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# What Lockbox and Disbursement Models Really Do

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## I. Introduction

MANAGEMENT of the firm's cash collection and disbursement systems is an important responsibility of the firm's treasurer. In fulfilling this responsibility, the treasurer's primary goal is to increase the amount of funds available for investment. On the cash collection side, this is frequently accomplished through the location of regional collection points known as lockboxes. Lockboxes increase the amount of funds available for investment by reducing the delay between the time the customer mails the check and the time funds are credited to the firm's account. On the disbursement side, the goal of increasing the amount of funds available for investment is achieved by disbursing funds from either "remote" or "control" locations. This serves to extend the delay between the time the firm mails the check and the time it is presented to the firm's bank for payment.

The fixed and variable costs of using bank provided lockbox and disbursement services are such that it is rarely profitable for a firm to have a lockbox or disbursement site in every city. Instead, the firm will work with a bank that does cash management studies to determine a smaller set of lockbox and disbursement locations that maximize the difference between total system benefits and costs.

A lockbox study consists of two parts. First, the study bank samples the firm's current remittances to determine the location of the firm's customers and the mail and availability times associated with the firm's current collection system. Second, the study bank designs an optimal lockbox system. If the float and cost savings of the new system are substantial, the firm will change its current system as recommended by the bank.

The problem of choosing the best set of lockbox sites is called the Lockbox Location Problem. This problem has been extensively studied by management scientists working in both academic and banking environments. Their work has produced a set of solution procedures used by most banks doing lockbox studies. Over the past seven years, the authors have been involved in the implementation of such solution procedures in over three dozen banks nationwide.

Disbursement studies are performed very much like lockbox studies. From a sample of the firm's checks, the bank estimates the average clearing times for the firm's current disbursing system. Using a database of nationwide clearing times and special optimization algorithms, the bank then determines the best set of disbursing banks as well as the assignment of vendors to these banks.

The problem of finding the best set of disbursing banks is called the

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Disbursement Bank Location Problem. Like the Lockbox Location Problem, this problem has been extensively studied in the literature. One of the formulations developed in this literature has been successfully implemented by the authors at a dozen banks.

This paper reviews the state of the art of lockbox and disbursement studies that banks perform for their corporate clients. In contrast to most academic papers on these subjects, we spend little time discussing optimization algorithms. Instead, we concentrate our attention on the issues involved in the practical implementation of lockbox and disbursement systems. In our opinion, banks who focus on these implementation issues offer far better cash management services to corporate clients than do those banks who worry about minor differences in algorithms.

## II. The Lockbox Location Problem

The Lockbox Location Problem can be described mathematically as follows:

$$\text{Minimize } \sum_{i=1}^{m}\sum_{j=1}^{n} A_{ij}X_{ij} + \sum_{j=1}^{n} F_jZ_j + \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}X_{ij} \tag{1}$$

$$\text{subject to } \sum_{j=1}^{n} X_{ij} = 1 \quad \text{for} \quad i = 1, \cdots, m \tag{2}$$

$$X_{ij} \leqslant Z_j \quad \text{for} \quad i = 1, \cdots, m; \qquad j = 1, \cdots, n \tag{3}$$

$$\sum_{j=1}^{n} Z_j \leqslant k \tag{4}$$

$$X_{ij} \geqslant 0 \quad \text{for } i = 1, \cdots, m; \qquad j = 1, \cdots, n \tag{5}$$

$$Z_j = 0 \text{ or } 1 \quad \text{for } j = 1, \cdots, n \tag{6}$$

where

$A_{ij}$ = $\alpha T_{ij} D_i$ the opportunity cost of \$-float arising when customer group $i$'s checks are sent to lockbox $j$,
$\alpha$ = the firm's annual opportunity cost of capital,
$k$ = the maximum number of lockboxes that can be selected,
$T_{ij}$ = average mail time in days from group $i$ to lockbox $j$ + average availability time in days from lockbox $j$ to group $i$'s bank,
$D_i$ = average daily incoming funds from group $i$,
$C_{ij}$ = variable cost of sending group $i$'s checks to bank $j$,
$X_{ij}$ = one if group $i$'s checks are sent to lockbox $j$, zero otherwise,
$F_j$ = the fixed charge per period for maintaining lockbox $j$,
$Z_j$ = one if lockbox $j$ is used, zero otherwise.

