4) we see that the greater absolute risk-aversion of domestic investors, the lesser weight will be attached to the return on investments in the domestic industry. Finally, we specify average return to risky investments. Since returns are uncorrelated, weights reflect the relative variance of the two investments:

$$\mu = \tilde{\sigma} \mu_I^{\tilde{\lambda}} + (1 - \tilde{\sigma}) \mu_F^{\tilde{\lambda}} \tag{5}$$

where

$$\tilde{\sigma} = \sigma_F^2/(\sigma_I^2 + \sigma_F^2)$$

The greater the variance of foreign investment, the smaller its weight in the average return. Using (5) and the definition of world wealth $W = \sum_i \sum_j V_{ij}$ we obtain the equilibrium risk-free interest rate as the difference between average return on risky investments and "world" relative risk aversion times a variance

term. We thus rewrite equation (7) in the paper as:

$$R = \mu - \lambda\sigma W = \mu - \gamma\sigma \tag{6}$$

where

$\lambda = \tilde{\lambda}\lambda_A$

$\sigma = \tilde{\sigma}\sigma_I^2$

$\gamma = \tilde{\lambda}\gamma_A + (1 - \tilde{\lambda})\gamma_D$ being the "world" relative degree of risk-aversion.

It is clear from (6) that an increase in μ_F increases μ and thus R. The effect of an increase in σ_F^2 on R is ambiguous. It will lower it if $\mu_F^{\lambda} + \sigma_I^2\gamma > \mu_I^{\lambda}$, which will always be true in the symmetric case. It is also clear from (6) that an increase in world wealth lowers R. An increase in γ will also lower R if absolute risk aversions do not change, or in the symmetric case. The same is true of an increase in γ_D (or γ_A) and of an increase in foreign wealth relative to domestic wealth. To see this, express γ as:

$$\gamma = \gamma_A\gamma_D(1 + \omega)/(\gamma_A + \gamma_D\omega) \tag{7}$$

where

$$\omega = W_A/W_D$$

Now, using (3) in (1) and (2), it is evident that non-zero net foreign investment in the domestic economy requires that technologies or preferences be different across countries:

$$N^D = \frac{1}{\lambda_A\sigma_I^2}(\mu_{AI} - R) - \frac{1}{\lambda_D\sigma_F^2}(\mu_{DF} - R) \tag{8}$$

Substituting from (6) into (8), we get:

$$N^D = M - (1 - \tilde{\lambda} - \tilde{\sigma})W = \Delta + (\gamma\tilde{\sigma}/\lambda) - (\gamma/\lambda_D) \tag{9}$$

where

$$M = \frac{1}{\lambda_A\sigma_I^2}(\tilde{\mu}_I - \mu_F^{\sigma}) - \frac{1}{\lambda_D\sigma_F^2}(\tilde{\mu}_F - \mu_I^{\sigma})$$

$$\tilde{\mu}_I = (1 - \tilde{\lambda}\tilde{\sigma})\mu_{AI} + \tilde{\lambda}\tilde{\sigma}\mu_{DI}$$

$$\tilde{\mu}_F = [1 - (1 - \tilde{\lambda})(1 - \tilde{\sigma})]\mu_{DF} + (1 - \tilde{\lambda})(1 - \tilde{\sigma})\mu_{AF}$$

$$\mu_F^{\sigma} = \tilde{\sigma}\mu_{DI} + (1 - \tilde{\sigma})\mu_F^{\lambda}$$

$$\mu_I^{\sigma} = \tilde{\sigma}\mu_I^{\lambda} + (1 - \tilde{\sigma})\mu_{AF}$$

In the symmetric case, the coefficient on world wealth goes to zero and $\Delta = 0$. Net foreign investment will be positive when $\Delta > 0$ and $\tilde{\lambda}$ and $\tilde{\sigma}$ are small, so that domestic investors are not very risk averse and the variance of return on

foreign investment is not too large. Under these conditions, an increase in μ_F will tend to decrease Δ (by increasing $\tilde{\mu}_F$ and μ_F^{γ}) and thus N^D, as stated in the paper for the symmetric case (Result 1).

When returns are weakly correlated with state-variable changes, so that, as a first approximation, $\mu_i = \mu_{AJ} = \mu_{Dj} = \mu_j^{\lambda} = \tilde{\mu}_j$ for $j = I, F$ (and $G = 0$ in the paper), then M in (9) becomes inversely proportional to λ

$$\lambda M = \frac{\mu_I - \mu_F}{\sigma_I^2 + \sigma_F^2} = \tilde{M} \tag{10}$$

In this case, we can rewrite net foreign investment as:

$$N^D = \frac{\tilde{M}}{\lambda} - (1 - \tilde{\sigma} - \tilde{\lambda})W \tag{11}$$

The above condition for an increase in σ_F^2 to decrease R (which was weaker than symmetry) carries over to the increase in N^D, as can be seen by noting that σ_F^2 only appears in denominator of (11) and that the numerator reduces to $\gamma\sigma_I^2 > 0$ in the symmetric case.

Differentiating (11) with respect to $\tilde{\lambda}$ and making the symmetry assumption implies a one-to-one increase in N^D/W, contrary to Result 3 of the paper, according to which there is no effect. Result 3 thus requires cross-country borrowing as well as symmetry.

Finally, using (7) in (11) and differentiating with respect to ω, with world wealth constant, we have a condition for an increase in N^D which, while always true in the symmetric case (or for that matter the logarithmic utility case where $\gamma_A = \gamma_D = \gamma = 1$), is weaker than the condition for Result 4.

$$\tilde{M}(\gamma_D - \gamma_A) + \gamma^2 > 0 \tag{12}$$

In particular, even if foreigners are more risk averse than domestic residents and the return to foreign investments is higher $\tilde{M} < 0$, condition (12) will hold and an increase in foreign wealth still increases net foreign investment in the domestic industry.

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