In this formulation, the objective is to minimize the opportunity cost of dollar float plus the fixed and variable costs of the lockbox system. Constraint set (2) insures that all checks are assigned to a lockbox, while constraint set (3), along with (6), insures that no check is sent to a "closed" lockbox. Constraint (4) is often used when the fixed charges $F_j$ are not easily obtainable.

Research on the Lockbox Location Problem falls into two categories: (1)

formulation and economic analysis and (2) mathematical optimization techniques. The papers by Kramer [15], Stancill [27], McAdams [19], Kraus, Jansen and McAdams [17], Shanker and Zoltners [26] and Maier and Vander Weide [20] fall in the first category. The papers by Baker, Maier and Vander Weide [1], Bulfin and Unger [3], Ciochetto, Swanson, Lee and Woolsey [5], Corneujols, Fisher and Nemhauser [7], Fielitz and White [11], Levy [18], Mavrides [22], Nauss and Markland [23] and Stone [31] fall in the second.

## III. Data Requirements and Issues

To solve the Lockbox Location Problem, the analyst needs to divide the firm's customers into "homogeneous" groups, obtain data on the fixed and variable costs of maintaining a lockbox system, the mail times from group $i$ to lockbox $j$, the availability times from lockbox $j$ to group $i$'s drawee bank, the mail and availability times for the firm's current system and the average daily incoming funds from group $i$, and find an efficient algorithm for evaluating the set of possible lockbox sites. At each stage the analyst must make important decisions that can have a significant effect on the value the firm receives from the lockbox system. As we consider each of these stages, we shall discuss the issues facing the analyst at that stage.

### 3.1 *Homogeneous Customer Groups*

Since the mathematical formulation does not address the issue of how to form homogeneous customer groups, the analyst must employ his judgment to design groups that are relatively homogeneous relative to mail and availability times, and, at the same time, are not so refined or complex to substantially increase the cost of analysis.

At one extreme the analyst can simply group customers by states or, possibly, 2-digit zip code areas. Simple groupings such as these significantly reduce the cost of collecting mail times and availability times, and by keeping the number of customer groups to a minimum, are both easy to analyze and implement.

The primary disadvantage of grouping customers by states or 2-digit zip codes is that it ignores the sometimes substantial differences between customers within the group. Such differences are especially pronounced when some of the firm's customers use controlled disbursing. Controlled disbursing gives the customer the ability to maximize disbursement float and to obtain timely information on check presentments. To work effectively, however, controlled disbursing often requires the customer to write checks on banks located in other regions of the country. Whereas a lockbox located in Los Angeles would no doubt best serve the customer who both mails and draws his checks from this city, a Chicago lockbox could very well be superior for a customer who draws checks on a North Carolina bank. Differences caused by the use of controlled disbursing cannot be ignored since one survey done by the authors has shown that more than 70% of the Fortune 500 firms use this practice.

At the other extreme, the analyst can treat each individual customer as a separate customer group. Although this has some merit in theory, it is prohibi-

tively expensive because it requires extensive data collection and complex implementation instructions for the firm's accounts receivable department.

A middle ground is to treat some of the firm's largest customers as single-member customer groups and to group all the remaining customers by 2- or 3-digit zip code areas. This alternative recognizes some of the more significant differences between customers in a state or zip code area, and yet is relatively easy to implement.

The analyst's choice of homogeneous customer groups can also have a significant effect in the design of lockbox systems for firms with many divisions. For such firms, it is both economically and administratively expensive to have each division instruct its customers to send remittances to, say, 10 lockbox sites. Accordingly, multi-division firms often instruct all customers of smaller divisions to remit to only one of the sites selected. By assigning all customers of this division to a separate customer group, it is possible to consider this practice within the framework of the overall optimization process.

### 3.2 *Fixed and Variable Costs*

The mathematical formulation requires the analyst to obtain information on the fixed and variable costs associated with each lockbox site. Since there are about 50 relevant lockbox cities with perhaps two or three banks in each city offering lockbox services, the analyst must obtain cost information from more than 100 banks. In gathering these costs, the analyst must recognize that bank charges for "good" customers may be significantly less than charges quoted to the general public; in addition, the charges may change frequently in response to market pressures.

Given the high cost of obtaining data on fixed and variable lockbox charges, the analyst must decide whether to obtain this information or not. Most analysts have decided not, partly based on their experience that float benefits between cities are frequently greater than cost differences between banks. These analysts approach the Lockbox Location Problem as follows:

1. Determine the dollar value of float benefits required to justify an additional lockbox site, for example a float reduction of $50,000 might be needed to expand the system by one additional bank.
2. Starting with one lockbox open, then two, etc. determine the optimal location of lockbox cities using city average float values. Stop this sequential analysis when the addition of one more city fails to meet the test in step 1.
3. Select candidate banks in each city determined by step 2 based on credit relationships, service quality, interface capabilities, size of the lockbox operation, and other sometimes intangible factors.
4. Reoptimize to determine specific banks and the assignment of customers to banks.

Special consideration is also given to existing lockbox bank relationships. At an early stage in the lockbox study, the firm's customers are reassigned within the existing system of lockbox banks. This so called "current system optimal" is much less costly to implement than the addition of new lockbox banks. Accordingly, improvements determined by the above four-step process are measured

against this current system optimal base instead of the float levels found in the system as currently operated.

### 3.3 *Mail and Availability Times*

Mail and availability times play a key role in the analysis of lockbox systems. With regard to these times the analyst faces several important issues.

1. In comparing the firm's current collection system to the "optimal" lockbox system, the analyst needs to decide whether to use Phoenix-Hecht Surveyed Mail Times or a sample of actual mail times to measure the mail float in the current system. (There is universal agreement that the mail times surveyed by Phoenix-Hecht, Inc., a Chicago-based consulting firm, are the best and most comprehensive times to use in finding the optimal system.) If the analyst chooses to use actual mail times, he must recognize that there are a great many measurement errors caused by the unscientific nature of the sampling process, e.g. the date may have been incorrectly stamped on the envelope, the date may have been stamped on the envelope at a late stage in the mail process, etc. If he chooses to use Phoenix-Hecht mail times, the analyst must recognize that these were collected under certain ideal circumstances that may not characterize the firm's actual experience. Phoenix-Hecht mail times, for instance, are measured from a central location within each customer region, whereas his customers may not remit from that central location.
2. With regard to availability times, the analyst must first decide whether to use availability times at all. Early implementations of the lockbox model ignored availability times on the grounds that availability differences from city to city were small, since the Federal Reserve guarantees a maximum of two day availability no matter where a check is deposited. If the analyst decides he wants to include availability times in the analysis, he must choose whether to measure them by quoted Federal Reserve availabilities, city average bank availabilities or individual bank availabilities. Since 1979 most analysts have relied on individual bank availabilities, a transition which caused significant changes in the solutions found (see the paper by Ferguson and Maier [8]). Individual bank availabilities tend to reduce the optimal number of lockboxes and to change their locations.
3. If the analyst decides to use both mail and availability times, he must also decide how to combine these times into a total collection time. A problem arises because of differences in the units by which mail and availability times are measured. For an insightful discussion of this issue the reader is referred to the paper by Nauss & Markland [24].

### 3.4 *Amount of Incoming Funds from Region i*

To estimate the amount of incoming funds from customer region $i$ the analyst typically collects a sample of the firm's incoming checks covering a one month period. For the purposes of analysis, the analyst wants to make sure that the sample is representative of the firm's long run collection experience. Since most

firms experience considerable seasonality and frequent changes in customer collection patterns, this is no simple matter.

One approach to this problem, developed by the First National Bank of Chicago and called Dynamic Lockbox Analysis, forecasts shifts in corporate remittance patterns as well as trends in mail and availability times during the initial design stage of the lockbox system.

### 3.5 *Summary*

The decisions the analyst makes regarding the design of the data collection process can have a significant impact on the quality of the solutions obtained. Early lockbox analyses relied on crude customer groupings, small unrepresentative samples of the firm's checks and unquestioning use of mail times to represent differences in city mail plus availability times. Recent improvements in data collection and a better understanding of the consequences of alternative data choices have greatly improved the quality of lockbox system design. We estimate these improvements to be approximately 10–15% of the total float savings in value.

## IV. Optimization and Heuristic Algorithms

Once the analyst has formed appropriate customer groups and gathered the requisite data, he must employ an algorithm to evaluate the float savings associated with each combination of lockbox sites.

In their award winning paper [7], Cornuejols, Fisher and Nemhauser analyze various exact and approximate algorithms to the Lockbox Location Problem. Their results may be summarized as follows:

1. It is difficult to find an algorithm that always produces an optimal solution to the Lockbox Location Problem in a reasonable amount of computer time. In fact, since the Lockbox Location Problem is NP-complete, there is no known algorithm to the problem with an upper bound on running time that is polynomial in the problem parameters.
2. A theoretical analysis of the worst possible case indicates that heuristic algorithms are capable of producing poor solutions.
3. However, experience with some reasonably realistic data sets indicates that heuristic algorithms produce very close-to-optimal solutions in practice. For instance, a simple 1-for-1 interchange heuristic achieves the optimal solution in 2/3 of the sample problems and is never more than 3% away from the optimal solution in the remaining sample problems.

The Corneujols, Fisher and Nemhauser conclusions on the value of heuristic procedures versus optimization algorithms are further supported by test results obtained by Baker, Maier and Vander Weide [1]. These authors test a sequential heuristic procedure that obtains an initial solution from a dynamic programming algorithm and improves this initial solution with 1-for-1 and 2-for-2 interchanges of lockboxes not in the solution with those currently in the solution. In no case did this heuristic procedure fail to obtain the optimal solution.

As noted above, the authors have implemented solution procedures for the

lockbox and disbursement location problems at over three dozen banks nationwide. We have generally recommended the use of the 1-for-1 and 2-for-2 interchange heuristics because they provide valuable information on both optimal and close-to-optimal solutions. However, we have also employed a Lagrangian Relaxation/Branch-and-Bound [22] procedure as a check on the heuristics. In the hundreds of studies we have done over the last seven years, we have never found a single instance in which the more elaborate optimization techniques found a superior solution to the heuristics.

The above results and experiences indicate that the literature's concern with algorithms has been misplaced. Given the errors in data noted earlier in this paper, minor differences in algorithms are insignificant, especially when experience has shown that fairly simple heuristics work just as well as the most sophisticated optimization techniques.

## V. The Disbursement Bank Location Problem

The mathematical description of the Disbursement Location Problem is similar to that of the Lockbox Location Problem (See Maier and Vander Weide [21]). The major difference is that optimization occurs over both mail points and drawee bank locations. Despite this difference, the solution algorithms for the Disbursement Location Problem are not significantly more complex than those for the Lockbox Location Problem.

The published research on the Disbursement Location Problem is concerned primarily with model formulation. The original formulation appears in a paper by Shanker and Zoltners [25]. The Maier and Vander Weide [21], Gitman, Forrester and Forrester [12] and Ferguson and Maier [9] paper contain modifications of this original formulation.

## VI. Data Requirements and Issues

To solve the Disbursement Location Problem, the analyst needs to divide the firm's vendors into homogenous groups; obtain data on the fixed and variable costs of maintaining a disbursement system, the mail times from regional offices to vendor groups, the presentation times from vendor groups to drawee banks, the mail and presentation times for the firm's current system and the average daily payments to vendor groups $j$; and find an efficient algorithm for evaluating the set of possible mail and drawee bank sites. Again the analyst must make important decisions at each stage of the analysis. In addition there are several problems with implementing solutions to the Disbursement Location Problem that are unlike any faced in implementing solutions to the Lockbox Location Problem.

### 6.1 *Maximizing Mail Times*

The original formulation [25] includes the possibility of maximizing mail times. This makes sense only if the recipient of a check uses the mail date to determine when an item is paid. This is by no means universal practice. A recent survey

(see Hill, et al. [13]) indicates that recipient firms use mail date less than 28% of the time. In addition, most firms are unwilling to incur the risk of vendor ill-will caused by mail time extension.

### 6.2 *Quality of Data*

Many of the same data issues that arise in the Lockbox Location Problem arise in the Disbursement Location Problem as well. First, the fixed and variable costs of setting up disbursement systems are hard to measure and the value of control and float extension far overshadows small cost differences that exist between banks. Moreover, as long as the payment system remains primarily paper based, the firm must write its checks on at least one bank. The elective feature of a lockbox is not a consideration when setting up a disbursement system.

Second, clearing time surveys are not nearly as accurate as the mail and availability time surveys used in the Lockbox Location Problem. This basic fuzziness of the database has discouraged analysis and made much of the existing analysis suspect. The major banks that do disbursing analysis report two to four times as many lockbox studies performed as disbursement studies. This fact is surprising when one considers that the disbursement problem is even more universally applicable than is the lockbox problem.

Finally, it should be noted that the same problems in defining homogeneous vendor groups exist here as they did the lockbox problem. Consequently, studies are performed in which vendors are grouped by states, zip codes and individually isolated. In addition, there is greater sophistication in handling individual vendors than in handling individual customers; some of the more advanced disbursement control systems assign individual vendors on a check-by-check basis.

### 6.3 *Federal Reserve Policies*

The solutions to the original formulation of the Disbursement Location Problem frequently produce an increase in "clearing system slippage." Clearing system slippage is defined as the difference between the clearing time experienced by the check writer and the availability time granted to the check depositor. The greater is this number, the greater the difficulty the clearing agent, principally the Federal Reserve, is having in moving checks through the system. It is argued that the Federal Reserve will concentrate on the points that have large slippage in its efforts to discourage remote disbursing and to reduce its own Federal Reserve float. Its activity in reducing Federal Reserve float has been very well publicized. In 1979 Fed float stood at 6.7 billion dollars on an average daily basis while in 1982 it has averaged less than 2 billion dollars per day.

An attempt to include in the analysis the risk that the Federal Reserve will further reduce clearing system slippage is described in Ferguson and Maier [10]. This paper suggests that the concept of the Efficient Frontier from Portfolio Theory can be applied to measure the risk/return trade-off faced by the corporate cash manager as he designs his disbursement system. Nine of the nation's leading cash management banks have adopted this technology as their consulting approach.

6.4 *Vendor Ill Will*

Attempts to implement the solutions to the Disbursement Location Problem may lead to deteriorating relationships with the firm's vendors or creditors. More than one firm has found that its vendors refuse to grant a discount for prompt payment when it uses a remote disbursement location. In one celebrated case, Merrill Lynch was sued by customers who were unable to get prompt refunds from security transactions. (Merrill Lynch had a practice of paying East Coast customers with checks drawn on West Coast banks and West Coast customers with checks drawn on East Coast banks. *Wall Street Journal* December 18, 1978, p. 14).

6.5 *Credit Problems*

Remote cities such as Helena, Montana; Reno, Nevada; and Midland, Texas frequently appear in the solution to the Disbursement Location Problem. Unfortunately, these cities are not always well suited as banking locations. For example, a firm will normally want its disbursing bank to be part of its credit group. This is because if for any reason there is a failure to fund the disbursing account, the disbursement bank can use its already preauthorized credit lines to extent credit to the firm and not have to return the items marked "insufficient funds", a situation most companies would consider unacceptable. However, banks are restricted by the Federal Reserve to lend no more than 10% of their equity capital to any one customer. Since banks in cities such as Helena and Reno have a small equity base, they cannot extend sufficient credit to fund the disbursements of large firms.

## VII. Conclusions

If there is a lesson to be learned from our experiences with lockbox and disbursement models, it is that the existing academic literature has been very misleading as to the true state of the art in this area. Perhaps the two most significant issues we have addressed in this paper are the importance of data and the over simplicity of the mathematical formulations.

The most important change in the lockbox design area over the past five years is the improvement in the precision of supporting databases. In particular, the evolution from mail time only to mail and bank availability analysis has had very major ramifications in the industry. The sensitivity of the problem to how customers are grouped and even the quality of the receivable sample cannot be overemphasized.

The simple model of the lockbox problem as originally proposed by Levy falls far short of the practical requirements of designing lockbox systems. What does one do in a multidivision environment? What happens if your customer base significantly uses remote disbursing? How do you incorporate seasonality? What happens if your collections are dominated by a few large items received on only a sporadic basis? These are some of the questions that must be and are addressed in practice.

The academic literature on the disbursement location problem has failed to

fully recognize the complexity of this problem. True, float is important. But so is control. Moreover, the environment today is so unpredictable that neither float nor control can be obtained without significant system risk in the form of instability.

In summary, the lockbox and disbursement problems remain two of the most widely applied management science models in the finance area. Unfortunately, the existing literature on these problems has been misleading on the issues that are really important to the practitioners who actually implement solutions.

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